Comparison between RTM gradient and PSPI gradient in the process of FWI

Sergio Romahn, Marcelo Guarido and Kristopher Innanen
CREWES, University of Calgary

Summary

Full waveform inversion (FWI) can be described as an iterative cycle of four steps. Firstly, we generate synthetic seismic data (modelled shots) from a smoothed initial model and obtain the difference among observed and modelled shots (data residuals). Secondly, we migrate the data residual (using the current velocity model) and stack. This step produces the gradient. Thirdly, we scale the gradient in order to create a velocity update. And finally, we obtain a new velocity model by adding the velocity update to the current velocity model. This new velocity model is used in the next iteration. The present work is focused in the second step of the cycle. We evaluated the suitability of producing the gradient by applying phase shift plus interpolation (PSPI) migration and compared it to the conventional gradient from reverse time migration (RTM). We used well-log information to convert the gradient into a velocity perturbation in the third step of the cycle. We found that the PSPI gradient is more sensitive to the initial velocity model than the one produced by RTM. PSPI would be a good option providing that the initial velocity model incorporates enough low frequency information. PSPI also showed a great sensitivity to the well-log interval that is used to calibrate the gradient, while the RTM gradient is more stable with short well-log coverages.

Introduction

Lailly (1983) described the inversion process as a sequence of pre-stack migrations of the date residuals. Tarantola (1985) showed the solution of the inverse problem as an iterative methodology that consists on a forward propagation of the actual sources in the current model and backward propagation of the data residuals. The correlation of the two fields yields to a correction of the model parameters. In other words, the full waveform inversion gradient is equivalent to a reverse time migration (RTM) of the data residuals. The idea that any kind of depth migration may be able to produce the gradient was introduced by Margrave et al. (2010, 2012). Several authors, such as Guarido et al. (2014) and Arenrin and Margrave (2015), have investigated this approach. Romahn and Innanen (2017) evaluated the use of well-log data to scale the PSPI gradient under different geological settings.

RTM vs PSPI

RTM was introduced by Baysal et al. (1983), Whitmore (1983) and Chang and McMechan (1986). It uses a finite-difference solution to the wave equation. A 1D mathematical representation of RTM is given in Equation 1.

$$IM(z) = \int d\omega\omega^2 G(z, z_s, \omega) \left[G(z_g, z, \omega) D^*(\omega)\right] \tag{1}$$

where $IM(z)$ is the prestack RTM image at depth z, $G(z, z_s, \omega)$ is the forward propagation of source field to depth z, $G(z_g, z, \omega)D^*(\omega)$ is the backpropagation of the measured data D into the medium to the same depth, and the integral is the correlation of the two. The FWI gradient *g* is shown in Equation 2.

$$g^{(n)}(z) = \int d\omega\omega^2 G(z, z_s, \omega \mid s_o^{(n)}) \left[G(z_g, z, \omega \mid s_o^{(n)}) \delta P^*(z_g, z_s, \omega \mid s_o^{(n)})\right] \tag{2}$$

where *g* is gradient, *n* is iteration number, $G(z, z_s, \omega \mid s_o^{(n)})$ is the forward propagation of source field to depth z through the model parameter $s_o^{(n)})$, $G(z_g, z, \omega \mid s_o^{(n)})\delta P^*(z_g, z_s, \omega \mid s_o^{(n)})$ is the backpropagation of the data residuals δP into the medium to the same depth. The difference between a prestack RTM seismic image and a RTM gradient is the data that we migrate. For the first case, we migrate the observed seismic shots, while for the second case we iteratively migrate the data residual.

Phase-shift migration was presented by Gazdag (1978). This frequency-wavenumber method is based on the exploding reflector model. It assumes that the sources are distributed along all reflectors, that the wavefield satisfies the scalar wave equation, and that the recorded seismic data are the values of the wavefield at the surface. We can downward extrapolate (downward continuation) the data to simulate a seismic section that would be obtained if the recording plane was at depth z. Extrapolating the wavefield backwards in time to *t = 0* when the sources where initiated, provides the migrated depth section. The phase shift of the Fourier coefficients, in the frequency-wavenumber domain, produces the downward extrapolation of source and receiver positions. Gazdag and Sguazzero (1984) conceived a generalization of this method that was called phase shift plus interpolation. PSPI addresses lateral velocity variations. We used the PSPI method modified by Ferguson and Margrave (2005). The algorithm accomplished prestack depth migration by the simultaneous downward continuation of shot records ($\psi^*_{s\downarrow}$) and model of the source wavefield ($\psi_{r(s)\uparrow}$), which is the upward-traveling receiver data from the same shot at the same depth. The PSPI gradient with a cross-correlation imaging condition is

$$IM(z) = \int \psi_{s\downarrow}{}^*(z,\omega)\psi_{r(s)\uparrow}(z,\omega)d\omega \qquad (3)$$

RTM applies two-way wave operators, while PSPI works with one-way wave operators. This fundamental difference allows RTM managing multiples, while PSPI only handles primaries. RTM is suitable for imaging complex geology such as salt bodies, while PSPI may have problems on this kind of geological settings. Computational cost and memory are important issues in RTM, while PSPI is relatively cheap.

Migration response

Figure 1 shows the two-layer model that was used to generate a single trace with a source-receiver pair separated by 1000 m. A minimum phase wavelet with a dominant frequency of 15 Hz was used as the seismic source. The RTM and PSPI migration of this trace are shown in figure 2. RTM computational time was 6 times longer than the time for PSPI migration. We can identify the following events: the source-and-receiver side reflection wavepaths B and C that are formed by convolving the scattered wavefields caused by the reflector and the forward and backward propagated wavefields that are built by the source and the receiver. Event A is the migration ellipse. The direct wave was overshadowed by the other events.

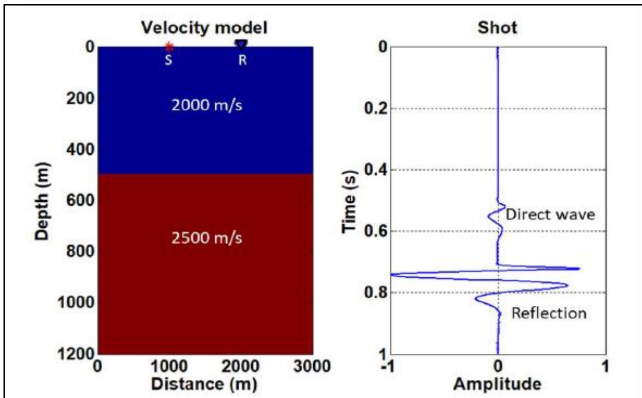


Fig. 1. Seismic trace generated by finite-difference modelling through a single interface model.

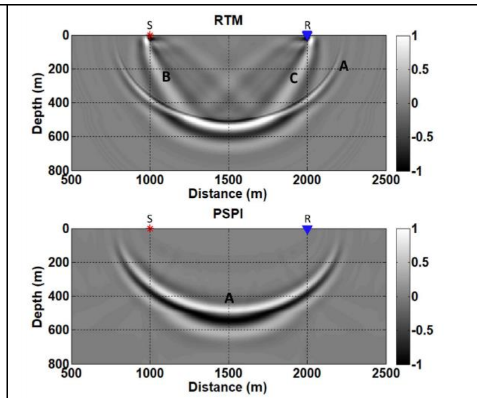


Fig. 2. PSPI and RTM applied to the 1000-m offset trace of figure 1.

It has been shown that the deconvolution imaging condition works as a gain correction. For the case of the FWI gradient, it does something similar to the main diagonal elements of the inverse Hessian matrix [Margrave et al. (2010) and Pan et al. (2014)]. We applied the deconvolution imaging condition in the examples shown in this work.

Inversion methodology

We inverted 81 synthetic shots generated by solving the acoustic wave equation by finite differences with constant density. We used a minimum phase wavelet with dominant frequency of 20 Hz. The source and receiver intervals are 100 and 10 meters, respectively. The maximum offset is 2000 m. The velocity model to be solved corresponds to a shallow syncline that constitutes a reservoir trap. The four steps of the inversion are: 1) Generation of synthetic seismic shots (modelled shots) from a smoothed initial model and calculation of the data residual (difference among observed and modelled shots). The first iteration was done with an initial velocity model that was constructed by applying a Gaussian smoother to the true velocity model with a half-width of 300 m. 2) In the second step we apply pre-stack depth migration of the data residual (using the current velocity model) and stack. We experimented with RTM and PSPI to migrate the data residuals in this stage of the process. We applied the multi-scale approach, where we start the inversion with low frequencies and introduce higher frequencies as we iterate. The frequency range for the first iteration was from 1 to 6 Hz. Then, we moved up the frequency band 1 Hz in each of the following iterations. The result of stacking the migrated data residuals is the gradient. 3) The third step consists in scaling or calibrating the gradient by using well-log velocity. This step produces a velocity update. The well calibration technique was described by Margrave et al. (2010) and Romahn and Innanen (2017). For this example we used the whole well interval to do the calibration. Later we will show the sensitivity of both migration gradients to the well interval coverage. 4) The last step of the cycle corresponds to the sum of the current velocity model and the velocity update, providing a new model that will be used in the next iteration.

Examples

The final inverted velocity models obtained by using PSPI and RTM gradients after 15 iterations are shown in figure 3. A seismic survey can be divided by three zones: the full fold zone, the migration apron which is inside the full fold area, and the innermost zone beyond the migration apron which is the domain of the interpreter. All layers laying in this zone should be considered full-fold and fully migrated (Cordsen et al., 2000). The RTM model shows the best performance inside the interpreter zone. The error increases as we go to the borders of the survey and where the layers dip to that direction. RTM showed to be more sensitive to the seismic coverage than PSPI. The PSPI inverted model shows a larger error in the area of the geological target, which suggests a poor performance of this migration method in the presence of high velocity contrasts.

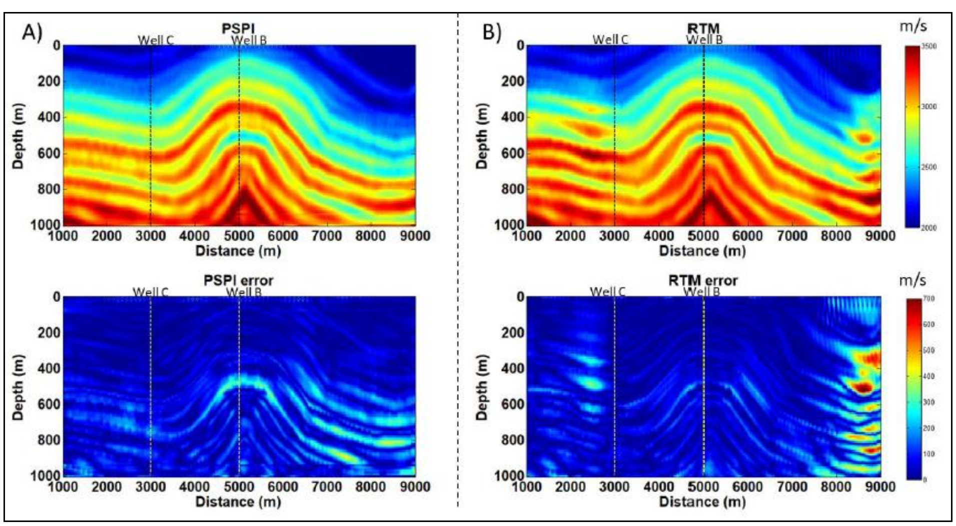


Fig. 3. Final inverted model after 15 iterations: A) PSPI and B) RTM.

Sensitivity to the initial velocity model

The previous results were obtained by using an initial velocity model derived from smoothing the true velocity model with a half-width Gaussian smoother of 300 m. We increased the half-width to 600 m in order to test the sensitivity of both migration gradients to the initial velocity model. The comparison between the two initial models and the inverted velocities in the calibration well location are shown in figure 4. PSPI inversion dramatically underperformed with a smoother initial model. RTM retrieves long wavelengths better than PSPI at a higher computational cost. The use of PSPI to obtain a satisfactory gradient will highly depend on how close the initial model is to the true model.

Sensitivity to the well interval coverage

We used the whole well interval from zero to 1000 m in the previous examples. In this section, we tested the sensitivity of RTM and PSPI gradients with progressively smaller well interval coverages for the first iteration. Figure 5 shows the inverted velocity in the blind well and the error for different well intervals. We found that the RTM gradient produces a similar effect with different well coverages. On the other hand, PSPI gradient performance significantly degrades as the well coverage is reduced. The well calibration technique applies a convolution filter to the gradient in order to derive the update. This matched filter affects the gradient phase. This suggests that the PSPI gradient need a phase correction, which will be strongly affected by the interval of calibration.

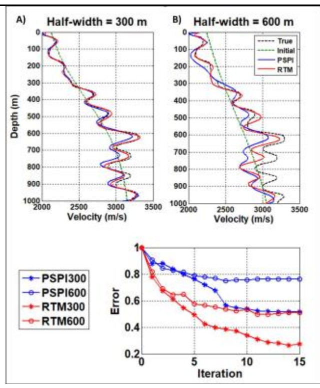
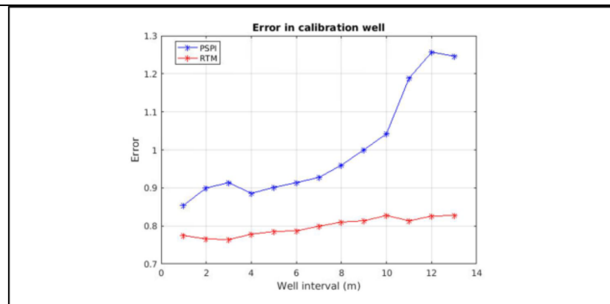


Fig.4. Sensitivity to the initial velocity model of PSPI and RTM. A) Gaussian smoother half-width = 300 m. B) Gaussian smoother half-width = 600 m.



Test	1	2	3	4	5	6	7	8	9	10	11	12	13
Well interval (m)	0-1000	0-750	50-750	100-750	150-750	200-750	250-750	300-750	350-750	400-750	500-750	550-750	600-750

Fig 5. Sensitivity of PSPI and RTM with well interval coverage for the first iteration (1-6 Hz).

Conclusions

PSPI is a one-way wave migration method, while RTM uses two-way wave operators to perform the migration. This difference makes RTM more expensive, but also capable to manage all the arrivals in the wavefield, including primaries and multiples. The FWI gradient is commonly obtained by applying RTM to the data residuals. We showed that PSPI is also suitable to produce the gradient; however, it is more sensitive to the initial model and the well interval coverage used for the calibration. Such characteristic will limit is applicability. RTM has the capability of recovering long-wavelength information; therefore, it is less sensitive than PSPI to a smoother initial model. The calibration of the RTM gradient with well information showed to be quite stable with smaller well interval coverages. In our synthetic example, RTM produced the smaller errors across the model and a superior result inside the full-fold and fully migrated zone. RTM showed to be more sensitive to the seismic coverage than PSPI. The better result provided by RTM comes with a higher computational cost. A migration of one shot with RTM took 6 times longer than PSPI. A hybrid inversion by using both methods is feasible and will save computational time, providing that we have enough well coverage to calibrate the PSPI gradient. RTM can be used in the first iterations when we use low frequencies to recover long wavelengths, and then PSPI can be used when we incorporate high frequencies to add detail to the model.

Acknowledgements

We thank the sponsors of CREWES for their support. We also acknowledge support from NSERC through the grant CRDPJ 461179-13. Author 1 thanks PEMEX and the government of Mexico for founding his research.

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