geoconvention 2018
Calgary • Canada • May 7-11

Viscoacoustic VTI and TTI wave equations and their application for anisotropic reverse time migration: Constant-Q approximation

Ali Fathalian and Kris Innanen
Department of Geoscience, University of Calgary

Summary

We investigated the simulation of viscoacoustic wave propagation and reverse time migration (RTM) in transversely isotropic (TI) media, vertical TI (VTI) and tilted TI (TTI), within approximating constant-Q. Reverse time migration (RTM) is based on two-way wave equation and has advantages over than other imaging methods. Such wave propagation can be modeled with a finite difference scheme by introducing a series of standard linear solid (SLS) mechanisms, and it can be carried out within a computationally tractable region by making use of perfectly-matched layer (PML) boundary conditions. The viscoacoustic wave equation for VTI and TTI mediums have been derived using the wave equation in anisotropic media by setting shear wave velocity as zero. Using the TI approximation and ignoring all spatial derivatives of the anisotropic symmetry axis direction leads to instabilities in some area of the model with the rapid variations in the symmetry axis direction. A solution to this problem is proposed that involves using a selective anisotropic parameter equating in the model to reduce the difference of Thompson parameters in areas of rapid changes in the symmetry axes. To eliminate the high-frequency instability problem, we applied the regularization operator and built a stable viscoacoustic wave propagator in TI media. After correcting for the effects of anisotropy and viscosity, the anisotropy RTM image in attenuation media with high resolution is obtained and compared with the isotropic RTM image.

Introduction

There are two ways to consider the anisotropic medium, the pseudo-acoustic wave equation and the pure acoustic wave equation. Alkhalifah (1998, 2000) derived the pseudo-acoustic wave equation from the dispersion relation by setting the shear-wave velocity along the anisotropy symmetry axis to be zero. To reduce the computational time, based on pseudo-acoustic approximation, Zhou et al. (2006b) and Duveneck et al. (2008) developed and simplified the pseudo-acoustic wave equation into two coupled second- order partial differential equations to account for VTI media. Although the VTI wave equation is used to image structures which have similar properties with a VTI media (Crampin, 1984), but may not be satisfied in anisotropic dipping layers. The TTI equations have been derived from VTI equations by assuming the symmetry axis is non-vertical and locally variable (Fletcher et al., 2008; Zhang and Zhang, 2008). The TI wave equations with the zero value of SV wave's velocity on the axis symmetry can't remove the effect of the residual shear wave, so the instability occurs. Fletcher et al. (2009) proposed the equations by adding non-zero S-wave velocity terms to solve the problem. To stabilize wave propagation and reduce shear wave artifacts the parameters models of anisotropy can be smoothing before numerical simulation, and setting $\varepsilon = \delta$ in the regions around source and areas with the high symmetry axis gradient (Zhang and Zhang, 2008; Yoon et al., 2010). However, to investigate the RTM images in anisotropic viscoelastic medium, generally, the focus is on the anisotropy or viscosity. In this work, we focus on both anisotropy and viscosity to obtain the accurate RTM images. In this paper, we investigate the simulation of wave propagation in anisotropic viscoacoustic medium within approximating constant-Q using a split-field PML equation in the time domain, and derive a viscoacoustic wave equation of VTI and TTI mediums.

Viscoacoustic VTI Media Equation

In 2D case, the first order acoustic wave equations of VTI media is expressed as follow (Duveneck et al., 2008)

$$\sigma_H = \rho V_P^2 \left[(1+2\varepsilon)\varepsilon_{11} + \sqrt{1+2\delta}\,\varepsilon_{33} \right],$$
$$\sigma_V = \rho V_P^2 \left[\sqrt{1+2\delta}\,\varepsilon_{11} + \varepsilon_{33} \right], \quad (1)$$

where σ_H and σ_V represent the horizontal and vertical stress components respectively. ε and δ are Thomsen parameters, and ε_{ij} are the diagonal elements of strain tensor. The first order differential equations of acoustic VTI media can be obtained by taking a time derivative of stress-strain relationship given in eq.1 and combining the result with the equations of motion. In order to introduce the PML boundary for such viscoacoustic waves, the first-order linear differential equations are modified using the complex coordinate stretching approach in frequency domain and transformed back to the time domain.

$$\partial_t u_x = \frac{1}{\rho}\partial_x \sigma_H - d(x)u_x,$$
$$\partial_t u_z = \frac{1}{\rho}\partial_z \sigma_V - d(z)u_z,$$
$$\partial_t \sigma_H = \rho V_P^2 \left[(1+2\varepsilon)\left[\left(\frac{\tau_\varepsilon}{\tau_\sigma}\right)\left[\partial_x(u_x + d(z)u_x^{(1)})\right] - r_H \right] + \sqrt{1+2\delta}\left[\partial_z(u_z + d(x)u_z^{(1)})\right] \right] - (d(x)+d(z))\sigma_H - d(x)d(z)\sigma_H^{(1)},$$
$$\partial_t \sigma_V = \rho V_P^2 \left[\sqrt{1+2\delta}\left[\partial_x(u_x + d(z)u_x^{(1)})\right] + \left(\frac{\tau_\varepsilon}{\tau_\sigma}\right)\left[\partial_z(u_z + d(x)u_z^{(1)})\right] - r_V \right] - (d(x)+d(z))\sigma_V - d(x)d(z)\sigma_V^{(1)}, \quad (2)$$

where the auxiliary variables $u_x^{(1)}$, $u_z^{(1)}$, $\sigma_H^{(1)}$, and $\sigma_V^{(1)}$ are the time-integrated components for velocity, pressure and memory variable fields. To avoid the high-frequency effect on reverse time propagation, the regularization must be considered. We construct a regularized equation based on equation 2 in viscoacostic VTI media

$$\partial_t \sigma_H = \rho V_P^2 \left[(1+2\varepsilon)\left[\left(\frac{\tau_\varepsilon}{\tau_\sigma}\right)\left[\partial_x(u_x + d(z)u_x^{(1)})\right] - r_H \right] + \sqrt{1+2\delta}\left[\partial_z(u_z + d(x)u_z^{(1)})\right] \right] - \left[\epsilon\rho V_P\sqrt{1+2\varepsilon}[\partial_t(u_x + d(z)u_x^{(1)})]\right] - (d(x)+d(z))\sigma_H - d(x)d(z)\sigma_H^{(1)},$$
$$\partial_t \sigma_V = \rho V_P^2 \left[\sqrt{1+2\delta}\left[\partial_x(u_x + d(z)u_x^{(1)})\right] + \left(\frac{\tau_\varepsilon}{\tau_\sigma}\right)\left[\partial_z(u_z + d(x)u_z^{(1)})\right] - r_V \right] - \left[\epsilon\rho V_P[\partial_t(u_z + d(x)u_z^{(1)})]\right] - (d(x)+d(z))\sigma_V - d(x)d(z)\sigma_V^{(1)}. \quad (3)$$

where ϵ is a small positive regularization parameter. The shear wave artifacts generated in an elliptic media ($\varepsilon \neq \delta$), and they can suppress at the source by design a small smoothly tapered circular region with $\varepsilon = \delta$ around the source. However, the shear waves that generated by source don't consider as the problem when the source located in the isotropic part of the model.

Viscoacoustic TTI Media Equation

The one simplest and most practical approximations for anisotropic media is VTI medium, while is only valid for simple geologic formations. In anticline structures and thrust sheets where sediments are steeply dipping, the VTI medium approximation is not useful because of non-vertical symmetry axis of the medium. Therefore, to consider such as areas is better to use the tilted transversely isotropic (TTI) media. The one way to calculate the TTI equations is to locally rotate the coordinate system of VTI medium. The rotation matrix as function of the polar angle and azimuth angle is defined as

$$\mathbf{R} = \begin{pmatrix} \cos\theta\cos\varphi & \cos\theta\sin\varphi & -\sin\theta \\ -sin\varphi & cos\varphi & 0 \\ \sin\theta\cos\varphi & \sin\theta\sin\varphi & \cos\theta \end{pmatrix} \tag{4}$$

where θ represent the tilt angle and φ represent the azimuth of tilt for TTI symmetry axis.

Synthetic RTM Examples

To verify the accuracy of the viscoacoustic wave equation in TTI media, the 2D data set modeled in an inhomogeneous TTI velocity model (Duveneck and Bakker, 2011) is tested (Figure 1a). There are some dipping anisotropic layers in velocity model that terminating against the salt body. The three anisotropy distributions are shown in Figure 1. The rapid variation of the tilt angle affected the TTI RTM images. The model grid dimensions are 700$\times$1200, and the grid size is 6.25m$\times$6.25m. The sampling interval is 0.8 ms, and the recording length is 6 s. We use as the source a zero-phase Ricker wavelet with a center frequency of 10 Hz. To remove the effect of S-wave that generated at the source the source is located in the isotropic part of the model, i.e., $\varepsilon = \delta$. Using the TI approximation and ignoring all spatial derivatives of the anisotropic symmetry axis direction leads to instabilities in some area of the model with the rapid variations in the symmetry axis direction (Duveneck and Bakker, 2011). The instability appears at the later time and can be solved with the smoothing of the model. Although smoothing the model would help in some models, but it is not useful for any models. However, Yoon et al. (2010) show that some spots of high symmetry axis gradient produce large instabilities and blows up the amplitudes of the wavefield. We can pick up the high gradient points by filtering the gradient of theta with a given threshold. In area with instability, the anisotropy can be taken off around the selected high gradient points which set $\varepsilon = \delta$ to suppress artifacts from the source point in an anisotropic medium.

Figure 2 shows the reverse-time migration results obtained using the split-PML field Viscoacoustic TTI equations (Figure 2b) and, for comparison, using Viscoacoustic VTI wave equations (Figure 2a). For the VTI migration, the θ set to zero and other parameters are same with the TTI model. The imaging of dipping layers such as salt flank is affected and mispositioned in the isotropic and VTI RTM images because of the presence of anisotropy. TTI RTM for both the salt body and dipping layers that terminating against the salt body is more accurate than VTI RTM.

Conclusions

Time-domain approximate constant-Q wave propagation involving a series of standard linear solid (SLS) mechanisms is investigated. The wave equations have been extended from isotropic media to transversely isotropic (TI) media including VTI and TTI media. For imaging application, the stability condition and the artifacts of shear wave triplications have been discussed. Results show that the stable anisotropic reverse time migration is accessible by taking off anisotropy around the selected high gradient points in areas of rapid changes in the symmetry axes. The TTI RTM image is more accurate than the VTI RTM and isotropic RTM images especially in the areas with strong variations of dip angle along the tilted symmetry-axis. Furthermore, the application of anisotropic equations to 3D RTM and field data and reduce computational time remains a challenge.

Acknowledgements

The authors thank the sponsors of CREWES for continued support. This work was funded by Maitacs, CREWES industrial sponsors and NSERC (Natural Science and Engineering Research Council of Canada) through the grant CRDPJ 461179-13.

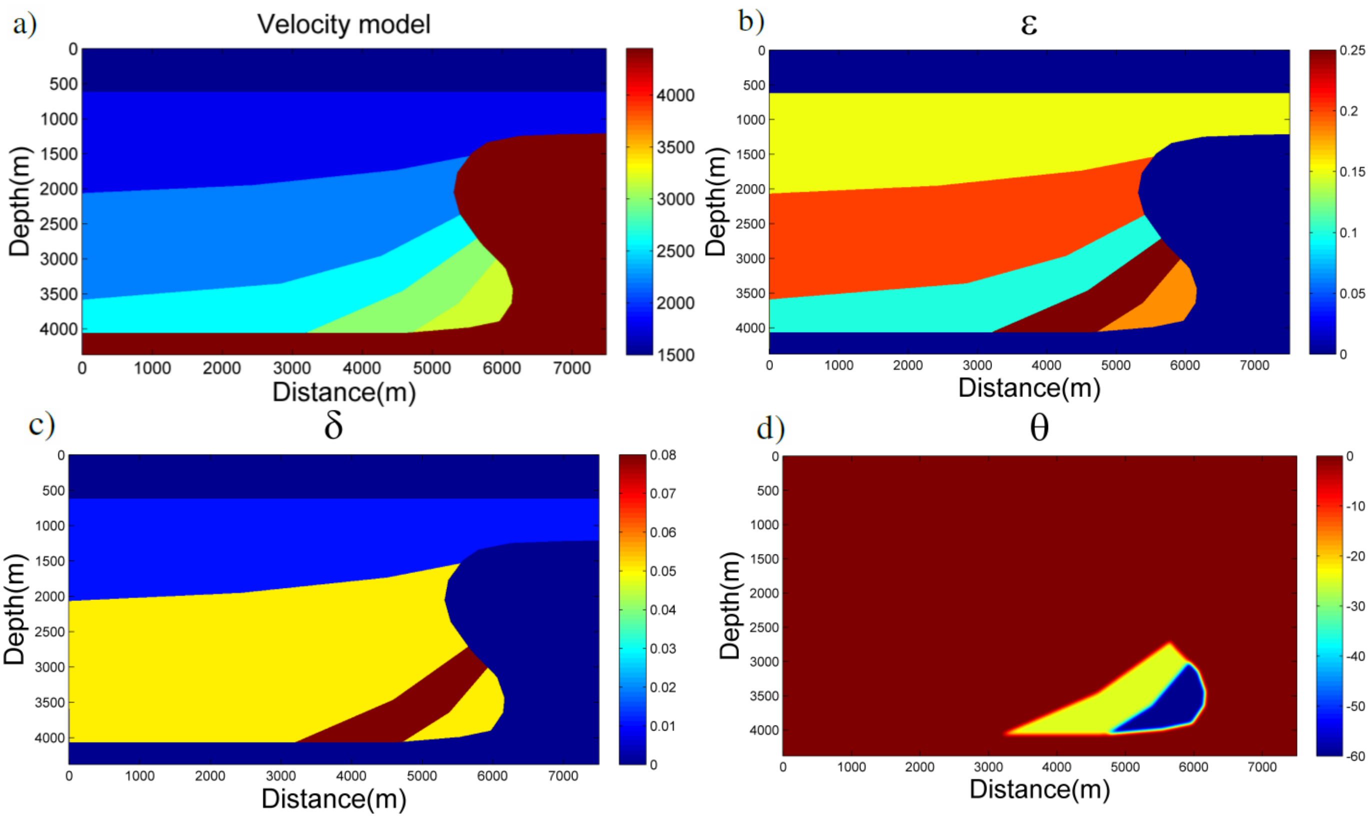

FIG. 1: Transversely isotropic velocity model (a), Thomsen's ε model (b), Thomsen's δ model (c) and Tilted dip angle along the tilted symmetry-axis (d).

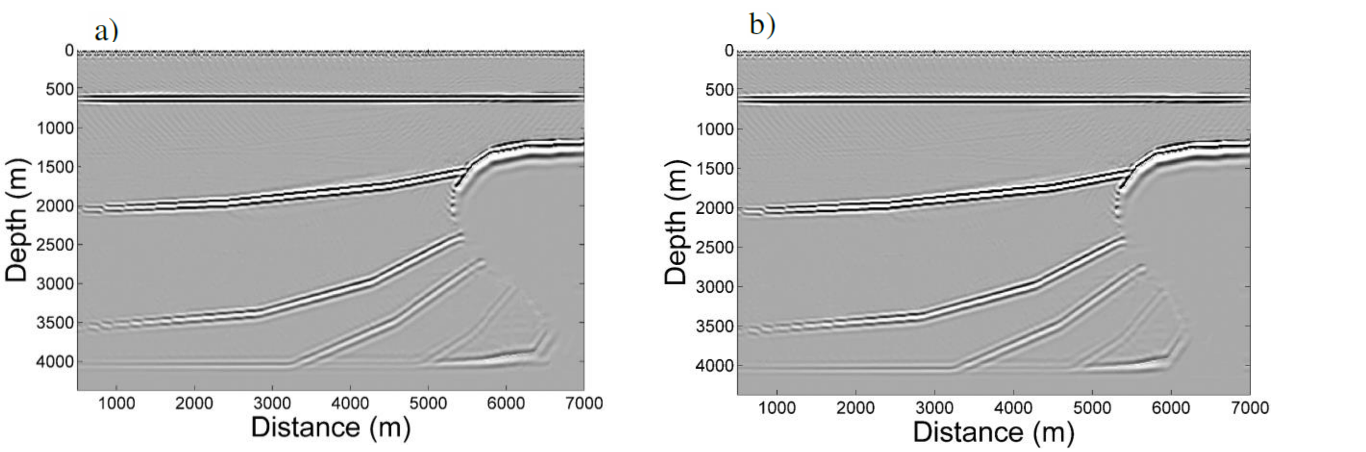

FIG. 2: Anisotropic reverse-time migration in attenuation medium. a) Anisotropic (VTI) Isotropic revers-time migration, and b) Anisotropic (TTI) Isotropic revers-time migration

References

Alkhalifah, T., 1998, Acoustic approximations for processing in transversely isotropic media: Geophysics, 63, 623–631.
Alkhalifah, T., 2000, An acoustic wave equation for anisotropic media: Geophysics, 67, 1304–1325.
Crampin, S., 1984, Anisotropy in exploration seismics, First Break, volume = 2, pages = 19–21.
Duveneck, E., and Bakker, P. M., 2011, Stable P-wave modeling for reverse-time migration in tilted TI media: Geophysics, 76, S65–S75.
Duveneck, E., Milcik, P., Bakker, P. M., and Perkins, C., 2008, Acoustic VTI wave equations and their application for anisotropic reverse-time migration: SEG Technical Program Expanded Abstracts, 27, 2186–2190.
Fletcher, R., Du, X., and Fowler, P. J., 2008, A new pseudo-acoustic wave equation for TI media: SEG Technical Program Expanded Abstracts, 27, 2082–2086.
Fletcher, R., Du, X., and Fowler, P. J., 2009, Reverse time migration in tilted transversely isotropic (TTI) media: Geophysics, 74, WCA179–WCA187.
Yoon, K., Suh, S., Ji, J., Cai, J., and Wang, B., 2010, Stability and speedup issues in TTI RTM implementation: SEG Technical Program Expanded Abstracts, 29, 3221–3225.
Zhang, H., and Zhang, Y., 2008, Reverse time migration in 3D heterogeneous TTI media: SEG Technical Program Expanded Abstracts,, 27, 2196–2200.
Zhou, H., Zhang, G., and Bloor, R., 2006b, An anisotropic acoustic wave equation for modeling in 2D TTI Media. In: Proceedings of the 76th Annual International Meeting, SEG, Expanded Abstracts 25, 194–198.