Multicomponent inverse scattering series internal multiple prediction part II: domains and implementions

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Summary

Identifying lower-higher-lower relationship is essential to inverse scattering series internal multiple prediction, which is more difficult for multicomponent predictions due to the wave-mode conversion of P- and S-waves. Since only conversions happened in the top layer can be handled by the algorithm, the input preparation for elastic internal multiple prediction becomes to an intractable problem. In paper of part I, we analytically analyzed the advantages and defects of input preparation using different methods, elastic stolt-migration, vertical traveltime stretching, and best-fitting velocity obtained by high resolution radon transform. In this paper, to examine the conclusions indicated previously, a synthetic model is utilized to implemented the multicomponent internal multiple prediction with different inputs generated by these approaches.

Prediction formulation: 1.5D case

Algorithm in k-z domain

For layered cases, we have, $k_{x_s}^{P}=k_{x_g}^{P}=k_{x_g}^{SV}$ with P-wave source only, and $k_{x_s}^{SV}=k_{x_g}^{SV}=k_{x_g}^{P}$ with S-wave source only. Assume $z_s=z_g$, prediction algorithm (Matson, 1997) for a layered case can be simplified as,

$$b_3^{ij}(k_g,\omega)=-\int_{-\infty}^{+\infty}\mathrm{d}z_1 e^{\mathrm{i}(\nu^m+\nu^i)z_1}b_1^{im}(k_g,z_1)\int_{-\infty}^{z_1-\epsilon}\mathrm{d}z_2 e^{-\mathrm{i}(\nu^n+\nu^m)z_2}b_1^{mn}(k_g,z_2)\int_{z_2+\epsilon}^{+\infty}\mathrm{d}z_3 e^{\mathrm{i}(\nu^j+\nu^n)z_3}b_1^{nj}(k_g,z_3) \qquad (1)$$

where, the input $b_1(k_g,z)$ is calculated as $b_1^{ij}(k_g,z)=\mathrm{i}2\nu^j D^{ij}(k_g,z)$, $\{i,j\}\in\{P,SV\}$. Here, j denotes the source side, and i represents the receiver side. $D^{ij}(k_g,z)$ is the migrated shot profile using elastic stolt migration in wavenumber manner with two constant background velocity for P- and SV-waves.

Algorithm in p-z domain

Similar to acoustic cases, algorithm can also be transferred into ($p-z$) domain by simply replacing the wavenumber variable (Sun and Innanen, 2016; 2017b). Its mathematical form is shown as,

$$b_3^{ij}(p_g,\omega)=-\int_{-\infty}^{+\infty}\mathrm{d}z_1 e^{\mathrm{i}(\nu^m+\nu^i)z_1}b_1^{im}(p_g,z_1)\int_{-\infty}^{z_1-\epsilon}\mathrm{d}z_2 e^{-\mathrm{i}(\nu^n+\nu^m)z_2}b_1^{mn}(p_g,z_2)\int_{z_2+\epsilon}^{+\infty}\mathrm{d}z_3 e^{\mathrm{i}(\nu^j+\nu^n)z_3}b_1^{nj}(p_g,z_3) \qquad (2)$$

where, the input $b_1(p_g,z)$ is calculated as $b_1^{ij}(p_g,z)=\mathrm{i}2q^j D^{ij}(p_g,z)$, $\{i,j\}\in\{P,SV\}$. $D^{ij}(p_g,z)$ is migrated shot profile using elastic stolt migration in horizontal-slowness manner with two constant background velocities for P- and SV-waves.

Algorithm in p-τ domain

As delineated in paper of part I, the plane wave domain algorithm can be obtained by performing one-to-one mapping between pseudo-depth and vertical traveltime which requires a time-stretched $\tau-p$ transformed data as the inputs. However,to avoid interpolation process in time-stretching, we can also achieve the same goal by modifying the integral limits with a traditional $\tau-p$ transformed data as input. Therefore, hereinafter, multicomponent prediction with time-stretching method signifies the algorithm with time-stretched integral limits and will only be performed in this way. The 1.5D plane wave domain algorithm (Sun and Innanen, 2017a) with time-stretching modified integral limits is shown as,

$$b_3^{ij}(p_g,\omega) = -\int_{-\infty}^{+\infty} d\tau_1^{im} e^{i\omega\tau_1^{im}} b_1^{im}(p_g,\tau_1^{im}) \int_{-\infty}^{\Upsilon(\tau_1^{im}|\tau_2^{mn})-\epsilon} d\tau_2^{mn} e^{-i\omega\tau_2^{mn}} b_1^{mn}(p_g,\tau_2^{mn}) \int_{\Upsilon(\tau_2^{mn}|\tau_3^{nj})}^{+\infty} d\tau_3^{nj} e^{i\omega\tau_3^{nj}} b_1^{nj}(p_g,\tau_3^{nj}) \quad (3)$$

where, the input $b_1(p_g,\tau)$ is obtained by $b_1^{ij}(p_g,\tau) = i2q^j D^{ij}(p_g,\tau)$, $\{i,j\} \in \{P,SV\}$. $D^{ij}(p_g,\tau)$ is the linear transformation of shot profile. The modified integral limits is described as,

$$\Upsilon(\tau_2^{mn} \mid \tau_1^{nj}) = \begin{cases} \tau_2^{mn}, & j=m; \\ \dfrac{\alpha+\beta}{2\beta}\tau_2^{mn}, & j=S \,\&\, m=P; \\ \dfrac{2\beta}{\alpha+\beta}\tau_2^{mn}, & j=P \,\&\, m=S; \end{cases} \quad (4)$$

Besides the prediction with time-stretching in plane wave domain, the implementation can also be performed with the best-fitting velocity model to adapt the monotonicity condition of vertical traveltime and actual depth. With the best-fitting velocity model, the modified integral limits for the plane wave domain prediction algorithm is delineated as,

$$\Gamma(\tau_2^{mn} \mid \tau_1^{nj}) = \frac{v^{mn}(p_g,\tau_2^{mn})}{v^{nj}(p_g,\tau_1^{nj})}\tau_2^{mn} \quad (5)$$

where, v^{xy} is the best-fitting velocity model obtained by high-resolution hyperbolic radon transform (Trad, 2003). By simply replacing $\Upsilon(\tau_2^{mn} \mid \tau_1^{nj})$ with $\Gamma(\tau_2^{mn} \mid \tau_1^{nj})$, the prediction algorithm with best-fitting velocity can be obtained. Based on the analytical analysis, we indicated in part I that prediction with best-fitting velocity model may requires a large research parameter. Because of the opposite sorting order of pseudo-depth, the multicomponent prediction with a cross-validate of vertical traveltime-stretching and best-fitting velocity model may allow a relative constant and smaller ϵ to identify the lower-higher-lower combinations.

Synthetic: input preparation

A layered synthetic model containing two interfaces was built to generate shot profile and then implemented the elastic algorithm to predict all multiples in shot gather. The model parameters are delineated in Figure 1, from top to bottom, P-wave velocities are [2000,3500,2500]m/s, S-wave velocities are [1200,2000,1300]m/s, and densities are [1.5,2.25,1.6] g/cm^3. A P-wave source is in the middle of the model, and receivers in 4m interval are arranged at same level of depth. With four absorbing boundaries (dashed line in the model shown in Figure 1), a multi-component shot gather is generated using finite difference, i.e., only primaries and elastic internal multiples appear in the seismic record. After collected the shot profile, it is decomposed into P- and SV-wave component using Helmholtz's method. The decomposed P- and S-wave components of data are illustrated in Figure 2.

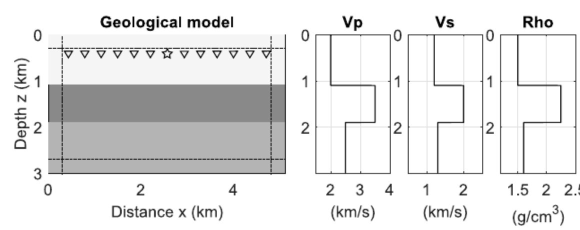

Figure 1. Geological model and model parameters.

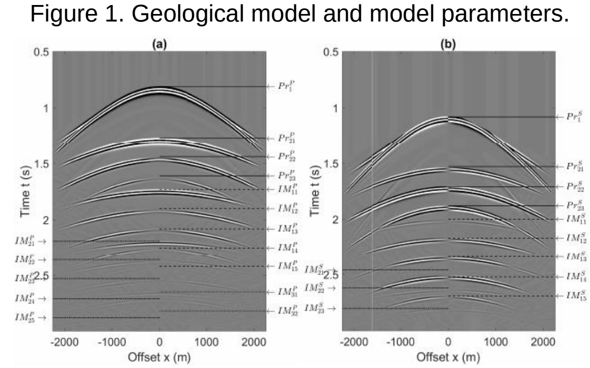

Figure 2. Helmholtz's deomposition of shot profile.

k-z domain Input using elastic stolt-migration

Figure 3 shows migrated images $R(k_h,z)$ both for P-P and P-SV reflections which are inputs for $k-z$

domain multicomponent prediction with multiplied weight factor $-2i\nu_s$. A clear image of P-P reflection is generated using elastic stolt migration with constant velocity. However, for P-SV reflection, there are some aliasing appear at the low pseudo-depth and the amplitudes of events at deep pseudo-depth are incorrect. To intuitively analyze the migrated images, the final step of elastic stolt migration is performed by stacking over k_h to achieve a single trace for a shot profile. Figure 4 indicates that elastic stolt migration is failed to migrate those primaries reflected by 2nd reflector into the same pseudo-depth in wavenumber domain.

p-z domain Input using elastic stolt-migration

To take advantages of plane wave on concentrating reflection, we also performed elastic stolt migration in horizontal slowness. The migrated images of P-P and P-SV reflections of a shot profile due to a P-wave source only are plotted in Figure 5, and the final migrated traces are plotted in Figure 6. Compared to wavenumber, elastic stolt migration in horizontal slowness provided much clearer migrated traces. However, these up-and-down hyperbola aliasing in P-P and ellipse aliasing in P-SV are fatal to the multicomponent prediction.

p-τ domain Input with modified integral limits

To simplify the preparation, instead of generating the appropriate inputs, prediction with modified integral limits are implemented with traditional $\tau - p$ transformation as inputs. The high-resolution hyperbolic radon was applied (Figure7) to achieve best-fitting velocity model (Figure 8). And then with modified limits in equation 5, multicomponent internal multiple prediction can be implemented in plane wave domain.

Synthetic: prediction and discussion

Since those aliasing events of inputs generated by elastic stolt migration gravely disordered the lower-higher-lower relationship of subevents, there is no point to implement the multicomponent prediction with elastic stlot migrated inputs in (k,z) and in (p,z) domain. In this section, we only performed the multicomponent prediction with modified integral limits, which provided by time-stretching, best-fitting velocity, and the intersection of time-stretching and best-fitting velocity.

Figure 9 shows multicomponent prediction with time-stretching method using a constant ò=96ms which is the width of a single wavelet. All internal multiples are predicted, but along with some undesired near-offset events. For example, two primary events are also predicted at the near-offset in P-P component, and one

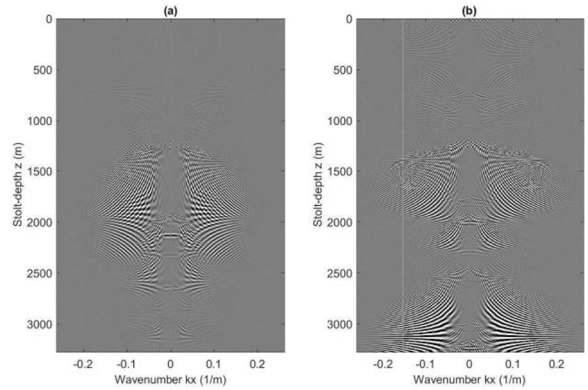

Figure 3. Input using elastic stolt migration in k.

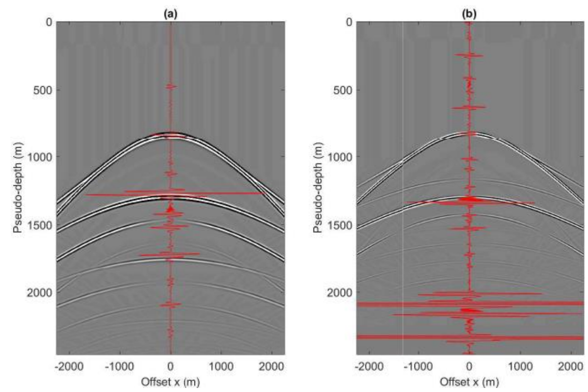

Figure 4. Final migrated traces in wavenumber.

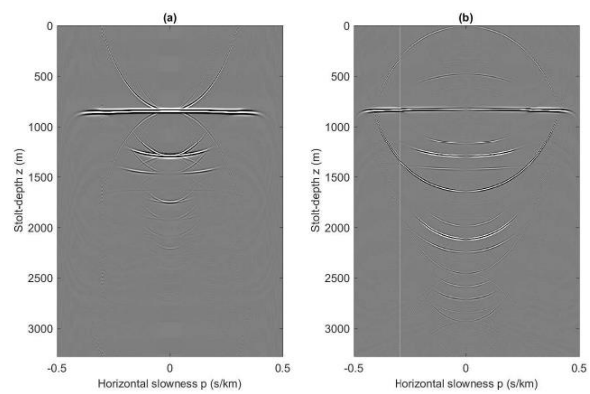

Figure 5. Input using elastic stolt-migration in p.

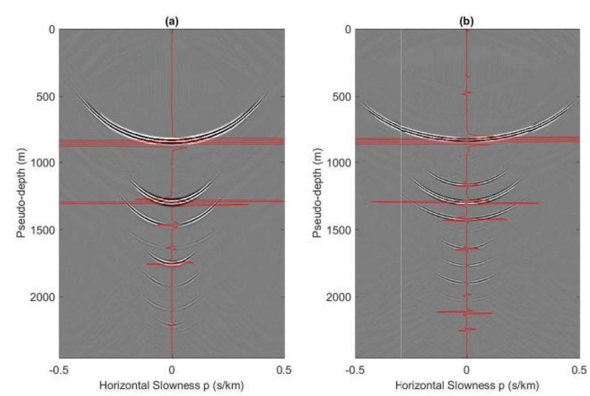

Figure 6. Final migrated traces in slowness.

primary appeared in prediction of P-SV component. These undesired events are generated by false combinations satisfying lower-higher-lower relationship. Figure 10 shows perferect prediction with best-fitting approach using ò=200ms, all multiples are well predicted and aliasing are eliminated. As discussed in Part I, best-fitting reference velocity model is an approximate solution for wave-mode conversion which requires a larger search parameter. To reduce the ϵ dependency of prediction algorithm, we also performed the prediction with the cross-validate integral limits using ò=96ms, shown in Figure 11, because of the opposite sorting order of time-stretching and best-fitting approach. As expected, even with a smaller search parameter, all internal multiples are predicted at correct traveltime.

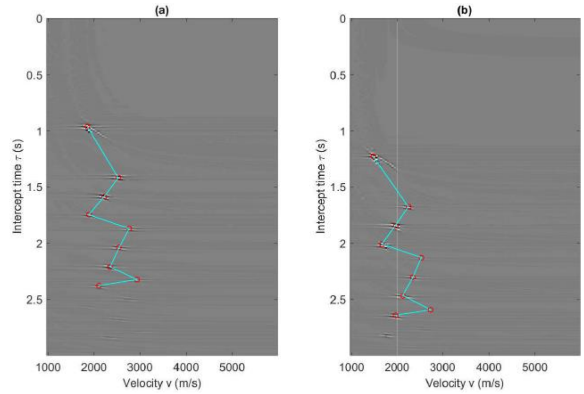

Figure 7. Hyperbolic radon transformation.

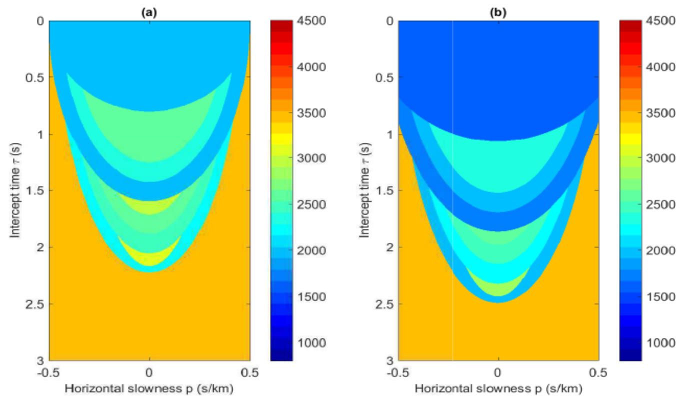

Figure 8. Best-fitting velocity model.

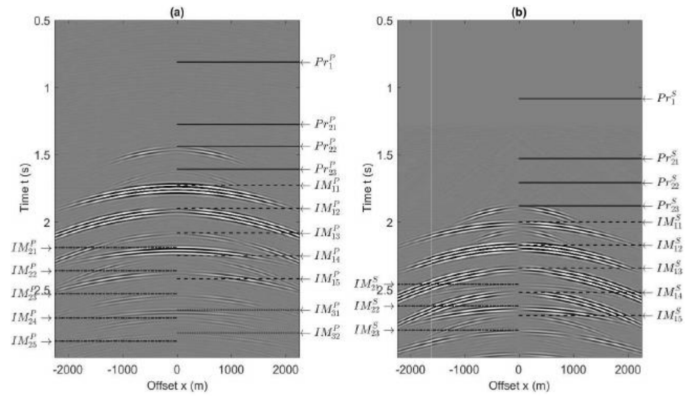

Figure 9. Prediction with time-stretching using ò=96ms.

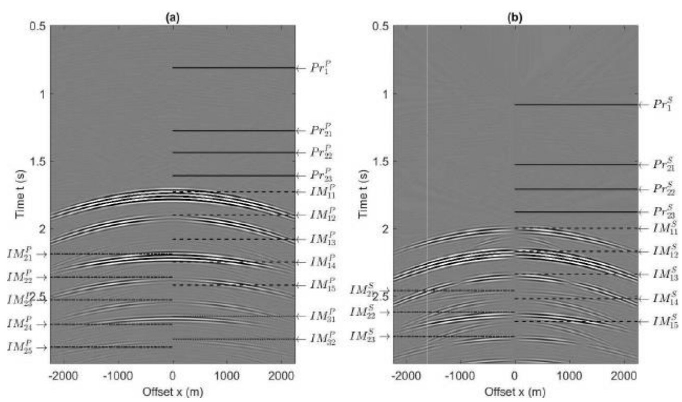

Figure 10. Prediction with best-fitting using ò=200ms..

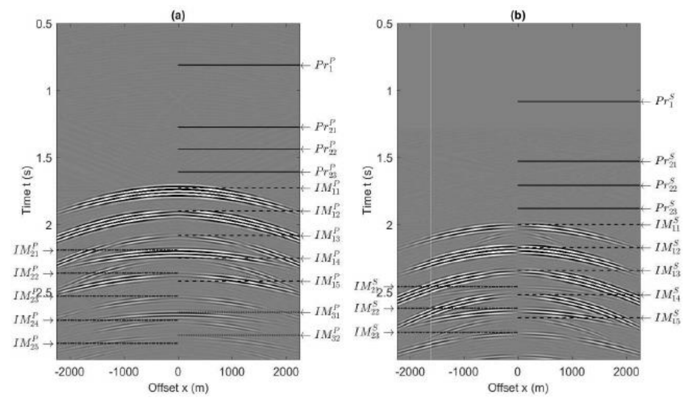

Figure 11. Prediction with cross-validation using ò=96ms.

Conclusions

With the quantitatively analysis, the elastic stolt migration with constant background velocity both in (k, z) domain and in (p, z) domain has been proven to be an inappropriate solution for elastic prediction. Time-stretching method provides an equivalent input as elastic stolt-migration do. Prediction with best-fitting approaches requires a relative larger search parameter. Take benefits of opposite sorting characteristic of time-stretching and best-fitting, their cross-validation produced the best internal multiple prediction on multicomponent data.

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