The convenient formulae of nonhyperbolic moveout
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Summary
Nonhyperbolic P-wave moveout for transverse isotropy with a vertical symmetry axis (VTI media) is controlled by the parameter $\eta$, or, alternatively, by the horizontal velocity $v_h$, which is also responsible for handling anisotropy in time-domain processing. Here, we recast the nonhyperbolic moveout equation, originally developed by Alkhalifa and Tsvankin, as a function of $v_h$ and normal moveout velocity $v_n$, and introduce formulae in a very simple form that provide a useful and practical tool for velocity analysis.

Introduction
Alkhalifa and Tsvankin (1995) showed that P-wave reflection moveout is largely controlled by just two parameter combinations—the normal moveout (NMO) velocity, $v_n$, from a horizontal reflector and the anisotropic coefficient $\eta$, which describes the nonhyperbolic portion of the moveout curve. The $\eta$ is defined as

$$\eta = 0.5 \left( \frac{v_h^2 - v_n^2}{v_n^2} \right)$$

where $v_h = v_n\sqrt{1 + 2\eta}$ is the horizontally propagating velocity. The normal moveout equation including reference to the anisotropic parameter and the velocities of $v_n$ and $v_h$ can be written as (Alkhalifa and Tsvankin, 1995)

$$T_x^2 = T_0^2 + \frac{x^2}{v_n^2} - \frac{2\eta x^4}{v_n^2[v_n^2T_0^2+(1+2\eta)x^2]}$$

(1)

Here $T_0$ denotes the two-way vertical travel time and $x$ is the offset between source and receiver. While equation (1) can be applied in seismic data processing, it lacks a clear physical understanding of how the anisotropic media affects the propagating wave. Because we know that in a VTI media, the propagating wave velocity varies with propagating angle, we can rewrite the normal moveout equation in a form where the velocity explicitly relates to the propagating angle. We will present new formulae and suggest that they provide convenient practical tools for velocity analysis.

Formulae derivation

Moveout angle-dependent velocity
First, we rewrite equation (1) as

$$T_x^2 = T_0^2 + \frac{x^2(T_0^2v_h^2+x^2)}{v_n^2(T_0^2v_h^2+(1+2\eta)x^2)} ,$$

(2)

Second, we define a pseudo-depth $z$ in terms of a vertical velocity and vertical propagating time as $z = T_0 v_n$.

Substituting $z = T_0 v_n$ into equation (2) yields

$$T_x^2 = T_0^2 + \frac{x^2(z^2+x^2)}{v_n^2(z^2+(1+2\eta)x^2)} ,$$

(3)

With simple trigonometry, we have
\[ \sin^2 \theta = \frac{x^2}{z^2 + x^2} \quad \text{and} \quad \cos^2 \theta = \frac{z^2}{z^2 + x^2}. \]

Where \( \theta \) denotes the propagating angle between anisotropic axis and propagation direction, then

\[ T_x^2 = T_0^2 + \frac{x^2}{v_R^2(1 + 2 \eta \sin^2 \theta)}, \quad (4) \]

Defining \( v(x)\theta = v_n^2(1 + 2 \eta \sin^2 \theta) \), we have the normal moveout equation

\[ T_x^2 = T_0^2 + \frac{x^2}{v(x)\theta}, \quad (5) \]

Though equation (5) is equal to equation (1), it is a simpler form than the standard normal moveout equation and with \( v(x)\theta \) we have a clearer physical meaning: velocity varied with changing angle. Obviously, when \( \theta = 0^\circ, v(\theta) = v_n, \theta = 90^\circ, v(\theta) = v_h \).

**Angle velocity**

The same as the standard normal moveout equation, equation (5) fits the travel time only within a small angle range. NMO based on ray tracing travel time calculations has been used to improve the result to larger angles (Sadri and Riahi, 2009 and Mukhopadhyay, Mullik, 2011), where the group velocity is required for applying Snell’s law in anisotropic media.

Rewriting equation (5) as

\[ T_x^2 = T_0^2 + \frac{x^2}{v_R^2(1 + 2 \eta \sin^2 \theta)} = \frac{x^2}{v_R^2(1 + 2 \eta \sin^2 \theta)} + \frac{x^2}{v_R^2(1 + 2 \eta \sin^2 \theta)} = \frac{x^2(1 + 2 \eta \sin^2 \theta) + x^2}{v_R^2(1 + 2 \eta \sin^2 \theta)} = \frac{x^2 + x^2 + 2 \eta \cos^2 \theta \sin^2 \theta}{v_R^2(1 + 2 \eta \sin^2 \theta)} \]

where \( D = \sqrt{z^2 + x^2} \) is the distance that the wave has traveled.

Applying \( \cos(\theta) = \frac{z}{D} \), we have

\[ T_x^2 = \frac{D^2(1 + 2 \eta \cos^2 \theta \sin^2 \theta)}{v_R^2(1 + 2 \eta \sin^2 \theta)} = \frac{D^2}{v_R^2(1 + 2 \eta \cos^2 \theta \sin^2 \theta)} \]

\[ v(\theta) = \frac{v_R^2(1 + 2 \eta \sin^2 \theta)(1 - 2 \eta \cos^2 \theta \sin^2 \theta)}{(1 + 2 \eta \cos^2 \theta \sin^2 \theta)(1 - 2 \eta \cos^2 \theta \sin^2 \theta)} \]

\[ v(\theta) \approx v_R^2(1 + 2 \eta \sin^2 \theta)(1 - 2 \eta \cos^2 \theta \sin^2 \theta), \quad (2 \eta \cos^2 \theta \sin^2 \theta)^2 \ll 1 \]

\[ v(\theta)^2 = v_R^2(1 + 2 \eta \sin^2 \theta - 2 \eta \cos^2 \theta \sin^2 \theta), \quad 4 \eta^2 \sin^4 \theta \cos^2 \theta \ll 1 \]

and finally, we have

\[ v(\theta)^2 \approx v_R^2(1 + 2 \eta \sin^4 \theta) \]

**Numerical comparison of \( v(\theta) \) and approximate \( v(\theta) \)**

To show the accuracy of equation (9), we use the following parameters comparing exact \( v(\theta) \) calculated from equation (8) (blue line) with approximated \( v(\theta) \) calculated from equation (9) (red line):

**Case 1:** \( v_n = 2000 m/s, \eta = 0.1, 0.2, 0.3 \)
Case 2: $v_n = 4000 \text{m/s}, \eta = 0.1, 0.2, 0.3$

Further derivations
With equation (9) for phase velocity, we can easily find:

**Derivative of** $v(\theta)$:

$$\frac{dv(\theta)}{d\theta} \approx \frac{4v_n \eta \sin^3 \theta \cos \theta}{\sqrt{1 + 2\eta \sin^4 \theta}}$$

**Group velocity**:

$$v_g^2(\theta) = v^2(\theta) + \left( \frac{dv(\theta)}{d\theta} \right)^2 \approx v^2(\theta) + \frac{(4\eta v_n \sin^3 \theta \cos \theta)^2}{(1 + 2\eta \sin^4 \theta)^2}$$

$$v_g(\theta) \approx v(\theta) \sqrt{1 + \frac{(4\eta v_n \sin^3 \theta \cos \theta)^2}{(1 + 2\eta \sin^4 \theta)^2}}$$

**Angle difference between directions of phase velocity and group velocity propagation**:

$$\tan(\phi - \theta) = \frac{1}{v(\theta)} \frac{dv(\theta)}{d\theta} \approx \frac{4\eta v_n \sin^3 \theta \cos \theta}{1 + 2\eta \sin^4 \theta}$$

Where $\phi$ denotes the propagating angle between anisotropic axis and group velocity propagation direction.

**Practical NMO application for equation (5)**

It was noticed that equation (1) was designed to ensure the correct behavior of the moveout for infinitely large offsets. Grechka and Tsvankin (1998) introduced an empirical factor $C=1.2$ into the equation to improve intermediate offsets moveout, i.e.

$$T_x^2 = T_0^2 + \frac{x^2(T_0^2 v_R^2 + x^2)}{v_R^2(T_0^2 v_R^2 + C(1 + 2\eta x^2))}.$$  

A similar idea was also presented by introducing a maximum limited offset $x_{\text{max}}$ to control the largest moveout offset behavior (Harlan, 1995)

$$T_x^2 = T_0^2 + \frac{x^2}{v(x)^2}, \quad \frac{1}{v(x)^2} = \frac{1}{v(x)^2_{\text{max}} \left[ 1 + 2\eta \frac{x_{\text{max}}^2}{v(x)^2_{\text{max}} + v(x)^2(T_0^2 \frac{x}{v(x)^2})} - \frac{x^2}{x^2 + v(x)^2 T_0^2} \right]}.$$  

The physical meaning of the modification is moving the maximum effect of $\eta$ on an infinitely large offset to a limited maximum offset, which is equivalent to moving the maximum effect of $\eta$ at $90^\circ$ to a limited maximum angle $\theta_{\text{max}}$. Therefore, we modify the angle moveout velocity $v(x)^2 = v_R^2(1 + 2\eta \sin^2 \theta)$ to $v(x)^2 = v_R^2(1 + 2\eta \sin^2 \alpha \theta)$, $\alpha = \frac{90}{\theta_{\text{max}}}$. 

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**Example**

We show the difference between standard NMO and one called the pseudo-velocity moveout method that is based on ray tracing. The idea of pseudo-velocity moveout is that in the standard NMO equation \( T^2_x = T^2_0 + \frac{x^2}{v^2} \), we calculate \( T_x \) by ray tracing with \( v(g(\theta)) \) and replace the corresponding velocity by pseudo-velocity by \( v(x)^2_{\text{pseudo}} = \frac{x^2}{T^2_{\text{ray}} - T^2_0} \). The NMO is then carried out by \( T^2_x = T^2_0 + \frac{x^2}{v(x)^2_{\text{pseudo}}} \). With this procedure, the ray bending is sufficiently considered.

**Conclusions**

The simplified formulae for wave-propagating velocity in VTI media can be useful in seismic data processing. Both moveout velocity and incident-angle-the dependent velocity are explicitly related to wave propagating angle, which are of clearer physical meaning regarding wave propagating in VTI media. Even though equation (5) is equivalent to the original moveout equation, the former has the advantage of computational efficiency. At the same time, the byproduct of calculated propagation angle may be useful when using this formula for migration. The approximate angle velocity is not only of high accuracy compared with the non-approximate formula, i.e. equation (8), but also has an advantage of substantially fewer calculations being required for the ray tracing.

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**References**