geoconvention
Calgary • Canada • May 15-19 2017

Preconditioning FWI via logarithmic velocity parametrization

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Summary

Full Waveform Inversion (FWI) is a powerful technique for estimating subsurface physical properties from seismic data. FWI is a local iterative optimization technique and many challenging problems must be solved before one can fully adopt FWI as part of standard industrial processing flows. A challenging problem is the lack of sensitivity of the gradient of the misfit function to model perturbations. The quality of the estimated model and convergence rate of the iterative solver depend on the model parametrization that one adopts for the FWI algorithm. As part of our efforts to alleviate practical shortcomings of FWI, we present a logarithmic velocity model parametrization that provides a good illumination compensation of subsurface model parameters and a fast convergence rate. Numerical results are presented to highlight the efficiency of the logarithmic velocity model parametrization.

Introduction

Full Waveform inversion (FWI) aims to estimate high resolution subsurface structures by minimizing the data misfit between observed and modelled seismograms through local iterative optimization techniques (Tarantola, 1987; Stekl and Pratt, 1998; Virieux and Operto, 2009). The majority of FWI algorithms depend on local iterative optimization methods, for example non-linear conjugate gradient, L-BFGS (Limited memory Broyden-Fletcher-Goldfarb-Shanno algorithm), Gauss-Newton or Full Newton (Hestenes and Stiefel, 1952; Nocedal, 1980; Pratt et al., 1998; Hu et al., 2011; Anagaw and Sacchi, 2013, 2014b).

Gradient-based methods have in general a slow convergence rate and cannot properly retrieved shallow and deeper parts of the reconstructed model parameters. A fast convergence rate and a good balance in amplitudes between the shallow and deeper parts of the model can be achieved by employing Newton-based optimization techniques. It has been shown that the convergence rate of Newton-based optimization engines are greatly influenced by the type of model parametrization that is adopted (Anagaw and Sacchi, 2014a). In this paper we develop and study an FWI algorithm that adopts a logarithm velocity model parametrization. Moreover, we compare its computational efficiency to the classical parametrization in terms of slowness or slowness squared (Anagaw and Sacchi, 2014a). The slowness model parametrization damps the amplitudes of the gradient in the deeper parts the model, whereas the slowness squared model parametrization provides less damping of amplitudes in deeper parts of the model. The slowness squared model parametrization is shown to be sensitive to noise in the data (Anagaw and Sacchi, 2014a). The logarithm velocity parametrization, on the other hand, provides a good trade-off in scaling the amplitudes of the gradient for both shallow and deep parts of the model. The use of different types of model parametrization can also be seen as introducing preconditioning to the gradient of the misfit function. The new logarithm velocity model parametrization leads to a gradient that is exponentially scaled making it more suitable obtaining models with well-balanced amplitudes at initial stages of the iterative inversion process. Numerical example demonstrates the benefit of adopting a logarithmic velocity parametrization. In our studies, we have adopted a quasi-Newton l-BFGS optimization method to approximate the inverse of the Hessian matrix (Nocedal, 1980). We have tested our algorithm with BP velocity model (Billette and Brandsberg-Dahl, 2005).

Theory

We start by indicating that our work pertains to acoustic FWI with frequency domain solvers (Pratt et al., 1998; Anagaw and Sacchi, 2014b). Waveform inversion often uses the least-squares misfit defined as the l_2 norm of the residual between the observed data d^{obs} and synthesized data d^{cal},

$$J(m) = \frac{1}{2}\sum_{\omega_i}^{N_\omega}\sum_{s,r}^{N_s,N_r}(d_{s,r}^{obs}(\omega_i) - ds,r^{cal}(\omega_i))^\dagger(d_{s,r}^{obs}(\omega_i) - d_{s,r}^{cal}(\omega_i)), \tag{1}$$

where $\dagger$ is the complex conjugate transpose, ω is the angular frequency and N_ω is the number of frequencies. The parameters N_s and N_r represent the number of sources and receivers, respectively. The minimization of the misfit function is a nonlinear problem where from measured wavefields one attempts to estimate the subsurface P-wave velocity model. The resulting nonlinear optimization problem is expressed via the following constrained optimization formulation

$$\begin{aligned} &\underset{m}{\text{argmin}} && J(m) \\ &\text{subject to} && A(m,\omega)p_s(\omega) = f(\omega)_s. \end{aligned} \tag{2}$$

Gradient-based minimization schemes such as the steepest descent and non-linear conjugate gradients (Nocedal, 1980) methods ignore the Hessian matrix. Gradient-based iterative methods have shown to converge slowly (Shin et al., 2001) and this becomes the reason one often prefers full Newton or Gauss-Newton methods to solve the FWI problem. However, we have recently shown that the convergence rates of these methods depend on the parametrization of the model that is adopted. Solving the iterative inversion with the full Hessian matrix is computationally expensive. Therefore, we adopt the quasi-Newton *l*-BFGS method that iteratively approximates the inverse of the Hessian matrix (Nocedal, 1980). We then update the model through the search direction g_m provided by *l*-BFGS

$$m_{k+1} = m_k + \alpha g_{mk}, \tag{3}$$

where g_{mk} is the gradient at the k^{th} iteration and the scalar parameter α is the step length that can be computed using a line search method. The gradient defines the direction in which the value of the misfit direction decreases. The preconditioning of the gradient in equation (3) is obtained by applying the following change of variable $m = \log(v)$. The latter results in the following

$$g_{mk} = e^{-2m}g_k, \tag{4}$$

where g_k is the gradient obtained by the cross-correlation between the forward and backward modelled wavefields and $p_k = e^{-2m}$ is the preconditioning induced by the logarithmic parametrization (Anagaw and Sacchi, 2014a).

Examples

For comparison purposes we have employed three model parameterizations and studied their numerical efficiency for velocity model building for the case of acoustic full waveform inversion. The three model parameterizations under considerations are slowness (v^{-1}), square of slowness (v^{-2}) and logarithm velocity ($\log(v)$). For our numerical examples, we consider a portion of BP velocity model (Figure 1). Figures 1(a) and (b) are the true and smooth initial models used for the inversion, respectively. The model is discretized with a uniform grid spacing of 24 m×24 m. A total of 75 sources and 150 receivers were used. The sources are placed one grid point below the surface and receivers are placed at the surface. A set of 10 discrete frequencies were selected between 3.0 Hz

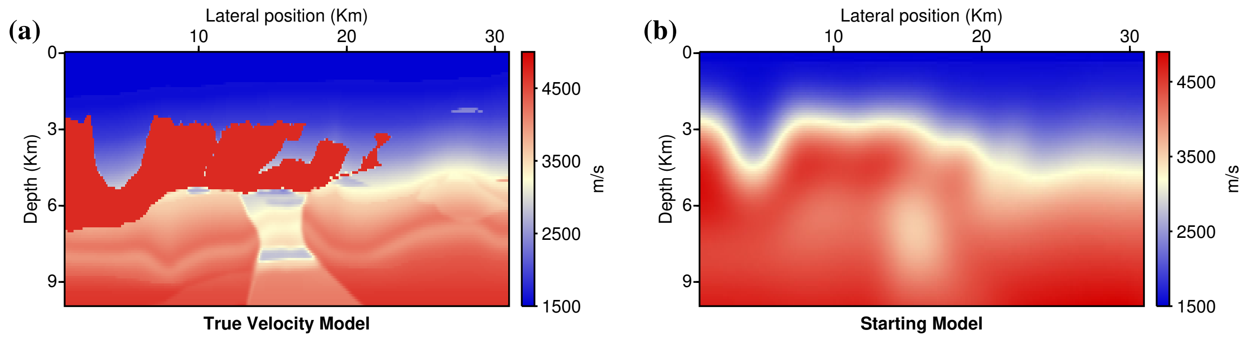

Figure 1: Portion of the original BP velocity model. True velocity model and (a) Smooth initial velocity model (b).

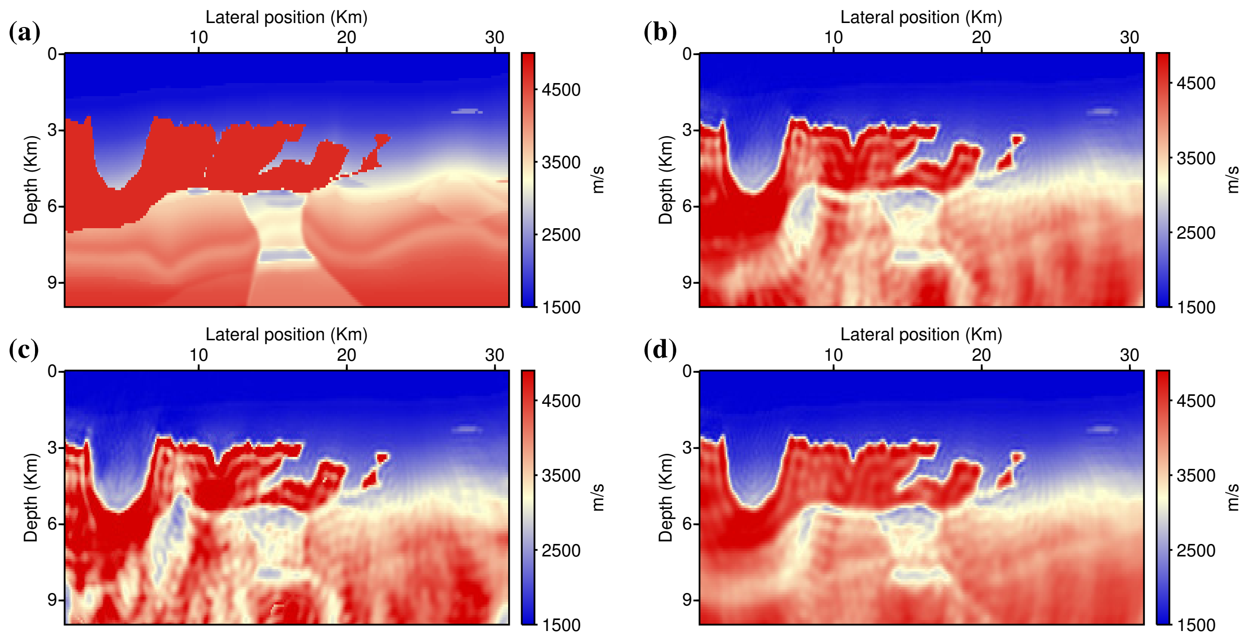

Figure 2: FWI results. (a) True model, (b) Reconstructed model via slowness parametrization (v^{-1}), (c) Reconstructed model via slowness squared parametrization (v^{-2}) and (d) Reconstructed model via the logarithm velocity ($\log(v)$) parameterization.

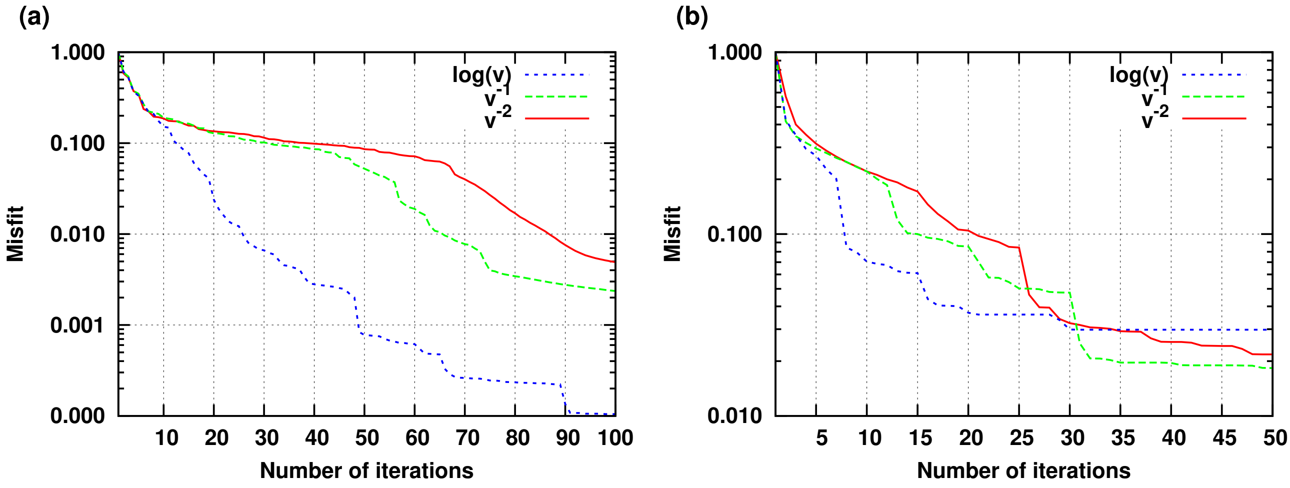

Figure 3: Relative data misfit reduction for our FWI code under three types of model parametrization. FWI for 6.0Hz (a) and 9.0Hz (b).

and 15 Hz, and the numerical inversion is then carried out in a sequential approach starting from low to high frequencies with the *l*-BFGS method.

Figures 2(b), 2(c) and 2(d) are the reconstructed velocity models obtained via our FWI algorithm using slowness, slowness squared and logarithm velocity parametrization, respectively. The three model parameterizations have permitted to properly retrieved shallow structures and the salt body. The results do differ in their quality of in the deep portions of the model. The slowness and logarithm velocity model parameterizations produce high-quality results in the deeper part of the model. However, the logarithm velocity model parametrization has produced a clearer image in areas of high velocity contrast and complexity.

Figures 3(a) and 3(b) portray convergence plots for two temporal frequencies (6.0 Hz and 9.0 Hz). The logarithmic velocity parametrization leads to a faster convergence of our FWI code. The new scheme updates properly the deeper and shallow parts of the model at early stages of the inversion which results in the high-resolution reconstructed velocity model displayed in Figure 2(d).

Conclusions

We have examined the efficiency of three model parameterizations that are available in our code for acoustic frequency domain FWI. The three model parameterizations yielded different results and convergence rates. A faster convergence rate and cleaner image is obtained by using the logarithm velocity model parametrization. This is merely due to the fact that the slowness model parametrization damps the amplitude of gradient in the deeper parts than in the case of slowness squared, whereas the logarithm velocity parametrization provides a good trade-off balance in scaling the gradient over different depth regimes. The logarithm velocity model parameterization provides a better preconditioning to the gradient of the misfit function. We have also found that the logarithmic velocity perturbation provides faster convergence than the slowness and slowness squared parametrization.

Acknowledgements

The authors are grateful to the sponsors of Signal Analysis and Imaging Group (SAIG) at the University of Alberta and NSERC for supporting this project.

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