Stabilization on Instantaneous Curvature

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Summary

The use of geometrical attributes is common procedure in seismic interpretation. As one of the geometrical attributes, instantaneous curvature describes a picked seismic surface by fitting a quadratic function to the inline and crossline coordinate. The accurate calculation of curvature helps improve the surface and fault characterization of the seismic data. Direct calculation of instantaneous curvature suffers from a numerical instability , and the generated singularities downgrade the results. This paper uses a weighted strategy to compute the instantaneous curvature which employs a first-moment formula to remove spikes and reduce rapid and confusing variations, thus enhancing the interpretability of the instantaneous curvature results.

Introduction

Instantaneous curvature is derived from three basic instantaneous attributes: instantaneous frequency, instantaneous wavenumber and instantaneous dip (Russell, 2012, Barnes, 1996, Al-Dossary and Marfurt, 2006). The performance of these instantaneous attributes controls the stabilization of instantaneous curvature. However, instantaneous frequency, instantaneous wavenumber and instantaneous dip are prone to spikes and noise.Furthermore, negative values in these attributes are common, which have uncertain physical meaning (Han and van der Baan, 2013). As a result, instantaneous curvature is often difficult to interpret, both qualitatively and quantitatively.

In this paper, we review a novel method to help stabilize instantaneous curvature. This method is an adaptive process based on first-moment formula. It effectively improves instantaneous frequency, instantaneous wavenumber and instantaneous dip by removing spikes and reducing rapid and confusing variations, and therefore can enhance the interpretability of instantaneous curvature. The real data examples provided illustrate the improved performance of this method in computing instantaneous curvature.

Theory

Instantaneous attributes (Taner et al., 1979) are derived from the seismic trace *x(t)* and its Hilbert transform *y(t)* by computing the analytic signal given by

$$z(t) = x(t) + iy(t) = R(t)\exp[i\theta(t)]. \tag{1}$$

where $R(t)$ and $\theta(t)$ are the instantaneous amplitude and instantaneous phase, respectively. Instantaneous frequency $f(t)$ is defined as the first derivative of instantaneous phase along the time direction,

$$f(t) = \frac{d\theta(t)}{dt}. \tag{2}$$

Applying the Hilbert transform to 3D seismic volume, instantaneous wavenumbers $k_x(t)$ and $k_y(t)$ are defined as the first derivative of instantaneous phase along inline direction and crossline direction (Barnes, 2000), where

$$k_x(t) = \frac{d\theta(t)}{dx}, \quad k_y(t) = \frac{d\theta(t)}{dy}. \tag{3}$$

Therefore, the inline instantaneous dip *p(t)* and crossline instantaneous dip *q(t)* can be obtained by calculating the ratio of $k_x(t)$ and $k_y(t)$ to $f(t)$,or

$$p(t)=\frac{k_x(t)}{f(t)},\quad q(t)=\frac{k_y(t)}{f(t)}. \tag{4}$$

All curvature methods are based on the initial paper by Roberts (2001) in which he used differential geometry to show that the curvature of some event in the earth's subsurface could be estimated from a time structure map by fitting the local quadratic surface given by:

$$z(x,y)=ax^2+by^2+cxy+dx+ey+f\ , \tag{5}$$

where Equation (5) is the sum of an ellipsoid given by the first three terms ax^2+by^2+cxy and a dipping plane given by the last three terms $dx+ey+f$. Al-Dossary and Marfurt (2006) and Russell (2012) linked these parameters with the inline and crossline instantaneous dips *p(t)* and *q(t)* as below:

$$a=\left(\frac{\partial p(t)}{\partial x}\right),b=\left(\frac{\partial q(t)}{\partial y}\right),c=\left(\frac{\partial p(t)}{\partial y}+\frac{\partial q(t)}{\partial x}\right),d=p(t)\ and\ e=q(t). \tag{6}$$

Equations (5) and (6) lead to the following quadratic relationship:

$$z(x,y)=\frac{1}{2}\left(\frac{\partial p(t)}{\partial x}\right)x^2+\frac{1}{2}\left(\frac{\partial q(t)}{\partial y}\right)y^2+\frac{1}{2}\left(\frac{\partial p(t)}{\partial y}+\frac{\partial q(t)}{\partial x}\right)xy+p(t)x+q(t)y+f\ . \tag{7}$$

Thus, various instantaneous curvature attributes can be created by combining the calculated parameters in equation (6), for example maximum instantaneous curvature and minimum instantaneous curvature.

Instantaneous curvature is derived from instantaneous dips, and instantaneous dips are directly controlled by the instantaneous frequency and instantaneous wavenumbers. However, singularities from instantaneous frequency and instantaneous wavenumbers are common, and the direct calculation can lead these attributes to fluctuate rapidly with spatial and temporal location (Han and van der Baan, 2013). Barnes (2000) proposed the weighted strategy for improving the performance on these basic instantaneous attributes, which utilizes the first-moment formula of a seismic trace (Mandel, 1974),

$$\frac{\int_{-\infty}^{\infty} f(\mathrm{t})\,\mathrm{R}(\mathrm{t})^2\,dt}{\int_{-\infty}^{\infty} \mathrm{R}(\mathrm{t})^2\,dt}=\frac{\int_0^{\infty} f(\omega)\,\mathrm{A}(\omega)^2\,df}{\int_0^{\infty} \mathrm{A}(\omega)^2\,df}=f_{weighted} \tag{8}$$

where $f(t)$ is instantaneous frequency, $R(t)$ is instantaneous amplitude, $f(\omega)$ and $\mathrm{A}(\omega)$ are frequency and amplitude in the Fourier domain, and $f_{weighted}$ is the weighted instantaneous frequency of the trace. The first and second expressions in Equation (8) state that the seismic trace in time domain is equivalent to its frequency domain response.

First-moment formula can be extended to a windowed analytic seismic trace. The instantaneous frequency weighted by the instantaneous power scaled by a window squared equals the average Fourier spectral frequency calculated in that window. We compute the weighted instantaneous frequency in a running window on a trace, which equals the centre frequency of the spectrogram. The window length determines the temporal resolution. Equation (8) also can be extended to compute the weighted inline and crossline instantaneous wavenumbers.

Examples

In this section, we compare the direct calculation with weighted strategy step by step, from instantaneous frequency and instantaneous dip to the final instantaneous curvature. The improvement found by using the weighted strategy is obvious and continues in every step.

First, we use a single seismic trace to compare the instantaneous frequency and weighted instantaneous frequency. The blue line in Figure 1 is instantaneous frequency calculated by equation (2). The direct calculation leads the instantaneous frequency to fluctuate rapidly, and introduces large numbers of nonmeaningful spikes and nonphysical negative values. The weighted instantaneous frequency (red line in Figure 1) is computed using a five sample running window in the time domain using Equation (8). It captures the main variation of the frequency change and reduces the spikes effectively. By using the new approach the spectrum now is always positive.

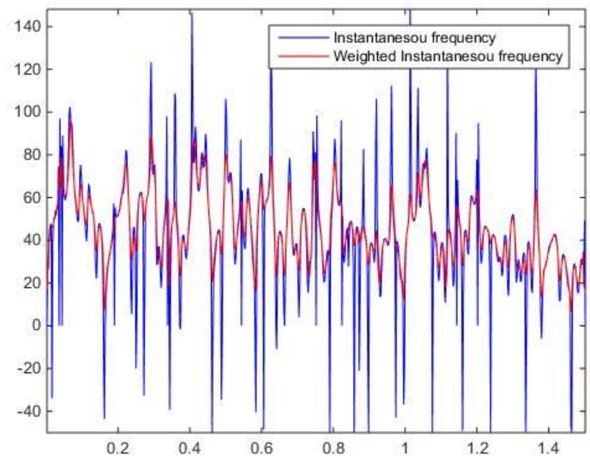

Figure 1: Blue line is instantaneous frequency, red line is weighted instantaneous frequency computed by 5 sample running window. The weighted instantaneous freuqency captures the variation of instantaneous frequency, and reduces the spikes.

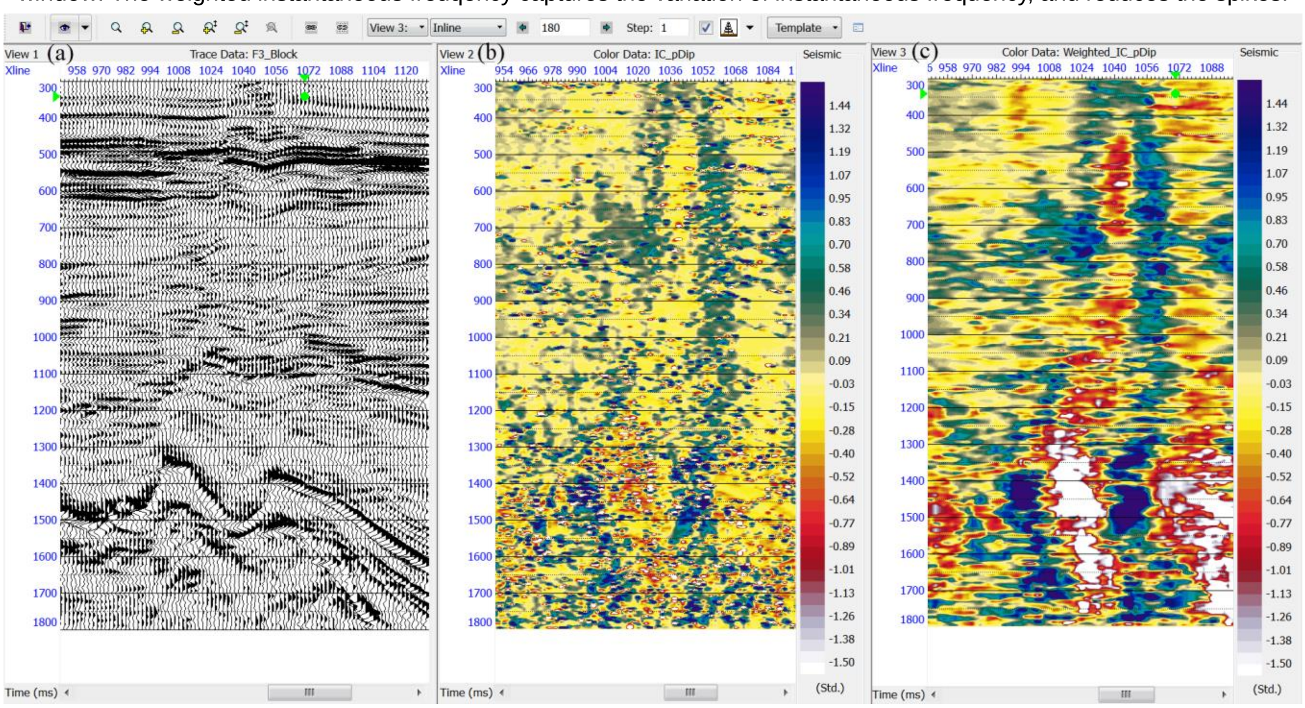

Figure 2: (a) is seismic data from North Sea, (b) is inline instantaneous dip and (c) is inline weighted instantaneous dip.

Next, Figure 2 shows a comparison of the inline instantaneous dip and weighted inline instantaneous dip (equation (4)). This attribute is the intermidiate step for calculating instantaneous curvatures. Figure 2(a) is a seismic dataset from the North Sea (the F3 Block from the Netherlands North Sea), where the complex geological structures involve faults, an anticline and a syncline. The interpretor expects a clear dip variation from these geological structures. Figure 2(b) shows the inline instantaneous dip, which is calculated as the ratio of instantaneous wavenumber to instantaneous frequency. Although the instantaneous dip roughly reflects the main trend of dip variation, the large number of spikes dominate the image, thus veil the detailed dip information. In comparison, the inline weighted instantaneous dip (Figure 2(c)) provides a better result. It is calculated as the ratio of weighted instantaneous wavenumber to weighted instantaneous frequency, which effectively reduces the spikes and unveil the detailed information, especially in the deep part. The dip variation (Figure 2(c)) matches the original seismic data, and highlights the geological structures.

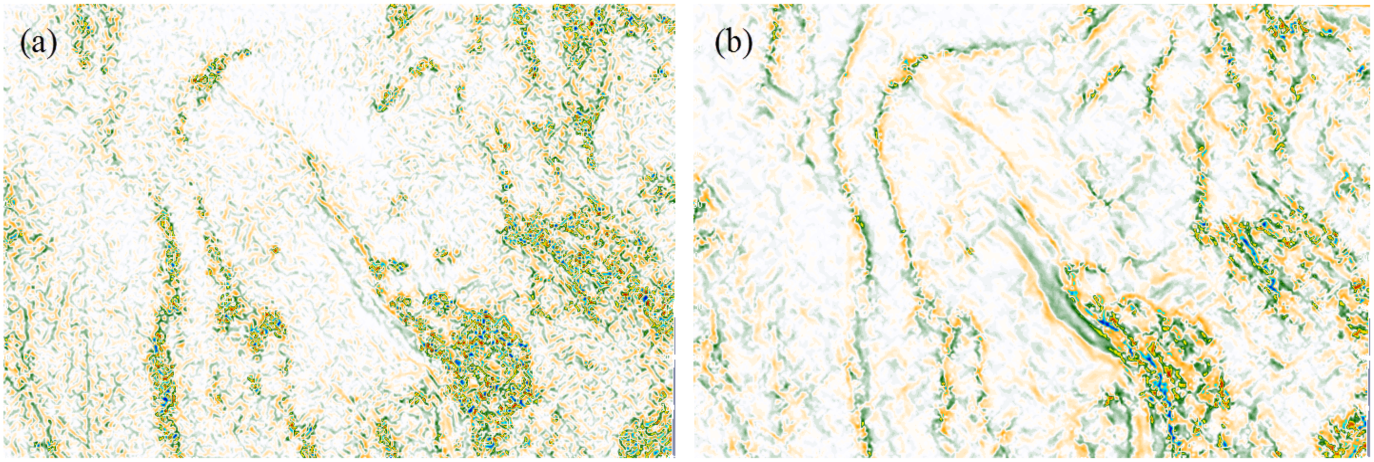

Figure 3: (a) is maximum instantaneous curvature, (b) is weighted maximum instantaneous curvature on a time slice. Weighted strategy enhances the continuous feature of image, and therefore highlight the fault information.

Lastly, Figure 3 compares the maximum instantaneous curvature and weighted maximum instantaneous curvature on a time slice from the North Sea dataset. The maximum instantaneous curvature, which is shown in Figure 3(a), seems noisier and does not highlight the faults clearly. This is mainly because of the large number of spikes introduced in the instantaneous dip calculation (see Figure 2(b)). The weighted maximum instantaneous curvature (Figure 3(b)) is a smoother image with more continuous fault features. The weighted strategy reduces the spikes during the calculation enhancing the interpretability of the instantaneous curvature.

Conclusions

The weighted strategy is an effective approach to improve the instantaneous curvature calculations. As discussed, this process is an adaptive process with a firm mathmatical foundation. Through smoothing the instantaneous frequency and instantaneous wavenumbers, this weighted strategy enhances the interpretability of instantaneous curvature. The real data examples shown here illustrate the improved performance of the method.

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