

## Multi frequency FX filter for seismic random noise attenuation

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### Summary

FX filter technique is widely accepted and used in the industry. The method is very effective in attenuating random noise. The physical meaning behind this method is that the signal is coherent and therefore, it is predictable. However, because conventional FX filter works on each frequency slice independently, therefore the information of cross coherent between frequencies cannot be used. In this paper, a multivariate autoregressive model is applied to FX filter (MFX). Instead of using only one frequency slice, a group of frequency slices are used in FX filter, which takes advantage of signal coherency among those frequencies. Numerical example shows that while this multi frequency FX filter can remove strong random noise, it can also preserve signal well.

### Introduction

Random noise attenuation via FX filter is widely accepted and used in the industry. The algorithm of FX is based on a single channel autoregressive (AR) model. The basic principle is that in F-X domain linear events or quasilinear events manifest themselves as a superposition of harmonics. If noise is taken into account, an optimal model to predict a superposition of harmonics is an AR model, which can be written as

$$x(\omega, i) = \sum_{k=1}^p x(\omega, i - k)a(k) \quad (1)$$

where  $\omega$  is the frequency,  $i$  and  $k$  denote the sample indices,  $a(k)$  is the single channel AR coefficient,  $p$  is the filter length and  $x(\omega, i)$  is a frequency – space domain data sample.

The physical meaning behind equation (1) is that retaining a few dominant harmonics can represent the spatial coherent energy (signal). However, as shown in equation (1), F-X works on each frequency slice independently and therefore, the information of signal coherence across frequencies is ignored. As a result, comparing to t-x domain predictive filter that can fully utilize the coherence information between traces, frequency – space domain filter produce less ideal result for random noise reduction (Abma and Claerbout, 1997).

Unlike single channel AR model that pursuit the coherent energy/signal inside the channel, multi autoregressive model (MAR) can take into account the coherence between channels. The early application of MAR in seismic de-noising can be found in the work of surface consistent prediction (Wang, 1995), where the frequency slice from multi gathers are used together for detecting the spatial coherence information, e.g. common offset coherent signal information. Application of MAR to simultaneously attenuate three-component seismic data can be found in publications, e.g. Leonard and Kennett (1999) and Naghizadeh and Sacchi (2012). In those applications, even if the contaminated noise in components may come from the same noise source, and this may imply that the noise in different components may correlated with each other, the results still shows that the noise reduction effects are significant. It is worthwhile to mention that these applications of MAR in seismic data is either taking the frequency slices from a group of gathers (Wang, 1995), where the coherence between common offset may degraded by complicated geological structure, or taking from different components as multi channels (Leonard and Kennett, 1999), where the noise in each component may correlated with each other.

The algorithm presented in this paper is another application of MAR that can be named as Multi frequency FX filter (MFX): a group of frequency slices are taken from the same gather and each frequency slice corresponds to one channel as that in MAR. The white noise in different frequency may not be correlated with each other, i.e. in this MFX system noise is random, which is an ideal situation for predictive technique of random noise attenuation.

### Method description

MFX is a vector version of prediction filter:

$$\mathbf{x}(\omega, i) = \sum_{k=1}^p \mathbf{x}(\omega, i - k) \mathbf{a}(k) \quad (2)$$

Equation (2) has the same form as equation (1) that is used in conventional FX filter. The difference is that  $\mathbf{x}(\omega, i)$  here is a vector containing a group of frequency components and  $\mathbf{a}(k)$  is a matrix: if the dimension of  $\mathbf{x}(\omega, i)$  is  $d$ , then  $\mathbf{a}(k)$  is a  $d$  by  $d$  matrix. As an example, assuming the dimension of  $\mathbf{x}(\omega, i)$  is two, i.e. only two frequencies are involved, and the filter length  $p$  is three, then equation (2) can be written as

$$[x_{\omega_1}(i), x_{\omega_2}(i)] = [x_{\omega_1}(i-1), x_{\omega_2}(i-1), x_{\omega_1}(i-2), x_{\omega_2}(i-2), x_{\omega_1}(i-3), x_{\omega_2}(i-3)] \begin{bmatrix} a_{11}(1) & a_{12}(1) \\ a_{21}(1) & a_{22}(1) \\ a_{11}(2) & a_{12}(2) \\ a_{21}(2) & a_{22}(2) \\ a_{11}(3) & a_{12}(3) \\ a_{21}(3) & a_{22}(3) \end{bmatrix} \quad (3)$$

In MFX, the number of coefficients to be determined in MAR is much larger than that for a single AR. However, solving a system of linear equations formed from a series of equations like equation (2) leads to a Toeplitz matrix equation system that can be efficiently solved via a modified Levinson recursion procedure (Wiggins and Robinson, 1957) or a direct multi channel predictive deconvolution solver developed by Posani and Ursin (2007).

It should be mentioned that white noise in different frequencies are not correlated with each other and therefore, the multi frequency AR equation system developed here is well designed for random noise attenuation. Whereas in the application to three component data, the 3-component geophones are located at the same place and the recorded noise may come from same sources and therefore, the noise in different component may be correlated, which may not be ideal for AR noise attenuation.

### Example

A synthetic data, figure 1, is used for comparison between MFX and FX noise attenuation methods. The data contains two linear events and two hyperbolic events. The results from FX and MFX are shown in Figure 2, where MFX produces a cleaner result than that of FX. Figure 3 depicts the differences between inputs and outputs of those two methods. It is to be noted that the result from FX contains some artificial background noise, whereas signal leakage in the result from MFX is less prominent, which may be attributed to the benefit of signal coherence among frequency slices.

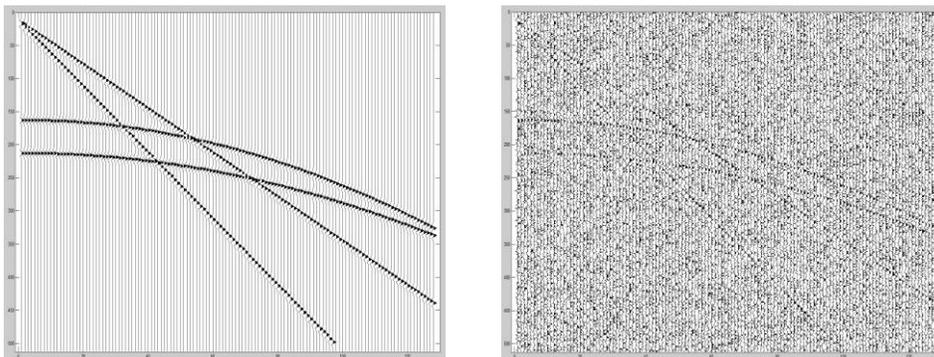


Figure 1. the synthetic data: without and with random noise, from left to right.

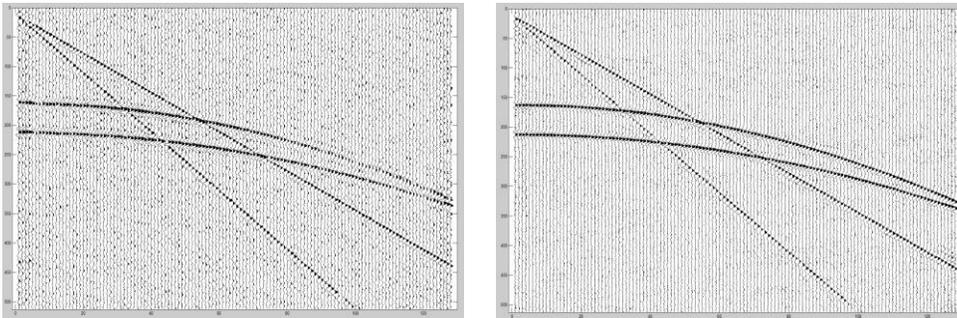


Figure 2. Noise attenuated via FX and MFX, from left to right.

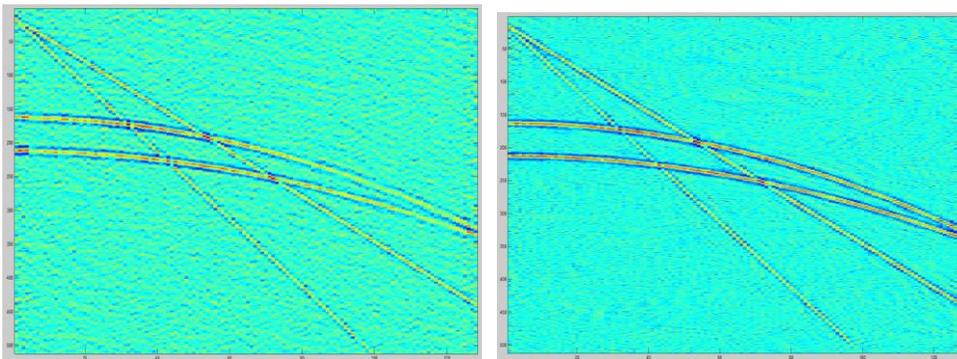


Figure 3. Image plot of Figure 2.

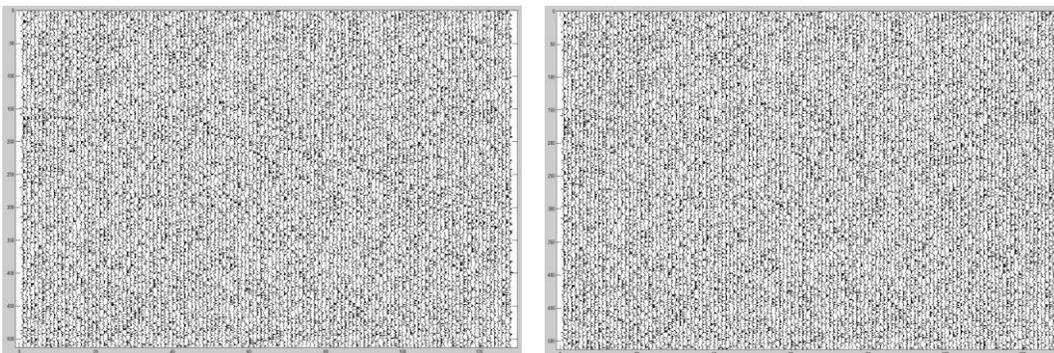


Figure 3. the differences between noisy input and the results from FX and MFX, from left to right.

## Conclusions

A multi frequency FX filter (MFX) is developed for seismic random noise attenuation. This method takes advantage of signal coherence across frequencies. Numerical example shows that MFX is suitable for strong white noise attenuation.

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## References

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