The signature of attenuation and anisotropy on AVO and inversion sensitivities

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Summary

We study the scattering of viscoelastic waves in an attenuative transversely isotropic media with vertical axis of symmetry (VTI). There are two approaches used to study the seismic wave scattering, the method of so-called Aki-Richards approximation based on the linearization of the exact solutions of the Zoeppritz equation, and, alternatively the Born approximation which is based on the first order perturbation theory. We consider to the VTI media with low loss attenuation and weak anisotropy such that the second and higher orders of quality factors and Thompson parameters are neglected. In a viscoelastic medium we have P-wave, SI-wave and SII waves, all with complex slowness vectors. We derived the expressions for potentials of scattering of PP-wave and show that our results cover the amplitude variation with offset equation.

Introduction

The weak contrast linearized reflection coefficients play a major role in inversion of seismic data as they contain unique information on sensitivity of the seismic data to the changes in earth properties (Beylkin & Burridge 1990; Tarantola 1984). The traditional way to compute the linearized reflection coefficients is based on the solution of the Zeoppritz equation assuming that properties across the boundary slightly change (Aki & Richards 2002).

The exact and approximate reflection and transmission coefficients have been derived for layered viscoelastic isotropic medium taking into the changes in the viscoelastic parameters for incident homogeneous wave (Ursin & Stovas 2002). The same problem for an inhomogeneous viscoelastic planewave interacting with a low contrast two layered isotropic viscoelastic media wherein the jumps in the inhomogeneity angle is accommodated recently have been derived (Moradi & Innanen 2016). It has been shown that these linearized reflection coefficients can be transformed into the viscoelastic scattering potentials as derived in the general volume scattering framework (Moradi & Innanen 2015). Cerven´y & Psenc´ık studied the homogeneous and inhomogeneous plane waves propagating in a viscoelastic anisotropic medium (Cerven´y & Psenc´ık 2005a; Cerven´y & Psenc´ık 2005b; Cerven´y & Psenc´ık 2008a). Linearized weak reflection coefficients for viscoelastic anisotropic media including the inhomogeneity angle of incident wave are derived based on the exact solutions of the Zoeppritz equations by Behura and Tsvankin (2006; 2009).

Understanding the scattering patterns induced by perturbations in medium properties is an essential prerequisite for AVO inversion and Full Waveform Inversion (FWI) (Virieux & Operto 2009; Fichtner 2010; Castagna1994). The Born approximation method based on the perturbation theory is an efficient method to evaluate the sensitivity kernel for FWI.

Insight into the seismic wave propagation in an attenuative anisotropic earth is of fundamental interest, our paper is a self-contained presentation of the scattering volume from inclusions both in anisotropic and viscoelastic properties. Comparing to the isotropic elastic medium derivation of such approximations is extremely complicated and needs the assumption of both weak anisotropy and low attenuation in lower and upper media. We assume (a) the anisotropy is weak, and (b) the media is low-loss attenuative.

Born approximation

The Born approximation method, based on perturbation theory, is an efficient approach to evaluate sensitivity kernels for many types of multi-parameter FWI. In this approach, the actual medium is decomposed into a reference medium (with slightly different properties from those of the actual medium), within which are perturbations which when combined with the reference properties recover the actual medium (Stolt & Weglein 2012). Figure (1.a) is a schematic description of the two layer viscoelastic VTI medium with weak anisotropy and attenuation. The properties can be divided into four parts: elastic properties including density, P- and S-wave velocities; viscoelastic properties including P- and S-wave quality factors; anisotropic parameters including three Thomsen parameters and three anisotropic viscoelastic Thomsen parameters. For a low contrast medium properties across the boundary are slightly different.

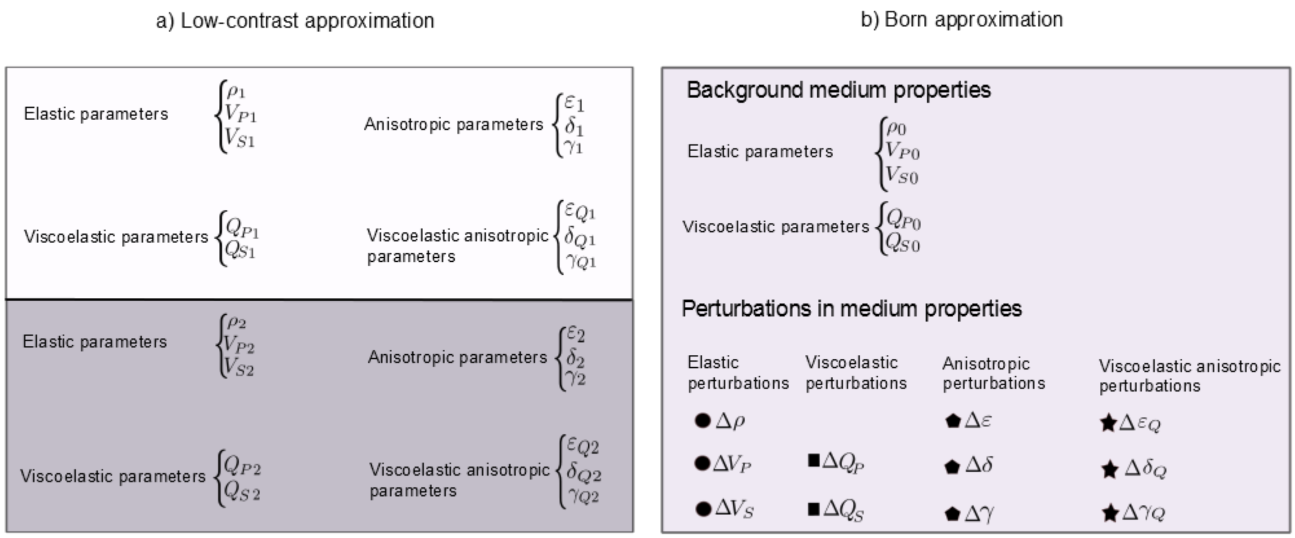

Figure 1. a) Schematic description of the two anisotropic viscoelastic layer Medium b) Diagram illustrating the mathematics of Born approximation based on the perturbation theory.

As a result a small portion of the incidence wave is reflected from the boundary and the majority transmitted to the lower medium. In this case we can linearized the reflection coefficients in terms of the first order perturbations in medium properties. On the other hand Figure (1.b) illustrate the configuration of the scattering in anisotropic viscoelastic medium in the context of Born approximation. The background medium is described by three elastic parameters P-wave velocity Vp_0, S-wave velocity Vs_0 and ρ_0; two viscoelastic parameters P-wave quality factor Qp_0 and S-wave quality factor Qs_0; Wave traveling in the reference medium interacts with these scatter points that randomly distributed. Technically an incident wave undergoes a sequence of multiple scattering events from the perturbations.

Examples

Let us consider the reference medium to be isotropic viscoelastic media filled with the perturbations in viscoelastic and anisotropic parameters. In this case the scattering potential breaks into the following components

$$[\mathrm{PP}] = [\mathrm{PP}]_{\mathrm{IE}} + [\mathrm{PP}]_{\mathrm{AE}} + i\,[\mathrm{PP}]_{\mathrm{IV}} + i\,[\mathrm{PP}]_{\mathrm{AV}}$$

with elastic, anisotropic, viscoelastic and viscoelastic anisotropic components

$$
\begin{aligned}
[\mathrm{PP}]_{\mathrm{IE}} &= [\mathrm{PP}]_{\mathrm{IE}}^{\rho}\frac{\Delta\rho}{\rho} + [\mathrm{PP}]_{\mathrm{IE}}^{\mathrm{Vp}}\frac{\Delta V_P}{V_P} + [\mathrm{PP}]_{\mathrm{IE}}^{\mathrm{Vs}}\frac{\Delta V_{\mathrm{S}}}{V_{\mathrm{S}}},\\
[\mathrm{PP}]_{\mathrm{AE}} &= [\mathrm{PP}]_{\mathrm{AE}}^{\varepsilon}\Delta\varepsilon + [\mathrm{PP}]_{\mathrm{AE}}^{\delta}\Delta\delta,\\
[\mathrm{PP}]_{\mathrm{IV}} &= [\mathrm{PP}]_{\mathrm{IV}}^{\rho}\frac{\Delta\rho}{\rho} + [\mathrm{PP}]_{\mathrm{IV}}^{\mathrm{Vs}}\frac{\Delta V_{\mathrm{S}}}{V_{\mathrm{S}}} + [\mathrm{PP}]_{\mathrm{IV}}^{\mathrm{Qp}}\frac{\Delta Q_{\mathrm{P}}}{Q_{\mathrm{P}}} + [\mathrm{PP}]_{\mathrm{IV}}^{\mathrm{Qs}}\frac{\Delta Q_{\mathrm{S}}}{Q_{\mathrm{S}}},\\
[\mathrm{PP}]_{\mathrm{AV}} &= [\mathrm{PP}]_{\mathrm{AV}}^{\varepsilon}\Delta\varepsilon + [\mathrm{PP}]_{\mathrm{AV}}^{\delta}\Delta\delta + [\mathrm{PP}]_{\mathrm{AV}}^{\varepsilon_{\mathrm{Q}}}\Delta\varepsilon_{\mathrm{Q}} + [\mathrm{PP}]_{\mathrm{AV}}^{\delta_{\mathrm{Q}}}\Delta\delta_{\mathrm{Q}},
\end{aligned}
$$

The elastic component $[\mathrm{PP}]_{\mathrm{IE}}$ is the function of fractional changes in density, P-wave and S-wave velocities. The anisotropic component $[\mathrm{PP}]_{\mathrm{AE}}$ desponds to the changes in Thomsen parameters ε and $\boldsymbol{\delta}$. The viscoelastic component variate with the fractional changes in P- and S-wave quality factors and S-wave velocity. Viscoelastic anisotropic components depend to the changes in Q-dependent Thomsen parameters ε_{Q} and $\boldsymbol{\delta}_{\mathrm{Q}}$. For normal incident the contributions from anisotropic and viscoelastic anisotropic are zero. In $[\mathrm{PP}]_{\mathrm{AE}}$ and $[\mathrm{PP}]_{\mathrm{AV}}$ components there is no influence of changes in vertical P- and S-wave velocities and corresponding quality factors. In figure 2 we plot the sensitivities of PP scattering potential to the changes in medium properties versus the incident angle.

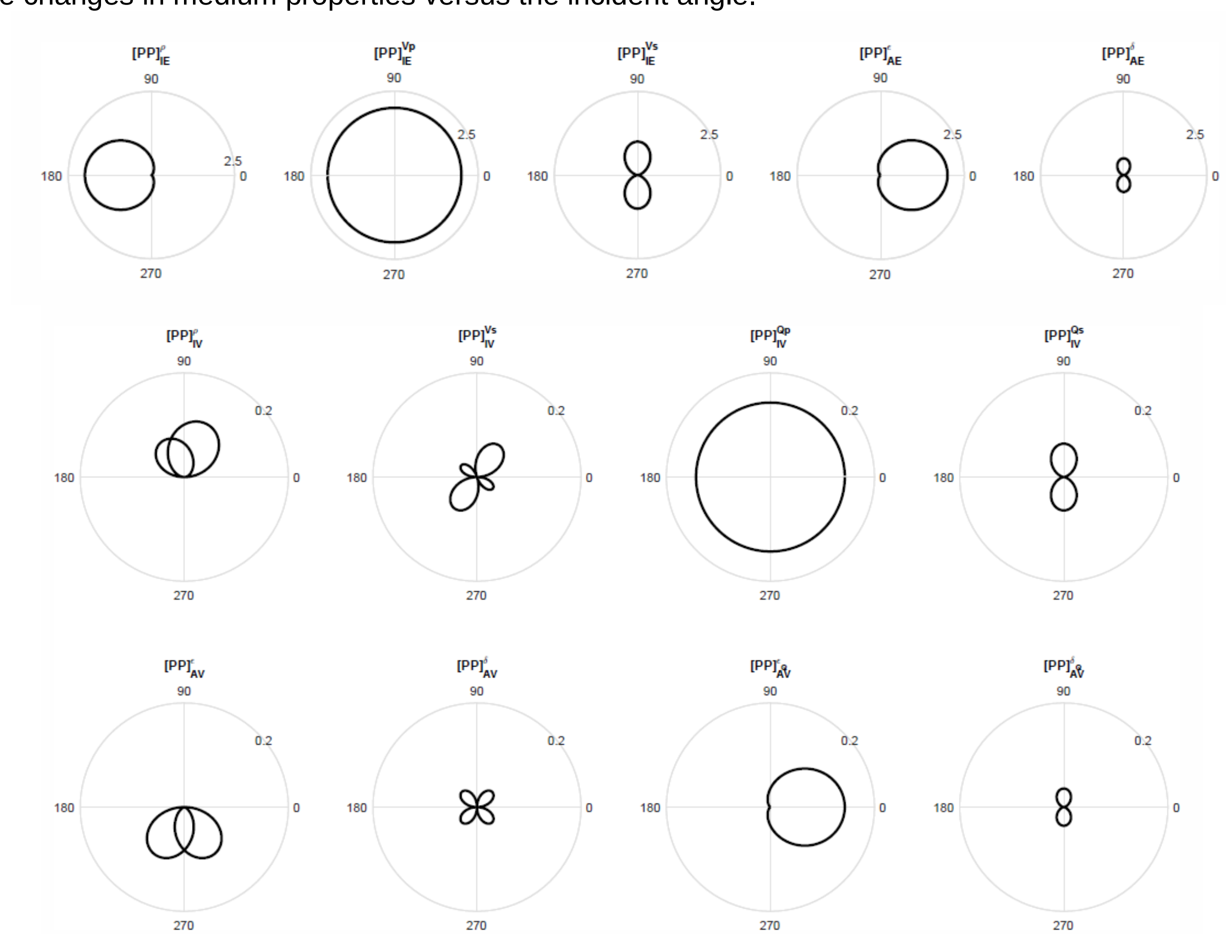

Figure 2. Sensitivity of the the P-to-P scattering potential to changes in properties versus incident P-wave phase angle.

Eventually, the linearized reflection coefficient emerged from the scattering potential is given by

$$
\mathrm{R}_{\mathrm{PP}} = \frac{1}{4\cos^2\theta_{\mathrm{P}}}[\mathrm{PP}] = \hat{\mathrm{A}}_{\mathrm{PP}} + \hat{\mathrm{B}}_{\mathrm{PP}}\sin^2\theta_{\mathrm{P}} + \hat{\mathrm{C}}_{\mathrm{PP}}\sin^2\theta_{\mathrm{P}}\tan^2\theta_{\mathrm{P}}
$$

where the coefficients are

$$\hat{\mathrm{A}}_{\mathrm{PP}} = \frac{1}{2}\left(\frac{\Delta\rho}{\rho} + \frac{\Delta V_P}{V_P}\right) - \frac{i}{4}Q_{\mathrm{P0}}^{-1}\frac{\Delta Q_{\mathrm{P0}}}{Q_{\mathrm{P0}}}$$

$$\hat{\mathrm{B}}_{\mathrm{PP}} = \frac{1}{2}\left[\frac{\Delta V_P}{V_P} - 4V_{\mathrm{SP}}^2\left(\frac{\Delta\rho}{\rho} + 2\frac{\Delta V_S}{V_S}\right) + \Delta\delta\right] - i\left[\frac{1}{4}Q_{\mathrm{P0}}^{-1}\frac{\Delta Q_{\mathrm{P}}}{Q_{\mathrm{P}}} - 2V_{\mathrm{SP}}^2 Q_{\mathrm{S0}}^{-1}\frac{\Delta Q_{\mathrm{S}}}{Q_{\mathrm{S}}}\right]$$
$$- i\left[2V_{\mathrm{SP}}^2(Q_{\mathrm{S0}}^{-1} - Q_{\mathrm{P0}}^{-1})\left(\frac{\Delta\rho}{\rho} + 2\frac{\Delta V_S}{V_S}\right) - \frac{1}{4}Q_{\mathrm{P0}}^{-1}\Delta\delta_Q\right]$$

$$\hat{\mathrm{C}}_{\mathrm{PP}} = \frac{1}{2}\left[\frac{\Delta V_P}{V_P} + \Delta\varepsilon\right] - \frac{i}{4}Q_{\mathrm{P0}}^{-1}\frac{\Delta Q_{\mathrm{P}}}{Q_{\mathrm{P}}} + \frac{i}{4}Q_{\mathrm{P0}}^{-1}\Delta\varepsilon_Q$$

A_{pp}, is called zero offset or normal incident reflection coefficient. It can be seen that this term depends only to the changes in density, vertical P-wave velocity and P-wave quality factor. Anisotropic and viscoelastic anisotropic properties does not influence this term. The second coefficient B_{pp} is called gradient which is sensitive to the changes in density, vertical P- and S-wave velocities, P- and S-wave quality factors, δ and δ_Q. The last term C_{pp} which is called curvature is influential for large angles of incidence. This term is affect by changes in P-wave related properties, vertical P-wave velocity, P-wave quality factor, ε and ε_Q.

Conclusions

Even for elastic medium exact reflection coefficients is a very complicated function of medium properties.

Nevertheless, under favorable conditions, if the changes in medium properties across the boundary are small and for small angle of incident, it is possible to find a reliable approximate solutions for reflection coefficients. Scattering potentials based on the Born approximation describes the low contrast layered medium reflection coefficients for scattering of seismic waves from complex structures including the anisotropy and attenuation. The advantages of using Born approximation is that it does not need the exact solutions of the wave equation. Our work is concerned with the scattering potential, relies on the perturbation theory, for scattering of viscoelastic waves in an anisotropic viscoelastic media. Instead of struggling with the mathematical difficulties of solutions of Zoeppritz equation and linearization, we employ the geometry of the Born approximation which provides a useful and simple language in which the amplitude variation with offset equations can be formulated effectively and clearly. When attenuation is included (to be specific let us say by adding the imaginary part to the stiffness tensor) there arise additional terms associated with the quality factors and anelastic Thomsen parameters. These extra terms can be seen as a deviation from the anisotropic stiffness tensor. In our calculations we assumed that the medium is weak anisotropic and attenuative, as a result inverse quality factors and Thomsen parameters are much smaller than unity. Also the fractional changes in properties are such small that higher orders can be neglected.

Acknowledgements

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