geoconvention
Calgary • Canada • May 15-19 2017

Theory and Application of Vector Singular Spectrum Analysis (SSA) for Multicomponent Seismic data Reconstruction

Mauricio D. Sacchi, Jinkun Cheng and Scott Janzen
Signal Analysis and Imaging Group, University of Alberta

Summary

We present a Singular Spectrum Analysis (SSA) filter (also called Cadzow filter) for vector field measurements. We use the vector autoregressive model to generalize the classical SSA method for scalar fields to vector fields. Multiple applications of Vector-SSA (V-SSA) are under study. In particular, the method can be used for 3C random noise attention and reconstruction of missing traces including 5D vector field reconstruction. We use synthetic examples to illustrate vector denoising and reconstruction.

Introduction

Singular Spectrum Analysis (also known as Cadzow filters) has become an important tool for denoising seismic data in the $f-x$ domain (Trickett, 2008; Oropeza and Sacchi, 2011). The SSA filter is also used in conjunction with data imputation algorithms to solve the seismic data reconstruction problem. Examples of the latter include 3D reconstruction (Oropeza and Sacchi, 2011; Naghizadeh and Sacchi, 2013) and 5D reconstruction (Trickett and Burroughs, 2009; Burroughs and Trickett, 2009; Gao et al., 2013). A robust version of the SSA filter has been proposed by Chen and Sacchi (2013). Furthermore, SSA has been modified to interpolate aliased data (Naghizadeh and Sacchi, 2013).

Current work on SSA filters has centred on scalar fields (single component data). In this presentation we investigate the application of SSA filters to vector field data. We pay particular attention to vector autoregressive models (Naghizadeh and Sacchi, 2012). The later leads to the generalization of the SSA filter to vector field data.

Theory

We first define a scalar field $P(x,\omega)$. For instance, one could have observed pressure or vertical particle velocity by a hydrophone or geophone, respectively. Similarly, we define a vector field as follows

$$\vec{P}(x,\omega) = \begin{bmatrix} P_1(x,\omega) \\ P_2(x,\omega) \\ P_3(x,\omega) \end{bmatrix} \tag{1}$$

where one understands that P_i indicates, for instance, the two horizontal components $(i = 1,2)$ and the vertical component $(i = 3)$ fields measured by a 3C receiver at spatial position x. Clearly, our measurement are in time but we prefer to simplify the analysis by working in the frequency domain with one monochromatic frequency component ω at a time. The discrete scalar field can be obtained by discretizing the spatial axis x as follows $x_n = (n-1)\Delta x$ where Δx is the spacing between measurements. We represent the scalar discrete field via

$$P(x_n,\omega) \equiv P_n, \quad n = 1,\dots .N$$

We can adopt a scalar autoregressive model (with no innovation) of order $p = 2$ to, for instance, represent the superposition of two complex sinusoids in x. This is equivalent to model two constant-dip events in $t-x$

$$P_n = a_1 P_{n-1} + a_2 P_{n-2}\,. \tag{2}$$

Similarly, we can define the discrete vector field at position $x_n = (n-1)\Delta x$

$$\vec{P}(x_n,\omega) \equiv \vec{P}_n, \quad n = 1,\dots N\,.$$

One can define a recursion for vector fields by adopting the so called vector autoregressive (VAR) process (Naghizadeh and Sacchi, 2012; Kamil et al., 2015)

$$\vec{P}_n = \mathbf{A}_1 \vec{P}_{n-1} + \mathbf{A}_2 \vec{P}_{n-2} \tag{3}$$

where $\mathbf{A}_1$ and $\mathbf{A}_2$ are 3×3 matrices. We have basically replaced the scalar AR coefficients a_i by matrices $\mathbf{A}_i$.

Let us consider the case with $p=1$. In this case only one matrix is needed by the vector AR representation $\mathbf{A} \in \mathscr{C}^{3\times 3}$

$$\vec{P}_n = \mathbf{A}\vec{P}_{n-1}\,. \tag{4}$$

We now continue with the usual SSA filter (Oropeza and Sacchi, 2011) where one first defines the Hankel matrix of the scalar spatial observations. However, we now re-define the Hankel matrix via entries that correspond to vector elements

$$\mathbf{H} = \begin{bmatrix} \vec{P}_1 & \vec{P}_2 & \vec{P}_3 & \vec{P}_4 \\ \vec{P}_2 & \vec{P}_3 & \vec{P}_4 & \vec{P}_5 \\ \vec{P}_3 & \vec{P}_4 & \vec{P}_5 & \vec{P}_6 \\ \vec{P}_4 & \vec{P}_5 & \vec{P}_6 & \vec{P}_7 \\ \vec{P}_5 & \vec{P}_6 & \vec{P}_7 & \vec{P}_8 \end{bmatrix}, \tag{5}$$

After substituting equation 4 into 5 , we arrive to the following expression

$$\mathbf{H} = \begin{bmatrix} \vec{P}_1 & \mathbf{A}\vec{P}_1 & \mathbf{A}^2\vec{P}_1 & \mathbf{A}^3\vec{P}_1 \\ \vec{P}_2 & \mathbf{A}\vec{P}_2 & \mathbf{A}^2\vec{P}_2 & \mathbf{A}^3\vec{P}_2 \\ \vec{P}_3 & \mathbf{A}\vec{P}_3 & \mathbf{A}^2\vec{P}_3 & \mathbf{A}^3\vec{P}_3 \\ \vec{P}_4 & \mathbf{A}\vec{P}_4 & \mathbf{A}^2\vec{P}_4 & \mathbf{A}^3\vec{P}_4 \\ \vec{P}_5 & \mathbf{A}\vec{P}_5 & \mathbf{A}^2\vec{P}_5 & \mathbf{A}^3\vec{P}_5 \end{bmatrix}. \tag{6}$$

The Hankel matrix of the vector field data modelled by 4 is a matrix of rank $r=1$. In essence, we can generalize the scalar SSA algorithm to a vector SSA algorithm by embedding vector measurements $\vec{P}_n$ in a Hankel matrix. We can show that for 3C data that consist of the superposition of p linearly polarized events in $t-x$, the rank of the vector field Hankel matrix is $rank = p$. This is an important result that permits the development of new Hankel-based algorithms for denoising and reconstruction of 3C seismic data.

The vector SSA filter can be reduced to the following expression (Oropeza and Sacchi, 2011)

$$\mathscr{F}[.] = \mathscr{A}\mathscr{R}\mathscr{H}[.] \tag{7}$$

where $\mathscr{A}$ is the un-Hankelization operator (averaging across anti-diagonals), $\mathscr{R}$ is the rank reduction operator and $\mathscr{H}$ is the process of forming the Hankel matrix from the vector field data. We clarify that the rank reduction operator ($\mathscr{R}$) can be implemented via the Singular Value Decomposition (SVD) or via a fast randomized (SVD). If we denote $\mathscr{T}$ the sampling operator, we can use the imputation algorithm described in Oropeza and Sacchi (2011) to reconstruct missing traces

$$\vec{P}^k = \alpha\vec{P}^{obs} + (1-\alpha\mathscr{T}) \circ \mathscr{F}[\vec{P}^{k-1}] \tag{8}$$

where k is iteration and $\alpha \in (0,1]$ is the trade-off parameter. The vector field data with missing traces is $\vec{P}^{obs}$.

Examples

We have created a synthetic example that consists of 4 linear events impinging on 3C geophones. The 4 dips are recorded via the components X, Y and Z. Each event has a different polarization in X, Y and Z components. The maximum amplitude of the noise-free data is 1 and the standard error of the noise in each component is $\sigma = 0.8$. We transform the data to the $f-x$ domain an apply two denoising algorithms to the multi-component data set. First, we apply the scalar SSA filter to each component separately (Oropeza and Sacchi, 2011). Then, we apply the vector SSA filter simultaneously to X,Y and Z. In all cases, we adopted $rank = 4$ and run the algorithms in the frequency range 0 to 100Hz. Figure 1 shows the spectrum of singular values of the Hankel matrix formed by the vector field data at 30Hz. Clearly, one can identify the presence of 4 waves immersed in noise. To quantify our findings, we have evaluated the quality of the reconstruction for each individual component

$$Q = 10log10(\frac{||D_0||^2}{||\hat{D}-D_0||^2}),$$

where $\hat{D}$ and D_0 indicate the estimated data after denoising and the true uncorrupted data, respectively. Figure 2 shows values of Q for SSA and V-SSA for three different signal-to-noise ratio scenarios. It is important to mention that in these examples V-SSA outperforms SSA by about 5dB in all tests.

We also tested vector-field SSA reconstruction. In the first case we tested the algorithm with noise-free data. The results are shown in Figure 3. In this case we choose $\alpha = 1$ which corresponds to the reconstruction of noise-free data. Choosing $\alpha = 1$ entails applying full re-insertion of the measured data in each iteration. We repeated the example but now we have contaminated the data with random noise ($\sigma = 0.4$). In this case we have adopted a reinsertion parameter $\alpha = 0.95$. Final results are portrayed in Figure 3. In both cases, we run the V-SSA from 0 to 100Hz for a maximum number of 30 iterations per frequency.

Conclusions

We have generalized SSA denoising and reconstruction to the vector field case. We first embed the 3C data into a Hankel matrix. Then, we apply rank-reduction and anti-diagonal averaging to estimate the denoised data. Like in the scalar case, noise increases the rank of the ideal Hankel matrix of the uncorrupted data. Then, rank reduction is used to attenuate noise. Dead traces also increase the rank of the Hankel matrix of the ideal fully sampled data. The SSA iterative imputation algorithm was also generalized to reconstruct vector field measuments.

References

Burroughs, L., and S. Trickett, 2009, Prestack rank-reducing noise suppression: Practice: SEG Technical Program Expanded Abstracts 2009, 3337–3341.

Chen, K., and M. D. Sacchi, 2013, Robust reduced-rank seismic denoising: SEG Technical Program Expanded Abstracts 2013, 4272–4277.

Gao, J., M. D. Sacchi, and X. Chen, 2013, A fast reduced-rank interpolation method for prestack seismic volumes that depend on four spatial dimensions: Geophysics, **78**, V21–V30.

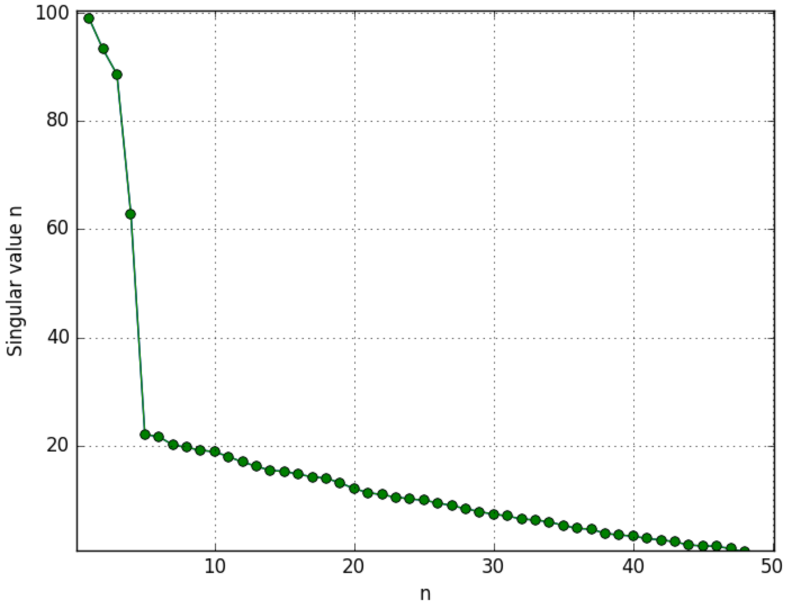

Figure 1: Singular values of the vector Hankel matrix for noise data and for one monochromatic frequency (30Hz). Four polarized waves of different dips are present in the data

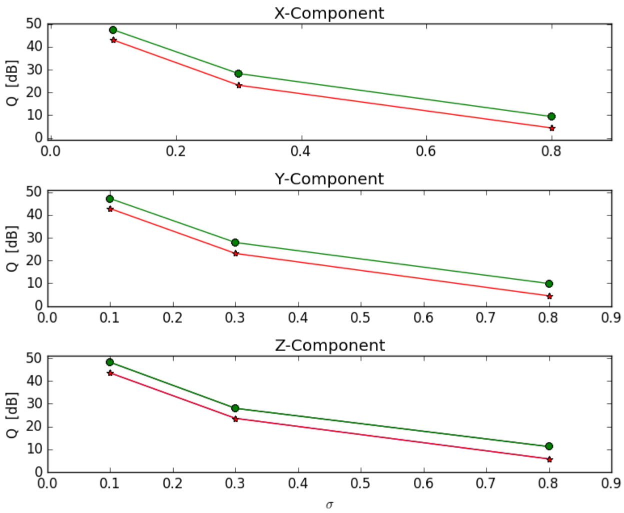

Figure 2: Quality of the reconstruction in dB. Circles correspond to vector reconstruction (V-SSA) and stars to scalar (SSA) reconstruction. In both cases a subspace of $p = 4$ eigen-images was used to denoise both scalar and vector Hankel matrices. A consistent gain of 5dB is obtained by using vector denoising (V-SSA)

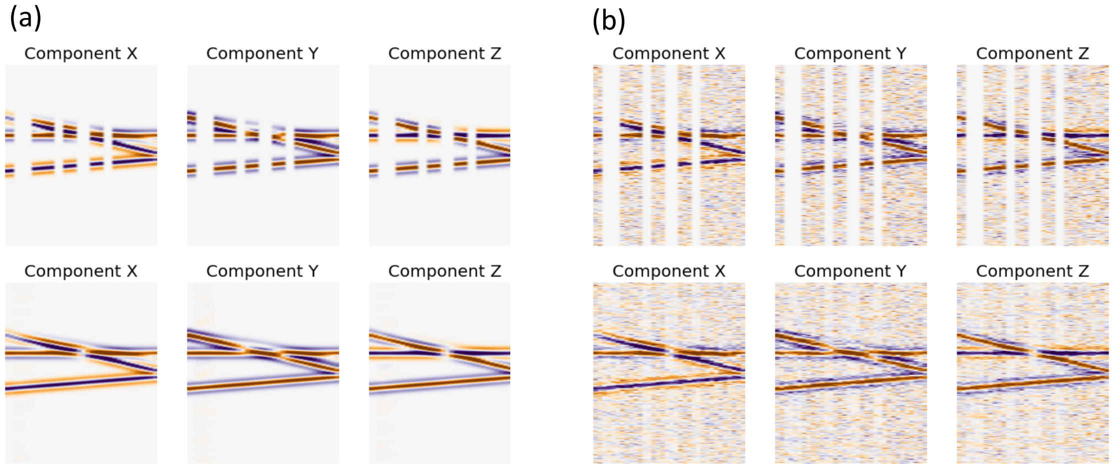

Figure 3: Denoising (a) and reconstruction (b) of 3C data via V-SSA.

Kamil, Y. I., P. L. Vermeer, and M. Vassallo, 2015, Noise attenuation in multimeasurement streamer data using weighted vector auto-regressive operators: SEG Technical Program Expanded Abstracts 2015, 4636–4640.

Naghizadeh, M., and M. Sacchi, 2013, Multidimensional de-aliased Cadzow reconstruction of seismic records: Geophysics, **78**, A1–A5.

Naghizadeh, M., and M. D. Sacchi, 2012, Multicomponent f-x seismic random noise attenuation via vector autoregressive operators: Geophysics, **77**, V91–V99.

Oropeza, V., and M. D. Sacchi, 2011, Simultaneous seismic data de-noising and reconstruction via multichannel singular spectrum analysis (MSSA): Geophysics, **76**, V25–V32.

Trickett, S., 2008, F-xy Cadzow noise suppression: SEG Technical Program Expanded Abstracts 2008, 2586–2590.

Trickett, S., and L. Burroughs, 2009, Prestack rank-reducing noise suppression: Theory: SEG Technical Program Expanded Abstracts 2009, 3332–3336.