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A comparative study on sensitivity analysis of frequency and time domain full waveform inversion

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Abstract

The aim of this paper is to compare time domain and frequency domain full waveform inversion (FWI). We compare these inversions against random coherent and incoherent noises with different signal to noise ratios, source wavelet inaccuracy and existence of gap in the survey data. In all cases, we find that time domain full waveform inversion is more sensitive to imperfections in the data and its result shows more deterioration than that of frequency domain full waveform inversion. All tests have been done on 2D synthetic data with 2D full waveform inversion code

Introduction

Full waveform inversion is a method of choice for solving the inverse problem using whole wavefield dataset. For the first time, Tarantola (1984) proposed the FWI formulation in time domain. Pratt (1990) developed and successfully applied to 2D BP synthetic data a frequency domain full waveform inversion. Since then full waveform inversion has been developed in either frequency domain or time domain (Bunks et al., 1995; Sirgue and Pratt, 2004; Brenders and Pratt, 2007b). In two dimensions, solving the sparse matrix arising from frequency domain FWI is not so expensive. Also, sparse solver approach has ability of inverting multiple shots simultaneously. However, in three dimensions the matrix inversion is time and memory- consuming. On the other hand, time domain has the advantage of applying time windowing on data for choosing selected arrivals in data as a function of time. In addition, time-domain FWI is more suitable for implementation on parallel processing computers such as GPGPUs.

Although there might be quite a few references on either time domain or frequency domain FWI, there hasn't been many studies on comparing these two methods when there is imperfection in the data. The aim of this study is to test and compare sensitivity of FWI in time and in frequency domain. For this propose we impose the data imperfections such as coherent noise, source wavelet errors, discrete and continues gaps in survey on synthetic data and present their impact on FWI results for both time domain and frequency domain inversion.

We decide not to add cycle skipping in the comparison study since there has been many studies of cycle skipping for both of these methods. Also for fair comparison we need fairly good inversion results for noise free inversions. This is obtained when we have lowest frequencies in the data.

In both frequency and time FWI, we use multi-scale FWI method and the misfit function was minimized via limited-memory Broyden-Fletcher-Goldfarb-Shanno (LBFGS) method. In frequency domain FWI we use perfectly matched layer (PML) for absorbing boundary condition and sponge is applied for time-domain FWI.

Error estimation

The aim of this study is to demonstrate how the data imperfections affects FWI in time and frequency domain. Since time and frequency domain FWI each gives different inversion result for noise free dataset shown in Figure (1). When we insert imperfection in the data, it will be difficult to measure the deterioration of the result if we compute the relative error base on exact model. Instead we measure the deterioration from noise free inverted model rather than exact model. Here is the step by step process,

1) Generate noise free synthetic data set and FWI on this dataset in time and frequency domain, run synthetic from the inverted model and call it time or frequency domain reference ynthetic dataset (R_t, R_f).
2) Add imperfection to time and frequency domain dataset, run inversion on imperfect data to find the inverted model, run synthetic code on inverted model and call it noisy synthetic dataset $(S_t, S_f)$and calculate error.

The error is calculated in percentage as follow,

$$Error_{t,f} = \frac{\|R_{t,f} - S_{t,f}\|_1}{\|R_{t,f}\|_1} \times 100 \quad (1)$$

Model and inversion parameters

we generate sensitivity analysis results by using 2D acoustic FWI on a subset of the North Sea model shown in Figure (1a). The geometry subset is 1600m long and 2000m deep with 8-m grid spacing. Initial velocity model was created as a 1D velocity model by averaging over every row of the true velocity model that is shown in Figure (1b). Synthetic data is generated with 32 shots and 189 receivers. Shots are located at lateral position between 48 and 1550 m with a 48-m shot spacing and lateral positions of receivers are between 40 and 1550 m and 8-m receiver spacing. Depth of all shots and receivers is 8m. The source time function is a Ricker wavelet with a 15-Hz central frequency and total recording time and time sampling are 3s and 0.001s respectively. We conducted the inversion process using multi-scale FWI, frequency band is from 0.33Hz to 20Hz, in 0.33Hz intervals that decomposed into 5 sub-band, with 15 iterations at each sub-band. The 0.33Hz frequency increment for frequency domain inversion corresponds to 3 seconds recording time for time domain FWI.

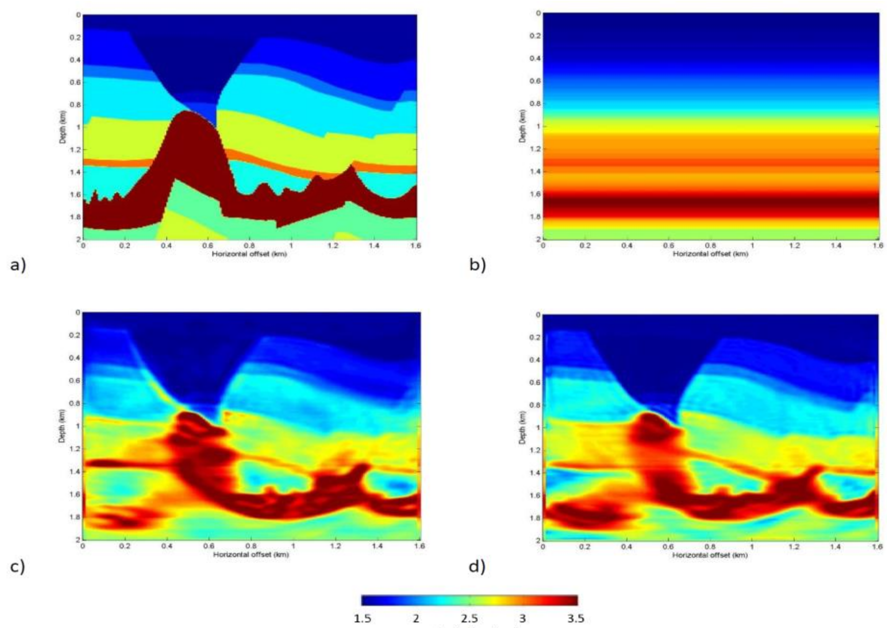

Figure 1. *(a) True velocity model. (b) Initial velocity model. (c) noise free FWI result for frequency domain. (d) noise free FWI result for time domain.*

Coherent noise

We can't deny the presence of noise in seismic data, wind motion or cable vibrations can generate random noise similar to incoherent noise. Some of noise sources create more coherent energy on the data and can be misinterpreted as true signal. Therefore, the sensitivity of FWI to coherent and incoherent noise have been examined both in frequency and time domain. To create coherent noise, we first generated a random Gaussian noise, then apply a smoothing filter on it with σ a control parameter defined as,

$$\sigma = \frac{L}{10S_L} \qquad (2)$$

In equation (2), L is the length of the model and S_L is the smoothing distance.

Gap in survey

In practical FWI, there are several challenges for collecting data, due to natural phenomena and equipment limitations. We assume there are gaps in survey (i.e., regions without any sources and receivers) and investigate their effect on inversion results. We impose two types of gap formations to create syntactic data. First gap is at the center of the model and second gap is two equally separated gaps with centers are located in one-third and two-third of the model. The length of total gap is equal for both types of gap, for example a 100m gap of first type could be comparable with 50-50 m gap of second type. Figure (2) shows the receiver data for both types of gaps with 400 m width.

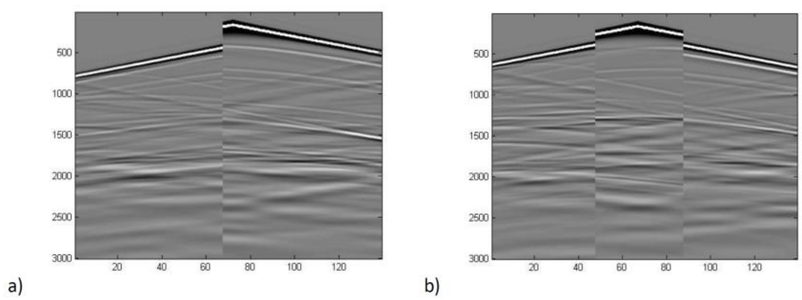

Figure 2. *receivers data for (a) 400 meter continous gap. (b) 400 meter discrete gap.*

Source wavelet error

We also examine the effect of phase and amplitude errors on source wavelet for different FWI methods. Here we test four different shifted phases (0,10,20,30) and for each phase we use two amplitudes (0.8, 1).

Discussion and Conclusion

In this study, we have designed different tests to analyze the effect of various errors on time and frequency domain full waveform inversion. Figure (3) shows the value of errors as described in equation 1. We can see from figure (3) that coherent noise has great destructive impact on time and frequency domain FWI, especially when SNR is low. The gap tests demonstrate that continuous gaps cause more deterioration into FWI results in both time and frequency domain. The source wavelet results show that both time-domain FWI and frequency domain FWI are sensitive to errors in amplitude and phase of source wavelet. The result of all tests shows more deterioration in time domain FWI than that of frequency domain. In other words, for every test which we conducted, we observed that frequency domain FWI is more preferable than time domain FWI in term of resistance to data imperfection.

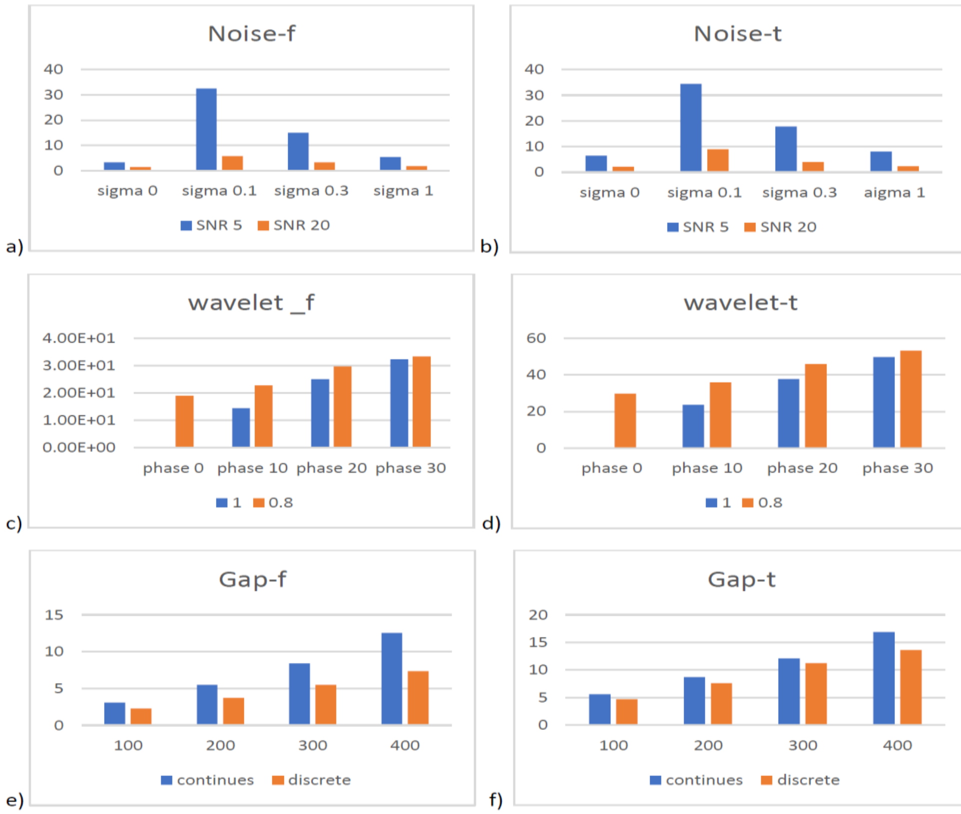

Figure 3. *Error calculated from equation (1), (a) frequency domain FWI result with noise error, (b) time domain FWI result with noise error, (c) frequency domain FWI result with source wavelet error, (d) time domain FWI result with source wavelet error, (e) frequency domain FWI result with gap, (f) time domain FWI result with gap.*

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