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An NPML Boundary Conditions for Second-Order Viscoacoustic Wave Equation in Time Domain

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Abstract

In the simulation of wave propagation phenomenon truncating the grid will lead to unacceptable reflection artifacts from boundary. One remedy is to extend simulation region by Perfectly Matched Layers (PML) that contains a special absorbing medium. In the PML region, a method of choice would be to solve Nearly Perfectly Matched Layer (NPML) absorbing boundary conditions which is contained coupled auxiliary differential equations. In this research NPML absorbing boundary conditions are derived for the second-order viscoacoustic wave equations for creating synthetic seismic. Numerical results of NPML are investigated on a constant velocity model and it is shown satisfactory damping of the reflected wave from boundaries for different value of attenuation factors.

Introduction

In the simulation of wave propagation in unbounded domains, Absorbing boundary conditions (ABCs) are used to absorb outgoing waves and avoid the reflections from the outer boundaries. In numerical implementation of wave propagation in acoustic and elastic media by finite difference method, various types of ABCs are used. Clayton and Engquist (Clayton and Engquist, 1977) introduced a type of ABCs based on a paraxial approximation of the elastic wave equation. Application of this type of ABCs performs unsatisfactory results in some cases and the artificial reflections are adsorbed imperfectly at the edges of the computational domain. Berenger (Berenger, 1994) introduced the perfectly matched layers (PML) for simulation of electromagnetic wave propagation. Instead of finding an absorbing boundary condition, Berenger found absorbing boundary layers. PML are layers of artificial absorbing material that control the reflection of waves from boundaries of simulation domain. The theoretical implementation of PML is based on stretching the coordinate variable from real to complex in absorbing region. This causes the energy of incident waves to exponentially decay when entering PML region, thus the reflections significantly attenuate. The PML is considered because of its highly effective, excellent absorption over a wide range of angles and insensitivity to frequency. The PML has been developed for elasticity (Chew and Liu, 1996), poroelasticity (Zeng and Liu, 2001) and anisotropic media (Becache and Joly, 2001). McGarry and Moghaddam (McGarry and Moghaddam, 2009) derived Nearly Perfectly Matched Layer (NPML) absorbing boundary conditions for the second-order scalar acoustic wave equation and for VTI pseudo-acoustic wave equations. In this work, NPML absorbing boundary conditions are derived for the second-order viscoacoustic wave equations and numerical results are presented using finite difference method. In viscoacoustic media, under constant quality factor Q (i.e., linear attenuation with frequency), following set of second-order viscoacoustic isotropic wave equations demonstrates the dispersion and attenuation effects of wave propagation (Robertsson et al., 1994),

$$\frac{\partial P}{\partial t} = -\frac{\partial[\kappa(1+\tau\exp(-\frac{\tau}{\tau_\sigma}))H(t)]}{\partial t} * \nabla\cdot v + f,$$
$$\frac{\partial v}{\partial t} = -\frac{1}{\rho}\nabla P, \tag{1}$$

where $\rho = \rho(x)$ is the density at the position x, $\kappa = \kappa(x)$ is the bulk modulus, $\mathrm{v} = \mathrm{v}(\mathrm{x}, \mathrm{t})$ is the particle velocity vector, $\mathrm{f} = \mathrm{f}(\mathrm{x}_s, \mathrm{t})$ is the source term at x_s, and H is the Heaviside function. The symbol $*$ stands for a time convolution. Here, $\tau = \tau_\varepsilon / \tau_\sigma - 1$ determines the magnitude of Q, where

$$\tau_\sigma = \frac{\sqrt{1+1/Q^2} - 1/Q}{\omega_0}, \text{ and } \tau_\varepsilon = \frac{1}{\omega_0{}^2\tau_\sigma},$$

are stress and strain relaxation times, and ω_0 is the central frequency of source wavelet. Above equation can be reformulated in the following form (Cheng et al., 2015)

$$\frac{1}{v^2}\frac{\partial^2 P}{\partial t^2} = (1+\tau)\rho\nabla\cdot(\frac{1}{\rho}\nabla P) - r + f,$$
$$\frac{\partial r}{\partial t} = \frac{\tau}{\tau_\sigma}\rho\nabla\cdot(\frac{1}{\rho}\nabla P) - \frac{1}{\tau_\sigma}r. \tag{2}$$

Derivation of NPML boundary conditions

Here, we show the derivation without the source term. Using the following complex coordinate stretching approach, (1) without the source term will be modified in the boundary region. In this regard, for the contraction of the field variables, we define

$$\frac{\bar{P}^\alpha}{\partial t} = -\frac{P}{\gamma_\alpha}, \text{ and } \frac{\bar{v}^\alpha}{\partial t} = -\frac{v}{\gamma_\alpha}, \tag{3}$$

where a bar denotes a field contracted in the direction α ($\alpha \in \{x, y, z\}$) and γ_α is the complex frequency-dependent contraction factor, given by

$$\gamma_\alpha = 1 - \frac{j\sigma_\alpha}{\omega},$$

where $j^2 = -1$, and ω is angular frequency and σ_α is a damping factor which controls attenuation of the acoustic wave within the PML. Replacing the original fields with the contracted fields in spatial derivative terms of the original equations we obtain the following equations for implementation within the PML

$$\frac{\partial P}{\partial t} = -\frac{\partial[\kappa(1+\tau\exp(-\frac{\tau}{\tau_\sigma}))H(t)]}{\partial t} * \left(\frac{\partial \bar{v}_x^x}{\partial x} + \frac{\partial \bar{v}_y^y}{\partial y} + \frac{\partial \bar{v}_z^z}{\partial z}\right), \tag{4a}$$

$$\frac{\partial \bar{P}^\alpha}{\partial \alpha} + \rho\frac{\partial v_\alpha}{\partial t} = 0. \tag{4b}$$

Transformation of equations (3) from the frequency to the time domain gives

$$\frac{\partial \bar{P}^\alpha}{\partial t} + \sigma_\alpha \bar{P}^\alpha = \frac{\partial P}{\partial t}, \tag{5}$$

$$\frac{\partial \bar{v}_\alpha^\alpha}{\partial \alpha} + \sigma_\alpha \bar{v}_\alpha^\alpha = \frac{\partial v_\alpha}{\partial t}. \tag{6}$$

Using (4b), we can rewrite (6) in the following form

$$\frac{\partial \bar{v}_\alpha^\alpha}{\partial \alpha} + \sigma_\alpha \bar{v}_\alpha^\alpha = -\frac{1}{\rho}\frac{\partial \bar{P}_\alpha}{\partial \alpha}. \tag{7}$$

Differentiating (4a) with respect to time, substituting from (6) and (4b), and using the bulk modulus $\kappa(x) = \rho(x)v(x)^2$ gives

$$\frac{1}{v^2}\frac{\partial^2 P}{\partial t^2} = -\frac{\partial G}{\partial t} * \rho\left(\nabla\cdot(\frac{1}{\rho}\nabla\bar{P}) + \frac{\partial}{\partial x}(\sigma_x \bar{v}_x^x) + \frac{\partial}{\partial y}(\sigma_y \bar{v}_y^y) + \frac{\partial}{\partial z}(\sigma_z \bar{v}_z^z)\right), \tag{8}$$

where $G = (1+\tau\exp(-\frac{\tau}{\tau_\sigma}))H(t)$. Computing $\partial G / \partial t$ and substituting into (8) results the following equation

$$\frac{1}{v^2}\frac{\partial^2 P}{\partial t^2} = (1+\tau)\rho\nabla\cdot(\frac{1}{\rho}\nabla\bar{P}) - \bar{r} + (1+\tau)\rho\left(\frac{\partial}{\partial x}(\sigma_x \bar{v}_x^x) + \frac{\partial}{\partial y}(\sigma_y \bar{v}_y^y) + \frac{\partial}{\partial z}(\sigma_z \bar{v}_z^z)\right) - \bar{R},$$

where $\bar{r}$ and $\bar{R}$ are memory variables given by

$$\bar{r} = \frac{\tau}{\tau_\sigma}[\exp(-\frac{\tau}{\tau_\sigma})H(t)] * \rho\nabla\cdot(\frac{1}{\rho}\nabla\bar{P}), \tag{9}$$

$$\bar{R} = \frac{\tau}{\tau_\sigma}[\exp(-\frac{\tau}{\tau_\sigma})H(t)] * \rho\left(\frac{\partial}{\partial x}(\sigma_x \bar{v}_x^x) + \frac{\partial}{\partial y}(\sigma_y \bar{v}_y^y) + \frac{\partial}{\partial z}(\sigma_z \bar{v}_z^z)\right), \tag{10}$$

Differentiating (9) and (10) in time, using the definitions of $\bar{r}$ and $\bar{R}$, assuming constant density and removing density altogether by defining $\bar{V}_\alpha^\alpha = \rho\bar{v}_\alpha^\alpha$ gives

$$\begin{aligned} \frac{1}{v^2}\frac{\partial^2 P}{\partial t^2} &= (1+\tau)\Delta\bar{P} - \bar{r} + (1+\tau)\left(\frac{\partial}{\partial x}(\sigma_x \bar{V}_x^x) + \frac{\partial}{\partial y}(\sigma_y \bar{V}_y^y) + \frac{\partial}{\partial z}(\sigma_z \bar{V}_z^z)\right) - \bar{R}, \\ \frac{\partial \bar{r}}{\partial t} &= \frac{\tau}{\tau_\sigma}\Delta\bar{P} - \frac{1}{\tau_\sigma}\bar{r}, \\ \frac{\partial \bar{R}}{\partial t} &= \frac{\tau}{\tau_\sigma}\left(\frac{\partial}{\partial x}(\sigma_x \bar{V}_x^x) + \frac{\partial}{\partial y}(\sigma_y \bar{V}_y^y) + \frac{\partial}{\partial z}(\sigma_z \bar{V}_z^z)\right) - \frac{1}{\tau_\sigma}\bar{R}, \end{aligned} \tag{11}$$

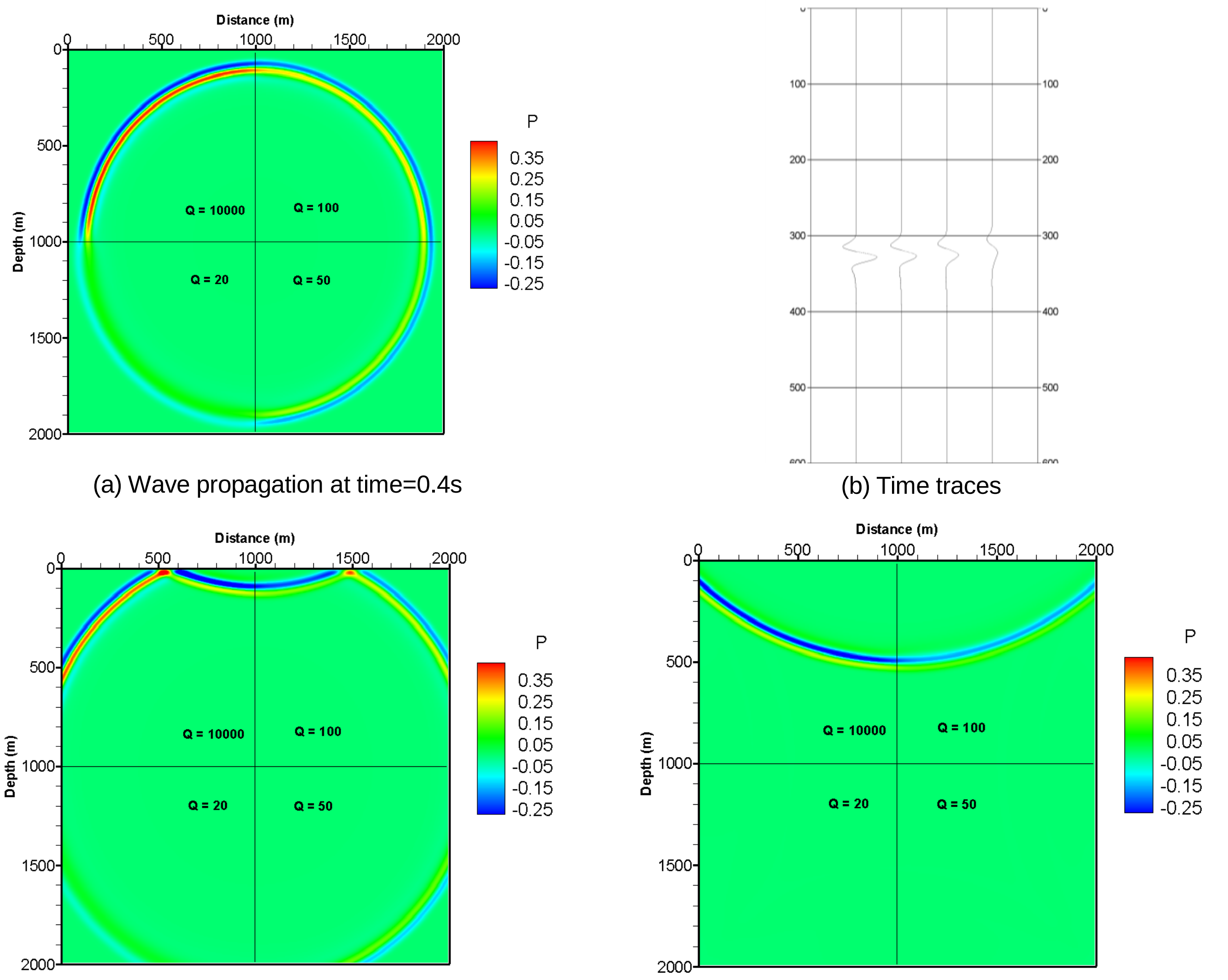
$$\frac{\partial \bar{V}_\alpha^\alpha}{\partial \alpha} + \sigma_\alpha \bar{V}_\alpha^\alpha = -\frac{\partial \bar{P}^\alpha}{\partial \alpha}. \tag{12}$$

Equations (11), (5) and (12) constitute the set of equations to be solved inside the PML. Within the model interior (2) is solved.

Numerical results

We consider the constant velocity model to investigate the accuracy of solution for the constant-Q wave equation by finite difference method. Model dimentions are 2km$\times$2km, and the source is located at point (1km, 1km), which is a Ricker wavelet with a center frequency of 30 Hz. Receivers are located at points (500m, 500m), (500m, 1500m), (1500m, 500m), and (1500m, 1500m). A PML absorbing boundary condition with 15 layers is applied to the sides and bottom of the model. Figures 1a and 1b show the effect of attenuation on amplitude and phase of a propagating seismic wave in a homogeneous medium with a background velocity of 2500 $\mathrm{m/s}$ and with value of density 1200 g/m^2 for different values of quality factor

(Q = 20 ,Q = 50, Q = 100 and Q = 10000). In Figure 1b time traces at receivers where Q factor decreases from left to right in millisecond unit are shown. The viscoacoustic wave field has the reduced amplitude and advanced phase in comparison with the viscoacoustic wave field with higher Q factor. Figures 1c and 1d show that incident wave to PML boundaries vanishes.



(a) Wave propagation at time=0.4s

(b) Time traces

(c) Wave propagation at time=0.48s

(d) Wave propagation at time=0.64s

Figure 1: Description of the separation of velocity dispersion (a) and amplitude reductions (b) at the same simulation specification. (c) and (d) incident wave to PML and free surface boundaries.

Conclusions

In this research Q model is used to present wave propagation in viscoacoustic media. In the PML region, NPML absorbing boundary conditions are derived for the second-order viscoacoustic wave equations for seismic synthetic modeling. Derived NPML equations closely resemble that of the original viscoacoustic equation. In the numerical results efficiency of derived NPML equations are shown. During propagation of the seismic wave, attenuation effect appears in the amplitude and phase of the wave.

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