geoconvention
Calgary • Canada • May 7-11 2018

Fracking / Brittleness Index: the effects of porosity, compressibility, storativity and elastic constants

Lateef Owolabi Lawal
College of Petroleum Engineering & Geosciences, KFUPM

Summary

The recent evolution of shale reservoirs has boosted a large interest in understanding the fracking/brittleness index of shale. From the shale exploration viewpoint, frackability/brittleness index measure the feasibility of hydraulically fracturing the tight shale. With the inventory of variant fracking/brittleness relations, it now appears that a complete understanding is precluded by lack of analytical method. To improve our understanding, the present paper extends the concept of dual-porosity to fracking index estimation. This methodology accounts for the coupling effect of hydraulic and elastic properties of the rock. With this idea, a simple and explicit equation is derived to quantify fracking/brittleness index in terms of the storativity and porosity of the matrix and fracture. The contribution of this work is to provide a physical and mathematical approach that would lead to a unique relation for the frackability/brittleness index.

Introduction

Fluid flow in low permeability reservoirs generally depends on naturally, chemically, and hydraulically induced fractures. The secondary process of dolomitization, acidization and rock fracturing would transform and redefined the flow characteristics of homogeneous and isotropic low permeability rock in Fig.1a by dividing it into matrix blocks and creating additional interconnected fractures and vugs as shown in Fig.1b. The idea of hydraulic fracturing is to transform a tight rock (Figure) into a matrix-fracture-vugs system (Fig 1b) with large volume of pressurized fluid. In principle, the hydraulic fracture induced would close on the withdrawal of the injected fluid pressure. This essential elasticity property implies the working assumption that the fracture pressure due to the injected fluid is the same as the closure pressure. The volume of injected fluid e.g. water required to cause the compression of the rock to induce mechanical discontinuity is characterized by sstorativity and fracture porosity. The storativity, ω represent the storage capacity of the individual matrix-fracture-vugs system. In groundwater hydrology, it is the quantity of water an aquifer produces due to compression (Korvin, 2016). The fracture porosity is the network of fracture as shown in Figure .

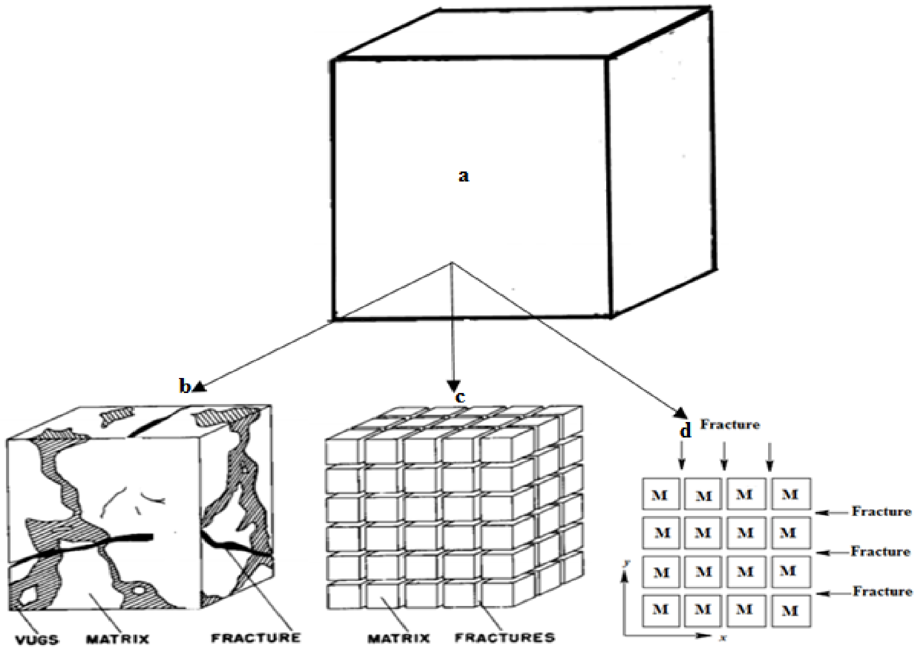

Figure 1: Conceptual Physical Model modified from (Warren and Root, 1963)

Theory and/or Method

The constitutive relation that characterizes storativity, ω of triple porosity system is defined as (Korvin, 2016):

$$\omega_m + \omega_f + \omega_v = 1 \qquad (2a)$$

$$\omega_m = \frac{\emptyset_m C_m}{\emptyset_m C_m + \emptyset_f C_f + \emptyset_v C_v} \qquad (2b)$$

$$\omega_f = \frac{\emptyset_f C_f}{\emptyset_m C_m + \emptyset_f C_f + \emptyset_v C_v} \qquad (2c)$$

$$\omega_v = \frac{\emptyset_v C_v}{\emptyset_m C_m + \emptyset_f C_f + \emptyset_v C_v} \qquad (2d)$$

where $\emptyset$ and C represent porosity and compressibility. The subscript m, f, v refer to matrix, fracture and vugs.

The fracture compressibility C_f is defined as:

$$C_f = \frac{1}{\emptyset_f}\frac{\partial \emptyset_f}{\partial p} \qquad (3)$$

The fracture porosity $\emptyset_f$ is defined according to (3) as:

$$\emptyset_f = \emptyset_f^{\circ} e^{C_f(p-p_{\circ})} \qquad (4a)$$

A Linear approximation of $(4a)$ by Taylor series truncation is given as:

$$\emptyset_f \approx \emptyset_f^{\circ}\left[1 + C_f(p-p_{\circ})\right] \qquad (4b)$$

The fracture porosity $\emptyset_f$ is defined in terms of the fracture width W_f and matrix length to L_m as:

$$\emptyset_f = \frac{W_f}{L_m} \tag{4c}$$

By applying (4c), (3) becomes:

$$C_f = \frac{1}{\emptyset_f}\frac{\partial \emptyset_f}{\partial p} = \frac{1}{\emptyset_f}\frac{\partial \emptyset_f}{\partial W_f}\frac{\partial W_f}{\partial p} = \frac{1}{\emptyset_f}\frac{1}{L_m}\frac{\partial W_f}{\partial p} \quad (5)$$

For elastic deformation, the stress ∂p due to the fluid pressure p in the x direction causes a strain in the y direction, expressed in terms of Poisson ratio v in (6a). Base on the theory of linear elasticity, the longitudinal contraction of the fracture width ∂W_f due to ∂p in the x direction is accompanied by the lateral expansion of the length matrix ∂L_m in the y direction as depicted in Figure such that $\partial W_f = \partial L_m$. The Poisson ratio, v defines the lateral strain ε_y relative to longitudinal strain as:

$$\varepsilon_y = -v\varepsilon_{\times} \tag{6a}$$

$$v = -\frac{\varepsilon_y}{\varepsilon_{\times}} \tag{6a}$$

where the minus sign in (2) is introduced to make v positive

$$\varepsilon_y = \frac{\partial L_m}{L_m} \tag{6b}$$

$$\varepsilon_{\times} = \frac{\partial p}{E} \tag{6c}$$

where E is Young's modulus

Rearranging (6), the contraction of the fracture width ∂W_f is expressed by:

$$\partial W_f = \partial L_m = \frac{v\, L_m \partial p}{E} \tag{6d}$$

By applying, $(6d)$ to (5) the fracture porosity defined as:

$$\emptyset_f = \frac{1}{E}\frac{v}{C_f} \tag{7a}$$

$$\frac{\emptyset_m}{\emptyset_f} = \frac{\emptyset_m E C_f}{v} \tag{7b}$$

Neglecting the vug in (2b) and rearranging, we have:

$$\emptyset_f C_f = \frac{\omega_f}{1-\omega_f}\emptyset_m C_m = \frac{\omega_f}{\omega_m}\emptyset_m C_m \quad (8)$$

The resulting equation by combining (7) and (8) is given as:

$$\frac{\omega_m}{\omega_f} = \frac{E}{v}\emptyset_m C_m = \frac{E}{v}\frac{\emptyset_m}{K} \tag{9}$$

where K is the bulk modulus (Recall, that in elasticity theory, bulk modulus is the reciprocal of compressibility).

Examples

Frackability/brittleness index may result from either fracture porosity or storativity. The first possibility is due to the presence of interconnected fracture networks which are characterized by fracture compressibility C_f, Young's modulus E and Poisson ratio v. In other words, frackability /brittleness index is sensitive the fracture porosity and it is quantify by (7b) in terms of the ratio of the matrix and fracture porosity. This relations shows that this index is directly proportional to the matrix porosity. The second possibility is the relative storage capacity of the matrix to the fracture. Thus, the index is the storativity ratio between the matrix storativity ω_m and fracture storativity ω_f as expressed in (9).

The equations in (8) and (9) suggested that the elastic constant of rock that relates stress to strain can be found useful in the geomechanical characterization of fracking/brittleness index for shale gas exploitation. However, the elastic constant template (ECT) as a tool is not unique and can be applied and interpreted differently. For a unified and meaningful interpretation of using any pair of the five elastic constants (λ, μ, E, K,v) of isotropic rock, a constitutive relation of elastic constants would be required. Exploring the relations between elastic constants, we found a simple relation to combine nicely the isotropic elastic constants as:

$$\mathrm{E}\lambda = 6K\mu v \qquad (10)$$

An explicit equivalence between the brittleness estimated from the isotropic elastic stiffness constants (E, v) and elastic compliance constants (λ, μ) based on (10) is given as:

$$\text{fracking index} = \frac{\omega_m}{\omega_f} = \frac{E}{v}\frac{\emptyset_m}{K} = \frac{6\mu\emptyset_m}{\lambda} = \frac{3\emptyset_m(1-2v)}{v}$$

Conclusions

The very small original permeability of unconventional shale makes hopeless any economic recovery of oil and gas without hydraulic fracturing. The volume of the pressurized fluid needed to create permeable fracture network depends on hydraulic and elastic properties of the rock. The fracking/brittleness index becomes important to identify, ‘’sweet spots’’ (zones of interest for initiation of hydraulic fractures) in order to optimize hydraulic fracturing design. A simple dual porosity model is used to idealize the complex fracture network to find the relationship between the fracking index, porosity, compressibility, storativity and elastic constant. This new finding will be found useful for meaningful estimation of fracking index and hydraulic fracturing applications.

Acknowledgements

This paper grew out of the author’s Master Thesis, *Characterization of Brittleness and Frackability for Unconventional Shale.* Many thanks to Prof. Gabor Korvin (retired in 2016) for his supervision and mentorship. The research facilities of King Fahd University of Petroleum and Minerals (KFUPM) are duly acknowledged.

References

Korvin, G., 2016. Permeability from Microscopy : Review of a Dream. Arab J Sci Eng 41, 2045–2046. doi:10.1007/s13369-016-2138-y

Warren, J.E., Root, P.., 1963. The Behavior of Naturally Fractured Reservoirs. Soc. Pet. Eng. 245–255. doi:10.2118/426-PA