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Dynamic Fluid Substitution in Porous Rock by Inversion for Mechanical Properties

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Summary

The conventional fluid substitution consists in predicting the bulk elastic modulus of an isotropic saturated rock (K_U) by using Gassmann's equation with fixed elastic moduli of solid grains (K_s), and the drained frame (K_D) but variable modulus of the pore fluid (K_f). By assuming "modified" (complex-valued and frequency-dependent) moduli K_D^* and sometimes K_f^*, such substitution is often also used to model the attenuation and wave-velocity dispersion in fluid-saturated porous rock. However, the use of frequency-dependent quantities in Gassmann's and related equations may be inaccurate, because this method only represents heuristic extension of certain known formulas outside of their scopes of validity. To obtain a rigorous and reliable fluid-substitution method, we note that Gassmann's equation follows from a much broader principle, which consists in matrix character of the elastic, anelastic, and inertial properties of most materials. This principle was recently utilized in the so-called General Linear Solid (GLS) model based on Lagrangian continuum mechanics.

By using the GLS approach, fluid substitution can be performed by constructing time- and frequency-independent models of porous rock with variable K_f. Such models can be used to explain any mechanical experiment with this rock, and in particular, for accurate modeling of attenuation and dispersion effects in boundless media or bodies of arbitrary shapes. As an example, a detailed fluid substitution model for Berea sandstone is inverted from published low-frequency attenuation/dispersion data. The new model can be extended to media with double or multiple porosities and used in many applications.

Introduction

When the properties of the fluid within the pores of a reservoir rock are changed, the saturated ("undrained") bulk modulus K_U of the rock changes accordingly. This change in K_U leads to changing velocities and attenuation of seismic waves, which can further be used for identifying the reservoir, inverting for its properties, or monitoring its time-lapse variations during, for example, water or CO_2 injection. The method that is commonly used for relating the changes in K_U to the type and amount of fluid within its pores is the Gassmann's equation (1951). This equation is based on assuming that the porous medium is elastic, monomineralic, and homogeneous. The fundamental meaning of Gassmann's equation consists in the existence of only three independent elastic properties of the material (the solid-grain (K_s), drained (K_D), and fluid moduli (K_f)), so that the undrained modulus K_U can always be derived from them (Berryman, 1999). Morozov and Deng (2016, 2018) emphasized that these three independent variables comprise a symmetric 2×2 matrix $\mathbf{K}^{\mathrm{B}}$ of elastic moduli. Therefore, Gassmann's equation represents only one of the consequences of this matrix character of the elastic bulk modulus for a two-phase medium.

Although Gassmann's equation was only derived for elasto-statics (Gassmann, 1951), it is often extended far beyond its domain, to predicting the frequency-dependent, complex-valued undrained modulus (denoted K_U^* here), which is used to describe the seismic-wave attenuation and velocity dispersion in fluid-saturated rock. For example, Gurevich et al. (2010) utilized a modified frequency-dependent (drained) rock-frame modulus K_D^* to predict the fully-saturated bulk modulus dispersion due to squirt-flow effects, and Mavko (2013) assumed a viscoelastic modulus K_f^* in the same way. However, such use of Gassmann's equation is only heuristic extrapolation of formulas to complex-valued material properties, not supported by physics, and the results of this extrapolation should not be accurate.

It is therefore desirable to obtain a rigorous, physically-consistent model of fluid substitution at nonzero frequencies. In this paper, we propose such an approach based on the General Linear Solid (GLS) framework by Morozov and Deng (2016). The model is quite general and should be applicable to most types of porous rock (Deng and Morozov, Geophysics, forthcoming).

Theory

In the GLS formulation (Morozov and Deng, 2016), the complete Biot's (1956) poroelastic theory is described by giving the Lagrangian density function (denoted L^{B} below) and the dissipation pseudo-potential (D^{B}) as algebraic quadratic forms with respect to a two-component vector-variable field $\mathbf{u}(\mathbf{x},t)$:

$$\begin{cases} L^{\mathrm{B}}\left\{\mathbf{u}\left(\mathbf{x},t\right)\right\} = \dfrac{1}{2}\dot{\mathbf{u}}_i^T\boldsymbol{\rho}^{\mathrm{B}}\dot{\mathbf{u}}_i - \left(\dfrac{1}{2}\boldsymbol{\Delta}^T\mathbf{K}^{\mathrm{B}}\boldsymbol{\Delta} + \tilde{\boldsymbol{\varepsilon}}_{ij}^T\boldsymbol{\mu}^{\mathrm{B}}\tilde{\boldsymbol{\varepsilon}}_{ij}\right), \\ D^{\mathrm{B}}\left\{\mathbf{u}\left(\mathbf{x},t\right)\right\} = \dfrac{1}{2}\dot{\mathbf{u}}_i^T\mathbf{d}\dot{\mathbf{u}}_i, \end{cases} \quad (1)$$

where the lowercase indices $i, j = 1,2,3$ (or x,y,z) denote the spatial coordinates, summations over repeated spatial indices are implied, matrix (boldface) notation is used with respect to the two-dimensional model space, and the superscripts ^{T} denote matrix transposes. The model vector $\mathbf{u}$ comprises two 3D spatial vectors denoted u_{Ji}, where the uppercase subscript $J = 1$ or 2 stands for the model space. The spatial vector u_{1i} is the observable displacement of the fluid-saturated rock, and u_{2i} denotes the relative filtration-fluid displacement multiplied by the initial-state (invariant) porosity ϕ: $u_{2i} \equiv -\phi\left(u_i^{\mathrm{fluid}} - u_{1i}\right)$. The model-space vector $\Delta \equiv \varepsilon_{kk}$ is the volumetric strain, and the zero-trace spatial tensor (vector in model space) $\tilde{\boldsymbol{\varepsilon}}_{ij} \equiv \boldsymbol{\varepsilon}_{ij} - \boldsymbol{\Delta}\delta_{ij}/3$ is the deviatoric strain. The 2×2 material-property matrices $\mathbf{K}^{\mathrm{B}}$ (bulk moduli), $\boldsymbol{\mu}^{\mathrm{B}}$ (elastic moduli), $\boldsymbol{\rho}^{\mathrm{B}}$ (density) and $\mathbf{d}$ (Darcy friction) were given by Bourbié et al. (1987) and Morozov and Deng (2016). The conventional empirical bulk moduli K_s, K_D, K_U, and K_f, the Biot-Willis parameter α, poroelastic modulus M, Skempton coefficients, and Gassmann's equation are all contained in matrix $\mathbf{K}^{\mathrm{B}}$ (Bourbié et al., 1987).

In addition to Biot's model (1), we also consider an additional scalar-variable vector $\boldsymbol{\theta}$ describing some internal deformations of the rock frame, such as dilatations of micropores or relative movements of mineral grains. Considering only "local" bulk deformations such as caused by squirt flows (Gurevich et al. 2010), the most general extension of Biot's model (1) becomes (Deng and Morozov, Geophysics, forthcoming):

$$\begin{cases} L = L^{\mathrm{B}} - \dfrac{1}{2}\boldsymbol{\theta}^T \mathbf{P}\boldsymbol{\theta} + \boldsymbol{\Delta}^T \mathbf{Q}\boldsymbol{\theta}, \\ D = D^{\mathrm{B}} + \dfrac{1}{2}\dot{\boldsymbol{\Delta}}^T \boldsymbol{\eta}_K^{\mathrm{B}} \dot{\boldsymbol{\Delta}} + \dfrac{1}{2}\dot{\boldsymbol{\theta}}^T \mathbf{P}' \dot{\boldsymbol{\theta}} - \dot{\boldsymbol{\Delta}}^T \mathbf{Q}' \dot{\boldsymbol{\theta}}, \end{cases} \tag{2}$$

where the 2×2 matrix $\boldsymbol{\eta}_K^{\mathrm{B}}$ describes the solid viscosity for Biot's rock (Deng and Morozov, 2016), the $N \times N$ matrix $\mathbf{P}$ describes the elastic (free) energy of the internal deformations $\boldsymbol{\theta}$, the $2 \times N$ matrix $\mathbf{Q}$ describes its elastic coupling to Biot's volumetric strains (Δ), and matrices $\mathbf{P}'$ and $\mathbf{Q}'$ have similar meanings for viscosity and viscous coupling. In the absence of additional constraints, the elements of vector $\boldsymbol{\theta}$ are subject to several linear transformations and scaling. By using this arbitrary scaling, variables $\boldsymbol{\theta}$ are selected so that the matrix $\mathbf{P}$ is diagonal with elements $P_{JJ} = K_D$ for any J.

The quadratic functional forms (2) allow straightforward derivations of all equations of motion for the anelastic rock, which further allow computation of the wavemodes and velocity-dispersion and attenuation spectra by means of (relatively) simple matrix calculations (Morozov and Deng, 2016, 2018). By fitting the predicted spectra to the results of low-frequency laboratory observations, elements of matrices $\mathbf{K}^{\mathrm{B}}$, $\boldsymbol{\eta}_K^{\mathrm{B}}$, $\mathbf{Q}$, $\mathbf{P}'$ and $\mathbf{Q}'$ can be obtained by nonlinear inversion, as outlined in the next section. The mechanical model can also be used for implementing finite-difference wavefield simulations and in many other applications.

Results

In this section, we invert for material properties K_D (included in $\mathbf{K}^{\mathrm{B}}$), $\mathbf{Q}$, and $\mathbf{P}'$ from low-frequency, Young's modulus attenuation/dispersion experiments with Berea sandstone reported by Tisato and Quintal (2013). Similar to these authors, the data were first transformed into bulk-modulus data assuming a constant shear modulus μ. The resulting frequency-dependent data ("effective", or apparent; see Morozov and Baharvand Ahmadi, 2015) for bulk-modulus dispersion ($\left|K_U^*(f)\right|$) and attenuation ($Q^{-1}(f)$) are shown by blue lines in Figure 1. The data were acquired at three water saturation levels of s_w = 62.4%, 86.6%, and 97.1% (Tisato and Quintal, 2013; Figure 1). Consequently, the variations of saturation can be viewed as fluid substitutions, with K_f given by Reuss averages of water and gas moduli: $K_f = \left[s_w K_w^{-1} + (1 - s_w) K_g^{-1}\right]^{-1}$. Note that the variations of the effective K_f for the tree levels of s_w are substantial (Table 1), and so the three effective "fluids" are contrasting to each other.

By inverting for the mechanical model (2) with four variables θ_J ($J = 1, 2, 3, 4$), we obtained models (Table 1) closely fitting the observations, particularly for larger saturations (red lines in Figure 1). All elastic properties of this model are fluid- (saturation-) independent (Table 1), which means that this model could be useful for quantitative modeling of geomechanical properties of this reservoir with any fluid. At the same time, viscous properties (elements of matrix $\mathbf{P}'$; Table 1) show substantial variations between the three saturation levels or fluids. This observation suggests tha the viscosity of the drained rock is still caused by trapped fluid within it.

Conclusions

A macroscopic model of fluid-saturated rock is developed based on the Lagrangian “General Linear Solid” (GLS) wave-mechanics framework proposed earlier. The model is illustrated by inverting low-frequency laboratory experiments with Berea sandstone. The resulting mechanical model is suitable for implementing wavefield simulations and for performing rigorous fluid substitution without assumptions about the validity of using viscoelastic moduli in Gassmann’s equations.

Table 1. Mechanical properties of Berea sandstone
derived from experimental data by Tisato and Quintal (2013)

Saturation-independent properties				s_w = 97.1%	s_w = 86.6%	s_w = 62.4%
K_s (GPa)	P_{JJ} (K_{d0}) [GPa]	Q_{1J} [GPa]	J (variable #)	P'_{JJ} [GPa·s]	P'_{JJ} [GPa·s]	P'_{JJ} [GPa·s]
36	2.60	0.84	1	15.990	0.217	1.478e3
		1.03	2	0.191	0.019	0.279
		1.05	3	0.028	3.97e-4	0.005
		4.17	4	0.001	3.26e-4	1.758e-5
Effective fluid modulus (GPa)				3.0e-3	7.46e-4	2.659e-4

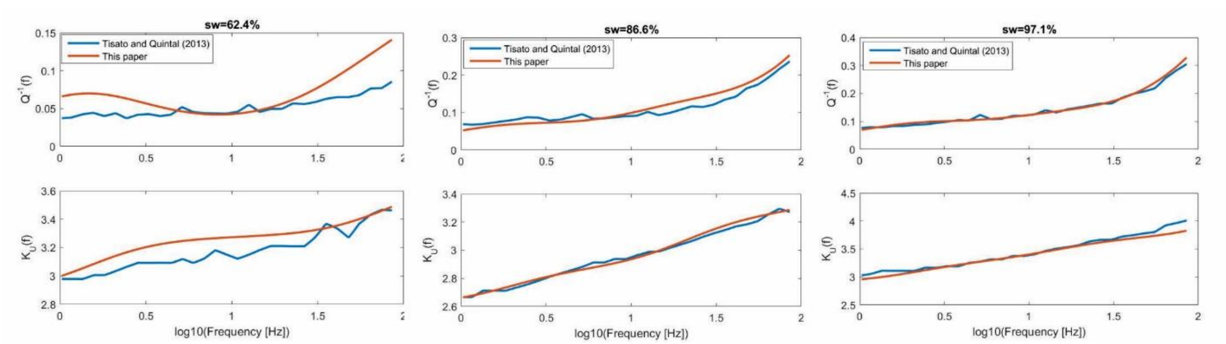

Figure 1. Dynamic fluid saturation results at 62.4%, 86.6% and 97.1% water saturation respectively. Blue lines are the experimental data by Tisato and Quintal (2013), and red lines are the predictions of the present model.

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