The convenient formulae of nonhyperbolic moveout

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Summary

Nonhyperbolic P-wave moveout for transverse isotropy with a vertical symmetry axis (VTI media) is controlled by the parameter η , or, alternatively, by the horizontal velocity v_h , which is also responsible for handling anisotropy in time-domain processing. Here, we recast the nonhyperbolic moveout equation, originally developed by Alkhalifah and Tsvankin, as a function of v_h and normal moveout velocity v_n and introduce formulae in a very simple form that provide a useful and practical tool for velocity analysis.

Introduction

Alkhalifah and Tsvankin (1995) showed that P-wave reflection moveout is largely controlled by just two parameter combinations—the normal moveout (NMO) velocity, v_n from a horizontal reflector and the anisotropic coefficient η, which describes the nonhyperbolic portion of the moveout curve. The η is defined as

$$\eta = 0.5\left(\frac{v_h^2 - v_n^2}{v_n^2}\right)$$

where $v_h = v_n\sqrt{1+2\eta}$ is the horizontally propagating velocity. The normal moveout equation including reference to the anisotropic parameter and the velocities of v_n and v_h can be written as (Alkhalifah and Tsvankin ,1995)

$$T_x^2 = T_0^2 + \frac{x^2}{v_n^2} - \frac{2\eta x^4}{v_n^2[v_n^2 T_0^2 + (1+2\eta)x^2]} \qquad (1)$$

Here T_0 denotes the two-way vertical travel time and x is the offset between source and receiver. While equation (1) can be applied in seismic data processing, it lacks a clear physical understanding of how the anisotropic media affects the propagating wave. Because we know that in a VTI media, the propagating wave velocity varies with propagating angle, we can rewrite the normal moveout equation in a form where the velocity explicitly relates to the propagating angle. We will present new formulae and suggest that they provide convenient practical tools for velocity analysis.

Formulae derivation

Moveout angle-dependent velocity

First, we rewrite equation (1) as

$$T_x^2 = T_0^2 + \frac{x^2(T_0^2 v_n^2 + x^2)}{v_n^2(T_0^2 v_n^2 + (1+2\eta)x^2)} \ , \qquad (2)$$

Second, we define a pseudo-depth z in terms of a vertical velocity and vertical propagating time as $z = T_0 v_n$,

Substituting $z = T_0 v_n$ into equation (2) yields

$$T_x^2 = T_0^2 + \frac{x^2(z^2 + x^2)}{v_n^2(z^2 + (1+2\eta)x^2)} \ , \qquad (3)$$

With simple trigonometry, we have

$$sin^2\theta = \frac{x^2}{z^2 + x^2} \quad and \;\; cos^2\theta = \frac{z^2}{z^2 + x^2}.$$

Where θ denotes the propagating angle between anisotropic axis and propagation direction, then

$$T_x^2 = T_0^2 + \frac{x^2}{v_n^2(1+2\eta sin^2\theta)}, \tag{4}$$

Defining $v(x)_\theta^2 = v_n^2(1 + 2\eta sin^2\theta)$, we have the normal moveout equation

$$T_x^2 = T_0^2 + \frac{x^2}{v(x)_\theta^2}, \tag{5}$$

Though equation (5) is equal to equation (1), it is a simpler form than the standard normal moveout equation and with $v(x)_\theta$ we have a clearer physical meaning: velocity varied with changing angle. Obviously, when $\theta = 0^o$, $v(x)_\theta = v_n$, $\theta = 90^o, v(x)_\theta = v_h$.

Angle velocity

The same as the standard normal moveout equation, equation (5) fits the travel time only within a small angle range. NMO based on ray tracing travel time calculations has been used to improve the result to larger angles (Sadri and Riahi, 2009 and Mukhopadhyay, Mullik, 2011), where the group velocity is required for applying Snell's law in anisotropic media.

Rewriting equation (5) as

$$T_x^2 = T_0^2 + \frac{x^2}{v_n^2(1+2\eta sin^2\theta)} = \frac{z^2}{v_n^2} + \frac{x^2}{v_n^2(1+2\eta sin^2\theta)} = \frac{z^2\left(1+2\eta sin^2\theta\right)+x^2}{v_n^2(1+2\eta sin^2\theta)} = \frac{\left(z^2+x^2\right)+2\eta z^2 sin^2\theta}{v_n^2(1+2\eta sin^2\theta)} = \frac{D^2+2\eta z^2 sin^2\theta}{v_n^2(1+2\eta sin^2\theta)}, \tag{6}$$

where $D = \sqrt{z^2 + x^2}$ is the distance that the wave has traveled.

Applying $\cos(\theta) = \frac{z}{D}$, we have

$$T_x^2 = \frac{D^2(1+2\eta cos^2\theta sin^2\theta)}{v_n^2(1+2\eta sin^2\theta)} = \frac{D^2}{v(\theta)^2} \tag{7}$$

$$v(\theta)^2 = \frac{v_n^2(1+2\eta sin^2\theta)}{(1+2\eta cos^2\theta sin^2\theta)} \tag{8}$$

$v(\theta)$, the angle-dependent velocity in VTI media. $v(\theta)$ can be further simplified by

$$v(\theta)^2 = \frac{v_n^2(1+2\eta sin^2\theta)(1-2\eta cos^2\theta sin^2\theta)}{(1+2\eta cos^2\theta sin^2\theta)(1-2\eta cos^2\theta sin^2\theta)}$$

$$v(\theta)^2 \approx v_n^2(1+2\eta sin^2\theta)(1-2\eta cos^2\theta sin^2\theta), \qquad (2\eta cos^2\theta sin^2\theta)^2 \ll 1$$

$$v(\theta)^2 = v_n^2(1+2\eta sin^2\theta - 2\eta cos^2\theta sin^2\theta - 4\eta^2 sin^4\theta cos^2\theta)$$

$$\approx v_n^2(1+2\eta sin^2\theta - 2\eta sin^2\theta cos^2\theta), \qquad 4\eta^2 sin^4\theta cos^2\theta \ll 1$$

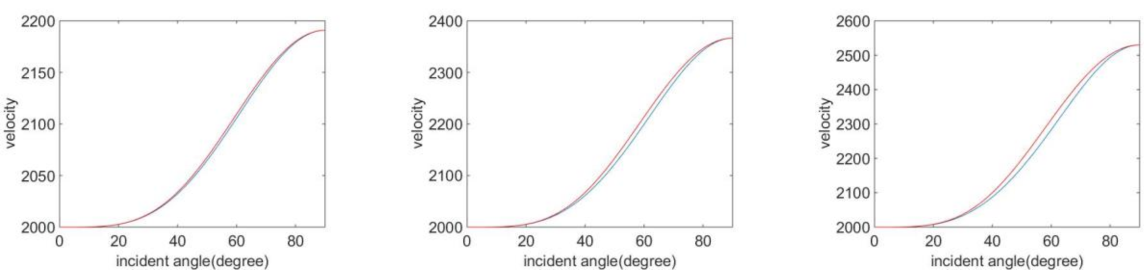
and finally, we have

$$v(\theta)^2 \approx v_n^2(1+2\eta sin^4\theta) \tag{9}$$

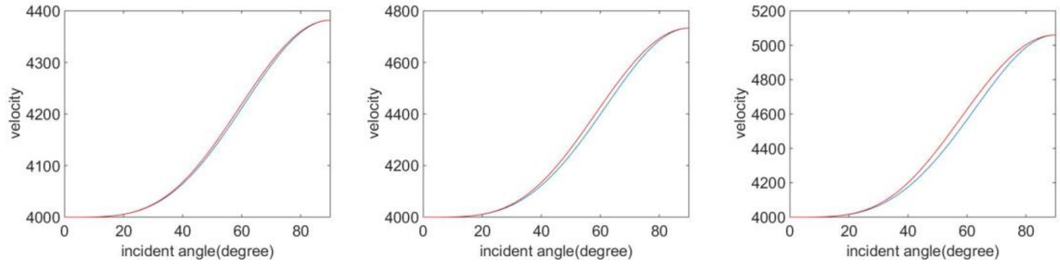
Numerical comparison of $v(\theta)$ and approximate $v(\theta)$

To show the accuracy of equation (9), we use the following parameters comparing exact $v(\theta)$ calculated from equation (8) (blue line) with approximated $v(\theta)$ calculated from equation (9) (red line):

Case 1: $v_n = 2000m/s,\ \eta = 0.1,\ 0.2, 0.3$



Case 2: $v_n = 4000m/s,\ \eta = 0.1,\ 0.2, 0.3$



Further derivations

With equation (9) for phase velocity, we can easily find:

Derivative of $v(\theta)$:

$$\frac{dv(\theta)}{d\theta} \approx \frac{4v_n \eta sin^3\theta cos\theta}{\sqrt{1+2\eta sin^4\theta}} \tag{10}$$

Group velocity:

$$v_g^2(\theta) = v^2(\theta) + \left(\frac{dv(\theta)}{d\theta}\right)^2 \approx v^2(\theta) + \frac{(4\eta v_n sin^3\theta cos\theta\ )^2}{(1+2\eta sin^4\theta)}$$

$$= v^2(\theta)\left(1 + \frac{(4\eta sin^3\theta cos\theta\ )^2}{(1+2\eta sin^4\theta)^2}\right)$$

$$v_g(\theta) \approx v(\theta)\sqrt{1+\left(\frac{4\eta sin^3\theta cos\theta}{1+2\eta sin^4\theta}\right)^2} \tag{11}$$

Angle difference between directions of phase velocity and group velocity propagation:

$$tan(\varphi - \theta) = \frac{1}{v(\theta)}\frac{dv(\theta)}{d\theta} \approx \frac{4\eta sin^3\theta cos\theta}{1+2\eta sin^4\theta} \tag{12}$$

Where φ denotes the propagating angle between anisotropic axis and group velocity propagation direction.

Practical NMO application for equation (5)

It was noticed that equation (1) was designed to ensure the correct behavior of the moveout for infinitely large offsets. Grechka and Tsvankin (1998) introduced an empirical factor C=1.2 into the equation to improve intermediate offsets moveout, i.e.

$$T_x^2 = T_0^2 + \frac{x^2(T_0^2 v_n^2 + x^2)}{v_n^2(T_0^2 v_n^2 + C(1+2\eta)x^2)},$$

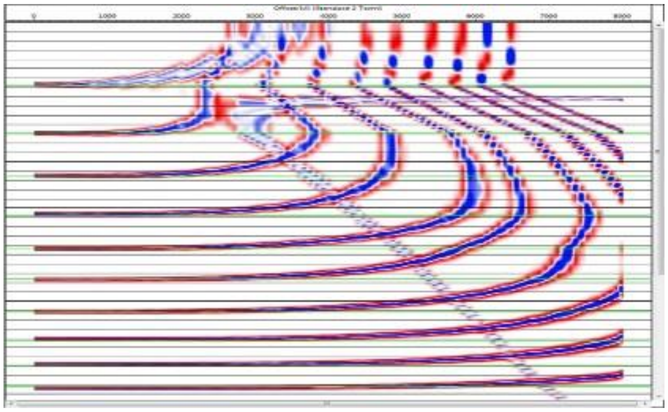
A similar idea was also presented by introducing a maximum limited offset x_max to control the largest moveout offset behavior (Harlan, 1995)

$$T_x^2 = T_0^2 + \frac{x^2}{v(x)_x^2}, \qquad \frac{1}{v(x)_x^2} = \frac{1}{v(x)_{x\ max}^2}\left[1 + 2\eta\left(\frac{x\_max^2}{x\_max^2 + v(x)_x^2 T_0^2} - \frac{x^2}{x^2 + v(x)_x^2 T_0^2}\right)\right].$$

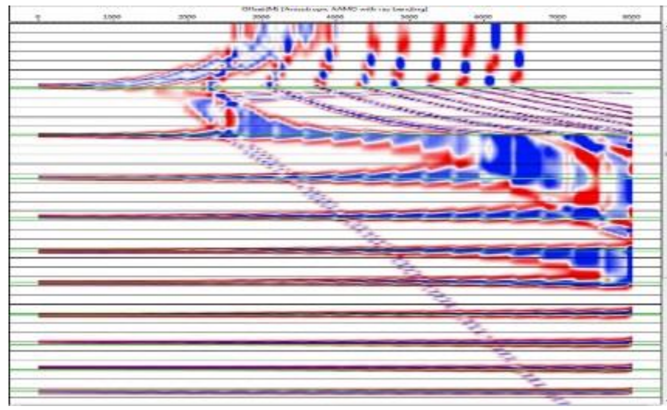
The physical meaning of the modification is moving the maximum effect of η on an infinitely large offset to a limited maximum offset, which is equivalent to moving the maximum effect of η at 90^o to a limited maximum angle θ_{max}. Therefore, we modify the angle moveout velocity $v(x)_\theta^2 = v_n^2(1+2\eta sin^2\theta)$ to $v(x)_\theta^2 = v_n^2(1+2\eta sin^2(\alpha\theta))$, $\alpha = \frac{90^0}{\theta_{max}}$.

Example

We show the difference between standard NMO and one called the pseudo-velocity moveout method that is based on ray tracing. The idea of pseudo-velocity moveout is that in the standard NMO equation $T_x^2 = T_0^2 + \frac{x^2}{v^2}$, we calculate T_x by ray tracing with $v_g(\theta)$ and replace the corresponding velocity by pseudo-velocity by $v(x)_{pseudo}^2 = \frac{x^2}{T_{x-ray}^2 - T_0^2}$. The NMO is then carried out by $T_x^2 = T_0^2 + \frac{x^2}{v(x)_{pseudo}^2}$. With this procedure, the ray bending is sufficiently considered.



Standard NMO



NMO with pseudo-velocity

Conclusions

The simplified formulae for wave-propagating velocity in VTI media can be useful in seismic data processing. Both moveout velocity and incident-angle-the dependent velocity are explicitly related to wave propagating angle, which are of clearer physical meaning regarding wave propagating in VTI media. Even though equation (5) is equivalent to the original moveout equation, the former has the advantage of computational efficiency. At the same time, the byproduct of calculated propagation angle may be useful when using this formula for migration. The approximate angle velocity is not only of high accuracy compared with the non-approximate formula, i.e. equation (8), but also has an advantage of substantially fewer calculations being required for the ray tracing.

Acknowledgements

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