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Comparison between time domain and frequency domain least-squares reverse time migration

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Summary

We propose a full waveform inversion (FWI) based LSRTM method in the frequency domain. We use the FWI formulation with the truncated Newton's method, to solve the linear equation which relates Hessian, model perturbation and the gradient by linear conjugate gradient method. We use simple layer models to compare the two formulations, LSRTM in time and frequency domain. Because of convergence problems that we have not solved yet, we get lower resolution images with the frequency domain FWI-LSRTM method. On the other hand, when the model is inaccurate, the reflector depth seems less affected in the frequency domain. The FWI-based LSRTM method seems to be more robust to velocity errors even if we don't correct the background model as usually done in FWI. Low frequencies seem to be less affected by the inaccurate velocities, and by model smoothness than the high frequencies, suggesting using methods from low frequencies to constraint the high frequencies can help to develop a more robust LSRTM.

Introduction

The recorded seismic data can be treated as the result of forward modeling problem and this is associated with solving the wave equation. For seismic migration, the migration operator is adjoint to the forward modeling operator. RTM, as a two-way migration method, migrates the data residual using a zero-lag cross-correlation on the forward and backward propagated wavefields. For conventional LSRTM methods in acoustic medium, the velocity model can be split into two parts: a long-wavelength component, which corresponds to the low frequency feature, and a short wavelength component, which corresponds to the high frequency feature in the model (Geng and Innanen, 2016). Based on this, the wavefield also consists of two parts: the incident wavefield using wavelets as the source and the scattered wavefield using data residual as the source. By the iterative algorithm of Born modeling and RTM, the reflectivity model is solved by the conjugate gradient method. However, this method depends largely on the initial model, such that if the initial model is inaccurate then the result is also wrong. Similar to LSRTM, FWI problems can be solved by a two-loop algorithm: the inner loop is to iteratively solve for the model perturbation and the outer loop is to update the current model and compute the synthetic data to get the new residual. The inner loop can be treated as the LSRTM problem (Chen and Sacchi, 2018). In fact, the gradient of the objective function can be proved to be equal to the image condition of RTM. Therefore, the optimization of the gradient is a way to implement LSRTM. By the Gauss-Newton approximation, using truncated Newton's method is a good way to solve this problem (Pan et al., 2017). The Hessian-vector product is calculated in each iteration in a matrix-free form. This method seems to be more robust to the inaccuracies in the velocity model although the convergence problems are not solved yet.

Theory

The acoustic wave equation in matrix form is

$$A(m,\omega)u(m,x_s,\omega) = f(x_s,\omega),$$

where $A(m,\omega) = (\omega^2 m(x) + \nabla^2)$ is the impedance matrix, m is the model. In Born approximation, the wavefield can be separated into 2 parts: the incident wavefield and the scattered wavefield.

$$A_0(m,\omega)u_0(m,\omega) = f(x_s,\omega),$$

$$A_0(m,\omega)\delta u(m,\omega) = \omega^2 \frac{2\delta v}{v_0^3} u_0(m,\omega).$$

$u_0(m,\omega)$ is the incident wavefield and $\delta u(m,\omega)$ is the scattered wavefield. Applying the adjoint state method on the two wavefields, we have the RTM imaging condition

$$m_{mig}(x) = \sum_{n_s}\sum_{n_\omega} \frac{1}{\omega^2} Re\left(\delta u G_0^\dagger(x_s|x) G_0^\dagger(x|x') f^\dagger(x_s,\omega)\right),$$

where $Re(\cdot)$ is the real part and $G_0^\dagger(x_s|x)$ and $G_0^\dagger(x|x')$ represent the conjugate transpose of the Green's functions, which illustrates the cross-correlation of two wavefields as the imaging condition of LSRTM. For the FWI-based LSRTM, the algorithm is formulated from the objective function of FWI. The gradient of FWI objective function is

$$g = R^T R \sum_{n_s}\sum_{n_\omega} \omega^2\, Re\left(G_0^\dagger(x_s|x) G_0^\dagger(x|x') f^\dagger(x_s,\omega)\delta u\right),$$

where R is the vector with receiver coordinates. Compare the gradient equation with the RTM imaging condition, we conclude that the imaging condition of RTM is equal to the gradient of the FWI objective function, except for the small coefficient changes. After expanding the objective function of FWI in Taylor series, we have the objective function of FWI-based LSRTM in image domain is

$$\phi(\delta m) = \frac{1}{2}||-g - H\delta m||^2$$

where H is the Hessian operator and the Hessian vector product using Gauss-Newton approximation is

$$H\delta m = u\frac{\partial A(m,\omega)}{\partial m} A^{-1}(m,\omega) R R^\dagger (A^{-1}(m,\omega))^\dagger (\frac{\partial A(m,\omega)}{\partial m})^\dagger u^\dagger \delta m$$

To solve the linear equation, we use linear conjugate gradient method to get δm iteratively.
For time domain least-squares reverse time migration, the cross-correlation image condition is

$$m_{mig}(x) = \sum_{n_s}\sum_{n_t} \phi_s(x,t)\phi_r(x,T-t),$$

which represents the process of the source wavefield $\phi_s(x,t)$ and the receiver wavefield $\phi_r(x,T-t)$ cross-correlation. This imaging condition is the same as the imaging condition in the frequency domain.

Results

In this section, we will show several tests to compare the time domain and the FWI-based frequency domain LSRTM for cases when the velocity model is wrong. The goal is to observe if either approach is more robust than the other. The model dimensions for all the tests are 126 points for the vertical axis and 384 points for the horizontal axis and the grid spacing is 8m. FIG. 1. shows a 2-layer model with velocities of 3000m/s for the upper layer and 4000m/s for the lower layer. The interface is at depth 560m and the wrong velocity model is set to a constant value of 3300m/s. The migration results are shown in FIG. 2. Because the velocity of the first layer is wrong (3300m/s instead of 3000m/s), the reflector depth is wrongly mapped for the time domain RTM, to around 600m instead of 560m, deeper as expected for velocity too fast. On the other hand, the FWI-based frequency domain LSRTM seems to provide the correct location, although the resolution is poor. The goal of this ongoing research is to find out why the frequency domain migration has this lower resolution but seems more robust than the time domain.

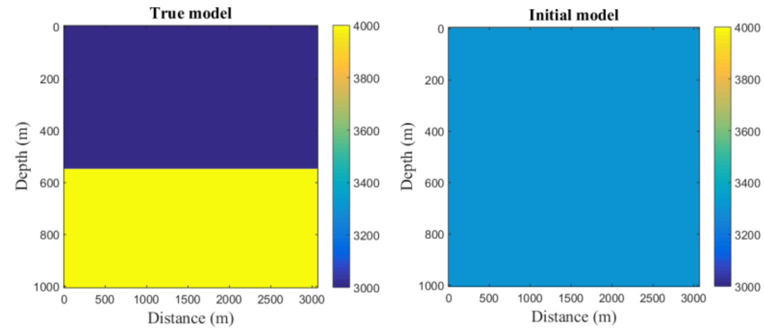

FIG. 1. The true model and the initial model for a 2-layer model

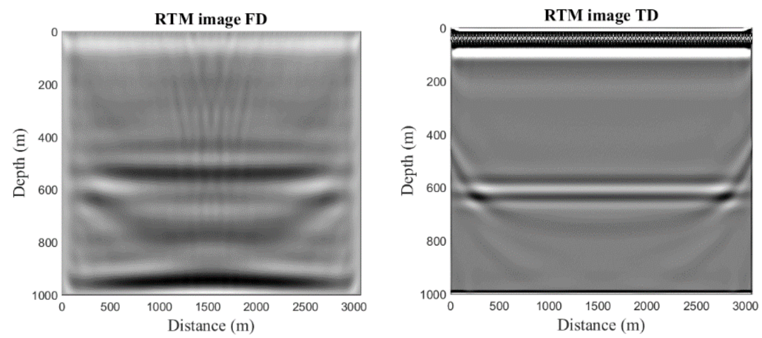

FIG. 2. RTM of a 2-layer model in frequency and time domain

In another test, we use the Marmousi model, again with wrong velocities. To create the wrong velocities, we apply a heavy smoothing. FIG. 3. shows the true model and the smoothed model. Comparing the results in different domains (FIG. 4. and FIG. 5.), the FWI-based LSRTM can locate the reflectors at approximately correct locations but suffers from the noise and convergence problems. The FWI-based LSRTM seems to have more details than the time domain version, in particular deeper in the section.

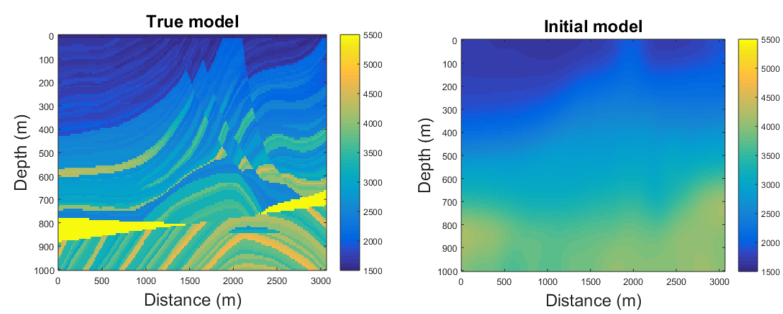

FIG. 3. The true model and the initial model of the resampled Marmousi model

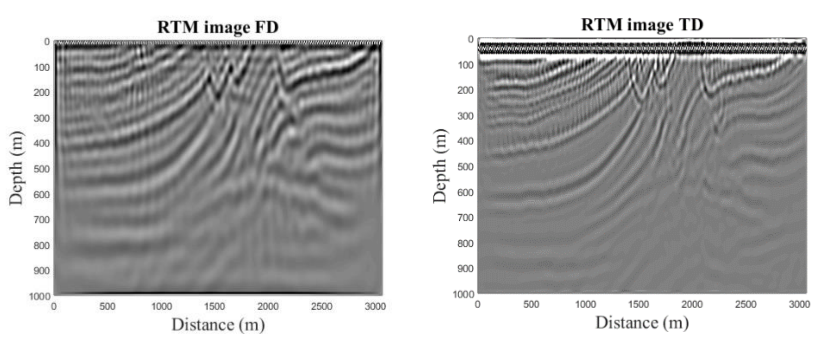

FIG. 4. RTM of the Marmousi model in frequency and time domain

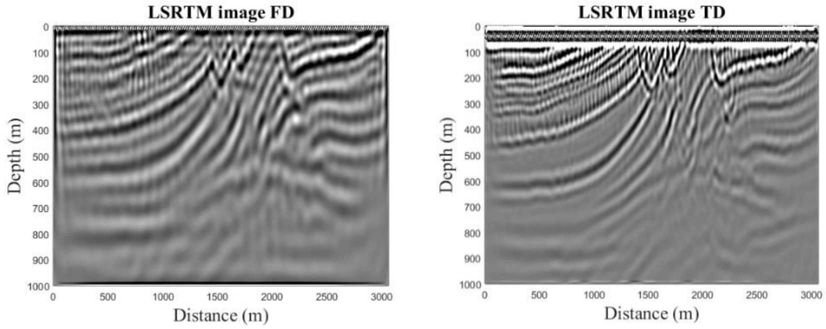

FIG. 5. LSRTM of the Marmousi model in frequency and time domain

Conclusions

We investigate an FWI-based LSRTM in the frequency domain. This method is based on the objective function of FWI and uses truncated Newton's method to solve for the model perturbation. We used two examples to compare these algorithms and understand the strength and weakness of each method. The conventional time domain LSRTM has a good convergence and sharper image when the velocity model is correct, but when the model is wrong, the reflectors are not correctly located. For the FWI-based LSRTM in the frequency domain with wrong velocities, although it suffers from convergence problems, it seems to locate the reflectors at the correct locations. We speculate that low frequencies seem to be less affected by the wrong velocities. If this is true, we expect to be able to develop an RTM in the frequency domain that uses information from the low frequencies to constrain the high frequencies. Also, we plan to investigate further how the convergence is affected in each case when the velocities are wrong.

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