Seismic investigation of lithological controls on effective stress

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Summary

The effective stress is an important concept for many subsurface engineering problems, where it represents the external load carried by the rock itself. It can be decomposed into two main components, which include the confining stresses and the pore pressure effects. The confining stresses can be estimated relatively easily with knowledge of the elastic properties. The pore pressure effects however, require knowledge of the Biot-Willis coefficient, which is a key component associated with the pore pressure term. To properly estimate the Biot-Willis coefficient, we require knowledge of the more fundamental building blocks of a rock mass that go beyond the elastic domain. These include the petrophysical properties from which components of the rock mass such as its pore space, frame and solid phase can be assembled from its constituent parts.

In this study, we estimate the effective stress from seismic data using a combination of elastic properties estimated from an AVO inversion and petrophysical parameters estimated from a rock physics inversion. We demonstrate thorough an example in the Montney in NE British Columbia, Canada.

Introduction

Knowledge of the effective stress field plays a vital role in many subsurface engineering problems from drilling to hydraulic fracturing in unconventional reservoirs. Assuming a relaxed basin with zero tectonic strain, the effective minimum horizontal stress is given by

\[
\sigma'_{h} = \frac{v}{1-v} \sigma_{v} + \alpha P_{p},
\]

where \(\sigma_{v}\) is the vertical stress, \(v\) is the Poisson’s ratio, \(P_{p}\) is the pore pressure and \(\alpha\) is the Biot-Willis coefficient. The first term on the right hand side of equation 1 represents the lithostatic loading term and can be estimated by integrating the density function to obtain the vertical stress multiplied by the scalar consisting of the Poisson’s ratio. The pore pressure term however, is more difficult to obtain and requires knowledge of the Biot-Willis coefficient, which is given by

\[
\alpha = 1 - \frac{K_{d}}{K_{s}},
\]

where \(K_{d}\) is the drained bulk modulus and \(K_{s}\) is the bulk modulus of the solid phase. Here, \textit{drained} refers to deformation such that pore pressure remains constant. The limiting cases for \(\alpha\) are then scenarios where the porosity is zero, representing the lower bound, and at critical porosity, above which the grains are suspended and no longer frame supported (Nur et al., 1998), representing the upper bound. At zero porosity, \(K_{d}=K_{s}\) and hence, \(\alpha=0\), resulting in a null pore pressure effect. Above the critical porosity, the grains are in full suspension and the medium becomes infinitely compressible (until grain contact is established at the critical porosity value). At this upper bound, \(K_{d}=0\) and hence, \(\alpha=1\), resulting in the pore pressure and confining stress having equal weights for their respective
contribution to the effective stress. This represents the maximum pore pressure effect. Using a value of one for the Biot-Willis coefficient is only valid for soft sediments near critical porosity. For most consolidated rocks, the Biot-Willis coefficient will be less than one. To determine the Biot-Willis coefficient, the drained and solid phase bulk modulus must be determined. This requires knowledge of the more fundamental building blocks of a rock mass that go beyond the elastic domain. Therefore, to properly estimate the Biot-Willis coefficient, we must determine the petrophysical properties from which components of the rock mass such as its pore space, frame and solid phase can be assembled from its constituent parts.

In this study, we estimate the Biot-Willis coefficient from seismic data via a rock physics inversion. First, we discuss the estimation of a calibrated rock physics model to provide a set of relationships between our elastic and petrophysical properties. Subsequently, we perform a rock physics inversion to estimate the total porosity and various mineral fractions, from which $K_d$ and $K_s$ can be obtained to compute the Biot-Willis coefficient. This was then used in conjunction with the elastic properties estimated from the AVO inversion to determine the effective stress.

**AVO and rock physics inversion**

First, we derive a set of rock physics relationships between our elastic and petrophysical properties. Here, we use a regression-based model to estimate the effective bulk modulus, $K$, and shear modulus, $G$, for a three mineral mixture of quartz, clay and limestone with varying porosity and fluid saturation values. Any variable that potentially influences the rock matrix is used as a regression variable (e.g. effective pressure or stress), which allows for calibration to log data capturing local trends of the field as well as enforcing theoretical models. The rock physics model for $M\{K, G\}$ is given by

$$\frac{1}{M + M_0} = \sum_i \frac{(1 - \phi)V_i}{M_i + M_0} + \frac{\phi}{M_{\text{fluid}} + M_0} \tag{3}$$

where $M_0$ is the regression variable, $M_i$ is the $i^{\text{th}}$ mineral moduli, $V_i$ is the $i^{\text{th}}$ mineral fraction, $M_{\text{fluid}}$ is the fluid moduli (zero for shear) and $\phi$ is the total porosity. The elastic and petrophysical logs are then used as inputs to estimate $M_0$ in addition to the moduli of the mineral end members of clay and limestone. Since clay and limestone are essentially “effective” end members consisting of various clay (e.g. illite and smectite) and carbonate (e.g. calcite and dolomite) minerals, these values must be determined from the data. Figure 1 shows the rock physics trend lines for total porosity and volume of limestone in acoustic impedance and Vp/Vs space.

![Figure 1: Rock physics trend lines for total porosity and volume of limestone.](image)
Next, we perform an AVO inversion to estimate the elastic properties and subsequently, using the rock physics model derived above, we perform a rock physics inversion to obtain the petrophysical parameters. Figures 2 and 3 show QCs of the AVO and rock physics inversion results, respectively.

![Figure 2: QC of inversion results at a well location for acoustic impedance, Vp/Vs and density.](image1)

![Figure 3: QC of the rock physics inversion results for total porosity, volume of clay and volume of limestone.](image2)

**Effective stress estimation**

With estimates of the total porosity and mineral fractions obtained from the rock physics inversion, we can compute the Biot-Willis coefficient using equation 3. The drained bulk modulus refers to deformation under constant pore pressure, therefore can be approximated by the bulk modulus of the porous frame. This is given by

$$\frac{1}{K_d + K_0} = \sum_i \frac{(1 - \phi)V_i}{K_i + K_0} + \frac{\phi}{K_0}. $$

(4)

Additionally, the bulk modulus of the solid phase represents the case of zero porosity, therefore is given by

$$\frac{1}{K_s + K_0} = \sum_i \frac{V_i}{K_i + K_0}. $$

(5)

Substitution of $K_d$ and $K_s$ into equation 2 then yields the Biot-Willis coefficient, which can be used in conjunction with the estimated Poisson’s ratio from the AVO inversion to determine the effective...
stress. Figures 4 and 5 show the confining stress (lithostatic loading term) and the effective horizontal stress (computed using a constant pore pressure of 30 MPa), respectively. The confining stress demonstrates a trend that is generally increasing with depth. The effective stress however, demonstrates larger variations that are due to the effect of the Biot-Willis coefficient, which is a manifestation of lithological and porosity variations. This is most evident in the Middle Montney where certain areas have similar effective stress values as the Upper Montney.

Conclusions
The effective stress field plays a vital role in many subsurface engineering problems including hydraulic fracturing of unconventional reservoirs. In order to estimate the effective stress, knowledge of the Biot-Willis coefficient is required. For consolidated rocks, the value of the Biot-Willis coefficient is between zero and one and is a function of the total porosity and mineralogy. Reservoir heterogeneity will therefore result in variations in the Biot-Willis coefficient and consequently, change the effective stress through the pore pressure effect. In this study, we estimate the Biot-Willis coefficient from seismic data through a rock physics inversion for total porosity and mineralogy. Using this approach, improvements in the estimation of the effective stress can be made by better understanding how pressurized fluids interact with the rock mass through the Biot-Willis coefficient.