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Barycentric Interpolation of Towed-streamer Position Data

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Summary

An efficient method for interpolating receiver positions of missing or decimated towed-streamer data is proposed. The method employs a family of high-accuracy stable barycentric rational interpolants suitable for irregularly spaced nodes. The interpolants are particularly well-suited for equidistant or quasi-equidistant nodes such as those encountered in vessel navigation plans. Robust near-optimal parameter selection and decimation rates are recommended based on numerical simulations. Performance metrics including file size, computation time, and resulting positional errors are compared for input datasets with varying decimation rates.

Introduction

Modern towed-streamer acquisitions include continuous line acquisition (CLA), simultaneous shooting, and multisource multistreamer configurations such as wide-azimuth towed-streamer, and coil shooting (IOGP, 2011; Vermeer, 2012). These configurations can generate substantial amounts of P190 (UKOOA, 1990) and/or P111 (IOGP, 2015) position data, where a few hundred thousand shot events are not uncommon.

A recent development is the decimation of receiver position data that finds application in vessel and streamer steering of 4D repeat surveys, fast transmittal of daily acquisitions, and data reduction in survey design software. The decimated datasets are sometimes called vessel navigation plans (VNPs) owning to their use in 4D repeats surveys. One decimation approach is to keep every Nth receiver along each streamer including the first and last receiver for $N \geq 1$. Datasets reconstructed from VNPs will exhibit positional errors, and so it is useful to know the impact of these errors. Small shifts in lateral recording position have been shown to limit survey repeatability, however they are of minor concern for exploration or reservoir characterization (Eiken et al., 2003). For a synthetic modeling study with matching shot positions, reasonable repeatability is attainable if receiver positions repeat within 10 m (Schonewille, 2003).

An efficient method is proposed for interpolating receiver position data of towed-streamer acquisitions. The paper is structured as follows. First, a method is proposed for reconstructing the position of missing receivers by interpolating their easting, northing and depth coordinates over receiver group numbers. Approximation error and parameter selection are discussed. Secondly numerical examples with different streamer shapes are shown. Thirdly performance metrics including file size, computation time and resulting positional errors are compared, and finally, conclusions are made.

Barycentric interpolation of receiver position data

The recently introduced family of barycentric rational interpolants are suitable for irregularly spaced nodes and possess several useful properties for interpolating towed-streamer data. The interpolants are linear in the data, guaranteed to have no poles in $\mathbb{R}$, well suited for equidistant and quasi-equidistant nodes, have arbitrarily high approximation orders, and are infinitely smooth (Floater and Hormann, 2007; Cirillo et al., 2017). The interpolants require $\Theta(n)$ time for construction, and $\Theta(n)$ time for each evaluation (Boost, 2018).

Given a function $f: [a, b] \rightarrow \mathbb{R}$, a set $X_n = \{x_0, x_1, \ldots, x_n\}$ of $n + 1$ real-valued interpolation nodes

$$a = x_0 < x_1 < \cdots < x_n = b,$$

and a parameter d with $0 \leq d \leq n$, the Floater–Hormann interpolant is given by (Floater and Hormann, 2007)

$$r\,(x) = \sum_{i=0}^{n-d} \lambda_i(x) p_i(x) \Big/ \sum_{i=0}^{n-d} \lambda_i(x), \qquad (1)$$

where

$$\lambda_i(x) = \frac{(-1)^i}{(x - x_i) \dots (x - x_{i+d})}$$

and p_i denotes a polynomial of degree $\leq d$ that interpolates f locally in the form

$$p_i(x_j) = f(x_j), \qquad j = i, \dots, i + d.$$

The interpolation problem is formulated on a per-streamer basis to permit flexible configurations. Hydrophone position data along a streamer can be represented by the four-tuple (g, x, y, z) where g is the group number, x and y are the map grid easting and northing coordinates, respectively, and z is the depth coordinate. For a streamer with $n + 1$ hydrophones, interpolation nodes are simply taken to be the group numbers, i.e., $X_n = \{g_0, g_1, \dots, g_n\}$. Sequential group numbering along a streamer of the form $X_n = \{1, 2, \dots, n+1\}$ is very common for both P190 and P111 file formats. We can also take $X_n = \{1, 2, \dots, n+1\}$ for reverse sequential group numbering of the form $\{ n+1, n, \dots, 1\}$ which is used for some streamer types. A consequence of these node choices is that hydrophone spacing along the streamer is assumed to be proportional to group numbering, such as in the ubiquitous case of equally spaced hydrophones with sequential group numbering. Variably spaced hydrophones can be represented using decimal-valued group numbering. Three separate univariate interpolants are constructed, one for each of the dependent tuple dimensions x, y, and z over streamer nodes X_n. Each interpolant is evaluated at the desired (or missing) group numbers.

Approximation error

The approximation error of the Floater–Hormann interpolants in equation (1)

$$e(x) = f(x) - r(x)$$

satisfies

$$|e(x)| \leq C h^{d+1}, \qquad x \in [a, b], \qquad (2)$$

for sufficiently smooth f, where the global mesh size h is the largest distance between neighboring nodes

$$h = \max_{0 \leq i \leq n-1} h_i\,, \qquad h_i = h_{i+1,i}, \qquad h_{i,j} = |x_i - x_j|.$$

For $d \geq 1$, the constant C is of the form

$$C = \begin{cases} (b-a)\dfrac{|f^{(d+2)}|}{d+2} & \text{for odd } n-d \\ (b-a)\left(\dfrac{|f^{(d+2)}|}{d+2} + \dfrac{|f^{(d+1)}|}{d+1}\right) & \text{for even } n-d. \end{cases} \qquad (3)$$

The interpolant r converges to f at the rate $O(h^{d+1})$ as h approaches 0. For the remaining case $d = 0$, an expression for C can be deduced from (Floater and Hormann, 2007) which is similar in nature to that of equation (3). For a given range of support $b - a$, we observe that the approximation error is proportional to h, and to the maximum of some higher-order derivatives of f where the order of the derivative is a function of d. For the example of $d = 1$, C is proportional to $f^{(3)}$ and possibly $f^{(2)}$. For straight-line streamers, $f^{(k)} = 0$ for $k \geq 2$ and thus $|e(x)| = 0$. Evidently, the approximation error of our interpolants depends on h, d, and most importantly, the shape of the streamers.

Selection of parameter d

Parameter d is the order of the interpolant. The interpolant is a blend of high-order polynomials and therefore can oscillate between nodes for large d, especially near the ends of the range (Baker and

Jackson, 2014). The $d = 0$ interpolant is essentially linear between nodes but smooth. The $d = 1$ interpolant is a quadratic polynomial yielding fits close to that of splines. Numerical examples show that a selection of d in the range of 3 to 5 is a reasonable choice. The $d = 3$ interpolant is a quartic polynomial which is found to perform well for all decimation rates and is a good default choice.

Automatic adjustment of a user-supplied parameter d_u downward according to $d = \min(n, d_u)$ can be a prudent action to aid the user in handling small n. This effectively clips d_u to the maximum possible d for a given n when needed. For example, a VNP with $n + 1 = 2$ nodes has a maximum possible d of 1. It is unlikely, but possible, for n to vary within a dataset, so this adjustment should be performed on each set of nodes (or streamer) to assure smooth operations for the user.

Numerical examples

Feathering example

Figure 1a shows a shot event with six streamers, 408 hydrophones per streamer, 12.5-m hydrophone spacing, and 75-m streamer spacing. Streamers exhibit feathering of approximately 340 m (or 3.8°). Figure 2 depicts curves of the mean and maximum position error of the reconstructed data for values of parameter d. Separate curves correspond to input datasets with different decimation rates.

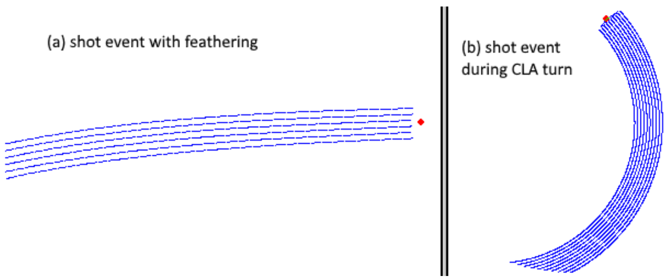

Figure 1. Two marine shot events. (a) Event with approximately 340m (or 3.8°) of feathering. (b) Event during a CLA turn of radius 5km.

First, we observe the existence of an optimal choice of d = 3 to minimize position errors. One interpretation is that the resulting interpolant order is flexible enough to represent the data, but small enough to avoid unwanted oscillations. Second, we observe that position errors are proportional to the decimation rate, i.e., error increases as decimation rate increases. A clustering of curves occurs over the decimation rate range from 2 to 40. For this range with d = 3, mean and maximum errors fall in the range of [0.0864, 0.5603] and [0.7071, 1.9849] m, respectively. On average for d = 3, the maximum error is 4.5 times that of the mean error and ranges from 3.2 to 8.1.

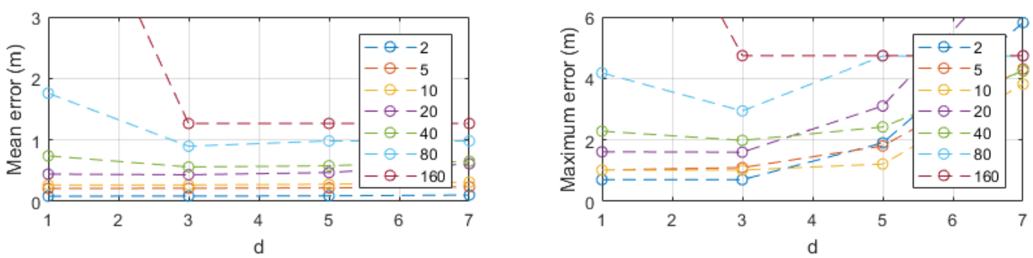

Figure 2. Mean and maximum position error of reconstructed data for the feathering example. Curves are shown for VNP-N decimated datasets for N = {2, 5, 10, 20, 40, 80, 160}.

CLA turn example

CLA is an acquisition technique in which line changes are incorporated in the survey area. In this way non-productive time associated with turns is virtually eliminated resulting in increased efficiency. Figure 1b shows a shot event with ten streamers, 800 hydrophones per streamer, 12.5-m hydrophone spacing, and 100-m streamer spacing. The streamers exhibit a turning radius of 5 km for this CLA turn. Figure 3 depicts curves of the mean and maximum reconstructed data position error.

Like the previous example, we observe that errors are minimized for d = 3 and that they are proportional to the decimation rate. Let us again observe curves over the decimation rate range from 2 to 40. For this range with d = 3, mean and maximum errors fall in the range of [0.0260, 0.0515] and [0.1414, 0.2236] m, respectively. On average for d = 3, the maximum error is 4.5 times the mean error and ranges from 3.4 to 5.5. Finally, we observe that errors for this CLA turn are smaller than those of the feathering example. The

CLA turn is very smooth exhibiting a consistent curvature (turn of 5 km) throughout each streamer. On the other hand, the streamer curvature of the feathering example increases towards the end of each streamer. Errors are reduced when streamers exhibit consistent curvature with minimal variations, e.g., during straight or coil sailings with minimal currents and localized streamer shapes.

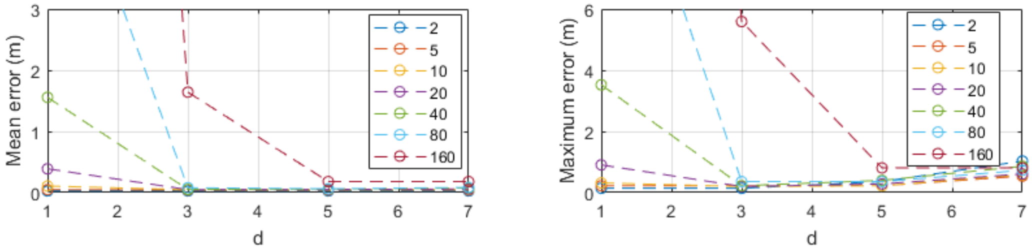

Figure 3. Mean and maximum position error of reconstructed data for the CLA turn example. Curves are shown for VNP-N decimated datasets for N = {2, 5, 10, 20, 40, 80, 160}.

Performance results

Table 1 shows performance of the proposed interpolation method with d = 3 for input datasets with different decimation rates. The decimated file size, reconstruction time, and positional errors are shown. All experiments were run with a seismic survey design software that implements decimation methods, the proposed interpolation method, and P190 import and export. To provide fair comparisons, the reconstruction time includes both the time it takes to read the P190 file and to interpolate.

Table 1. Performance of the interpolation method for feathering example using input datasets with different decimation rates.

Number of Nodes	Decimated P190 File Size		Reconstruction Time		Position Error	
	(Kilobytes)	(% Reduction)	(Milliseconds)	(% Reduction)	Max (m)	Mean (m)
408 (full)	66	0	28.65	0	0	0
205 (VNP-2)	34	48.5	31.51	-10.0	0.7071	0.0864
83 (VNP-5)	14	78.8	28.12	1.8	1.1000	0.2135
42 (VNP-10)	7	89.4	26.62	7.1	1.0198	0.2673
22 (VNP-20)	4	93.9	25.37	11.4	1.6000	0.4334
12 (VNP-40)	3	95.4	25.27	11.8	1.9849	0.5603
7 (VNP-80)	2	97.0	25.17	12.1	2.9428	0.8973
4 (VNP-160)	2	97.0	25.10	12.4	4.7381	1.2667

We observe that decimated file size and reconstruction time exhibit logarithmic behaviour topping out with reductions around 97 and 12%, respectively. Clearly file size improves the most of the two metrics. We also observe that reading a decimated P190 file with 205 nodes and performing the interpolation is 10% slower than simply reading the undecimated P190 file. This indicates that a certain rate of decimation must be performed to achieve speed improvements.

Evidently, a decimation rate within the range from 10 to 40 provides the best compromise. For VNP-20 decimation, file size and reconstruction time are reduced by 93.9 and 11.4%, respectively, at the expense of mean and maximum errors of 0.4334 and 1.600m, respectively. Even for the extreme VNP-160 decimation, we only sacrifice mean and maximum errors of 0.4334 and 4.7381m, respectively.

Conclusions

An efficient method for interpolating receiver positions of missing or decimated towed-streamer data was proposed. The method employs barycentric interpolants that can accommodate regularly and irregularly spaced nodes such as those encountered in VNPs. A value of d = 3 was found to minimize position errors of the reconstructed data for two examples. For VNP-40 decimation followed by d = 3 interpolation, the average file size and reconstruction time for the two examples are reduced by 95.4 and 11.8%, respectively, at the expense of mean and maximum errors of 0.3059 and 1.1042m, respectively. Smooth streamer shapes aid in achieving these results. It is recommended to reduce the decimation rate when dealing with localized shapes in the streamers, e.g., catenary streamer shapes caused by steering devices.

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