Velocity model building by slope tomography

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Summary

Slope tomography method uses slopes and traveltimes of locally coherent reflected events to estimate the macro velocity model from reflection data for depth imaging and full waveform inversion (FWI). Without the requirement of picking traveltimes on continuous reflection events, slope tomography is operationally more efficient than traditional reflection tomography. It is computationally more efficient than migration velocity (MVA), because it estimates the global velocity model simultaneously without layer stripping and expensive depth migration iterations. We review the slope tomography method and evaluate its effectiveness using a numerical example.

Introduction

An accurate starting model is very important to depth migration and full waveform inversion (FWI). Depth migration can update the velocity model iteratively using migration velocity analysis (MVA) methods; however, each iteration requires an update to the velocity model and depth migration process. The goal of FWI is to converge to the global minimum of the objective function and to arrive at the correct model. However, FWI is an ill-posed problem, its solution often represents only a local minimum. Therefore, an accurate initial model can improve the efficiency and accuracy of depth migration and FWI. Traditional reflection tomography methods inverse traveltime picks to a velocity model and requires difficult interpretive traveltime picking of continuous reflection events. Tomographic CDR method is one several slope tomography methods. It uses the concept of controlled directional reception (CDR) method and ray parameters of the waves transmitted from a shot to a receiver to invert for the velocity model. The ray parameters of a locally coherent reflection event can be picked interactively or automatically on localized slant stacks of shot and geophone gathers. Here, we review the slope tomographic CDR method and evaluate its effectiveness and applicability using numerical examples.

Tomographic CDR method

The tomographic CDR method (Sword 1987) characterizes a locally coherent event with the source position x_s , receiver position x_g , traveltime T_{sr} and ray parameters p_s at the source and p_r at the receiver (Fig. 1). These parameters are referred to as the reciprocal parameters. They can be determined using localized slant stacks on common shot and common receiver gathers. A locally coherent event is associated with a ray segment pair that is characterized by the reflector or diffractor position X, ray shooting angle θ_s , θ_g, and traveltime T_s and T_g. If we ray trace from x_s and x_g using ray take off angles associated with ray parameters p_s and p_r, the rays will meet when the sum of Ts and T_g equals T_{sr}. However, if an incorrect velocity model is used in the ray tracing, the rays will not meet (Fig.2) and the distance error X_{err} will be non-zero:

$$Xerr = x_g - x_s \tag{1}$$

X_{err} is used in the cost function $J(v)$ for CDR tomographic inversion:

$$J(v) = ||\, X_{err}||^2 \tag{2}$$

Damping factors λ_x and λ_z are added to avoid rapid changes in velocity:

$$J(v) = ||\, X_{err}||^2 + \lambda_x^2 ||\, \partial v^2/\partial x\ || + \lambda_z^2 ||\, \partial v^2/\partial z\ || \tag{3}$$

The tomographic inversion problem is solved by finding the value of v that minimize the cost function $J(v)$. This is done by solving the following least squares system:

$$A^{(k)}\,\Delta v = \,-X_{err}^{(k)} \qquad , \tag{4}$$

where $X_{err}^{(k)} = X_{err}(v^{(k)})$ and is computed by ray tracing the current velocity model and takeoff angles computed from p_s and p_g, Δv is the value to update the velocity model with and $A^{(k)}$ is the Fréchet derivative matrix:

$$A_{ij}^{k} = \left(\frac{\partial X_{err}(j)}{\partial v_i}\right)_{v=v^k} \tag{5}$$

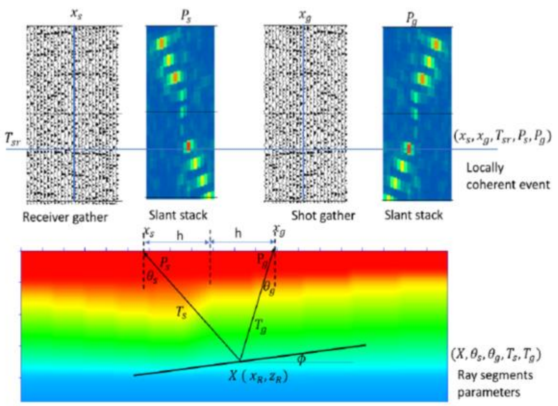

Figure 1: A locally coherent event can be picked on the localized shot and receiver slant stacks. The event is characterized by the traveltime T_{sr} and the ray parameters p_s and p_r and is associated with a ray segment pair in the velocity model.

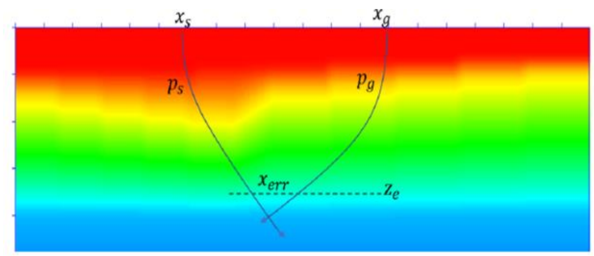

Figure 2: If error exists in the velocity model, rays traced from source and receiver will not meet at the depth where the sum of ray traced times equals the measured time.

Fréchet Derivatives for Tomographic CDR method

The major step in solving equation (4) is building the Fréchet derivatives $\frac{\partial X_{err}(j)}{\partial v_i}$, which can be rewritten as:

$$\frac{\partial X_{err}(j)}{\partial v_i} = \frac{\partial X_{err}(j)}{\partial \Delta X_{\,i}}\frac{\partial \Delta X_i}{\partial v_i} + \frac{\partial X_{err}(j)}{\partial \Delta t_{\,i}}\frac{\partial \Delta t_i}{\partial v_i} \;. \tag{6}$$

The first term in equation (6) can be defined by considering the ray path geometry in figure 2 and definition of $Xerr$ in equation (1),

$$\frac{\partial X_{err}(j)}{\partial \Delta X_{\,i}} = -1 \quad \text{for source ray path,}$$
$$\frac{\partial X_{err}(j)}{\partial \Delta X_{\,i}} = \quad 1 \quad \text{for receive ray path.}$$

To derive the third term $\frac{\partial X_{err}(j)}{\partial \Delta t_{\,i}}$, we rewrite it as:

$$\frac{\partial X_{err}(j)}{\partial \Delta t_i} = \frac{dX_{err}(j)}{dZe}\frac{d\text{Ze}}{d\Delta t_i} \quad , \tag{7}$$

where Ze is the depth of the end points of the rays as show in figure 2, and

$$\frac{dX_{err}(j)}{dZe} = \frac{dX_{ge}}{dZe} - \frac{dX_{se}}{dZe} \quad \text{, and} \tag{8}$$

$$\frac{dX_{err}(j)}{dZe} = \tan\theta_{ge} - \tan\theta_{se} \tag{9}$$

By rewriting $\frac{dZe}{d\Delta t_i}$ as

$$\frac{dZe}{d\Delta t_i} = \frac{1}{dt/dZe} \quad \text{,and} \tag{10}$$

$$\frac{dt}{dZe} = \frac{1}{v_{se}}\frac{1}{cos\theta_{se}} + \frac{1}{v_{ge}}\frac{1}{cos\theta_{ge}} \tag{11}$$

Substitute equation (9) and equation (11) into equation (7):

$$\frac{\partial X_{err}(j)}{\partial \Delta t_i} = -v_{se}v_{ge}\frac{\sin\theta_{ge}cos\theta_{se} - \sin\theta_{se}cos\theta_{ge}}{v_{se}cos\theta_{se} + v_{ge}cos\theta_{ge}} \tag{12}$$

The second and fourth terms in equation (6) can be computed by perturbing the velocity model and compute the differences in ray path position and travel time.

Stereotomography

Stereotomography was proposed by Billette and Lambaré (1997) as a generalized slope tomography method. Stereotomography is developed within the frame work of Hamiltonian ray theory, paraxial ray theory ad general inverse problem theory. Stereotomography uses the same data space d and data picking concept as tomographic CDR method. For a dataset with N picked events, d is represented as:

$$d = \left[\left(X_s, X_g, P_s, P_g, T_{sr}\right)_n\right]_{n=1}^{N}. \tag{13}$$

However, the model space is expanded to include reflection/diffraction point X, ray scattering angles θ_s, θ_g , and one-way traveltime T_s and T_g (Fig. 3) :

$$m = \left[\left[\left(X, \theta_s, \theta_g, T_s, T_g\right)_n\right]_{n=1}^{N}, [C_m]_{m=1}^{M}\right]. \tag{14}$$

The l_2 $norm$ cost function for the inversion of the mode parameter is

$$J(m) = ||\ \Delta d\ ||^2, \tag{15}$$

where

$$\Delta d(m) = d_{obs} - d_{calc}(m) \tag{16}$$

The model update Δm can be found by solving the linear system of equations

$$J(m)\Delta m = \Delta d(m), \tag{17}$$

where $J(m)$ is the Fréchet derivative matrix:

$$J(m) = \frac{\partial(X_s, X_g, p_s, p_g, T_{sr})}{\partial(X, \theta_s, \theta_g, T_s, T_g, C_m)} \tag{18}$$

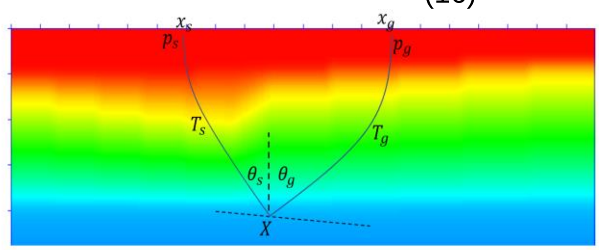

Figure 3: Model space of stereotomography $(X, \theta_s, \theta_g, T_s, T_g, C_m)$.Forward modeling is done by shooting rays toward source and receiver from X.

Examples

To validate the tomographic CDR method and its ability to reconstruct the velocity model, we test the tomographic CDR method using a lateral homogeneous model V(z) and a lateral heterogeneous model V(x,z). The first test traces the ray paths with different shooting angles through the lateral homogeneous model V(z) at six different depth levels. Ray parameters, source and receiver position at surface and the sum of the travel times of the source and ray path are used as input to the tomographic CDR method. Model V(z) and the actual ray paths are show in figure 4a. Starting velocity for this test is 1000 m/s. Figure 4b shows the initial ray paths using the input ray parameter, surface position and two-way time data and the constant starting velocity. Figure 4c shows that the tomographic CDR method is able to recover the velocity model by minimizing the errors in the end points of the ray pairs.

The second test traces the ray paths with different shooting angles through the lateral heterogeneous model V(x,z) at six different depth levels and different X locations. Ray parameters, source and receiver positions at surface and the sum of the travel times of the source and ray path are used as input to the tomographic CDR method. The model V(x,z) and the actual ray paths are shown in figure 5a and 5b. The initial starting velocity for this test is 1300 m/s. Figure 5c shows the initial ray paths using the input ray parameters, surface positions and two-way time data and the constant starting velocity. Figure 5d shows that the reconstructed velocity model after 10 iterations.

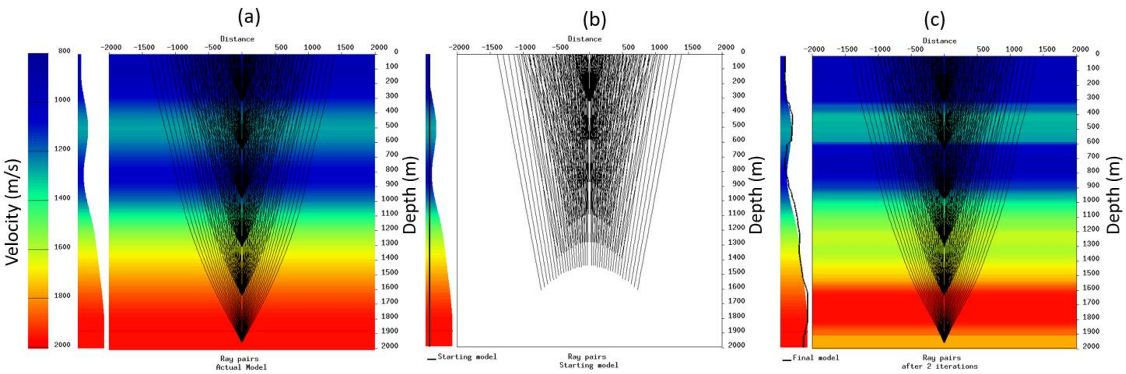

Figure 4: (a) Ray pairs from actual V(z) model, ray parameters, two-way traveltimes and surface positions are used in CDR tomography method. (b) Starting model of 1000 m/s and associated ray pairs. (c) CDR tomography solution after 2 iterations and associated ray pairs.

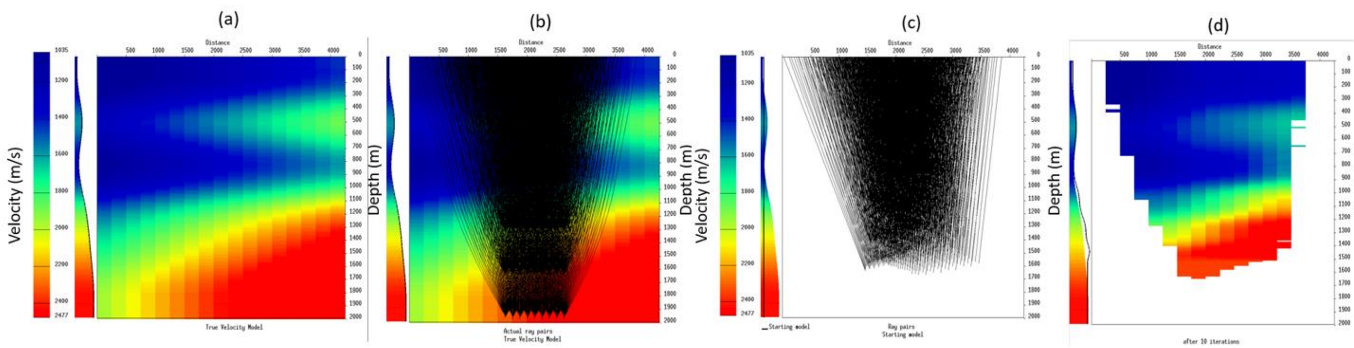

Figure 5: (a) V(x,z) model. (b) Ray pairs from true velocity model. (c) Starting model of 1300 m/s and associated ray pairs. (d) Stereotomography solution after 10 iterations.

Conclusions

We reviewed the tomographic CDR method and stereotomography method. We verified the ability of tomographic CDR method to recover velocity information using ray parameters and associated travel time and shot and receiver position using laterial homogenous and heterogenous velocity mode. The test results confirm that the algorithm can recover the velocity information; however, it is limited by the lateral and depth converage of the input data. This study shows that the slope tomography method, including tomographic CDR method and stereotomography method reviewed in this paper, has the potential of constructing starting model for depth imaging or full waveform inversion.

Acknowledgements

This work was funded by CREWES industrial sponsors and NSERC (Natural Science and Engineering Research Council of Canada) through the grant CRDPJ 46119-13.

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