Improving efficiency of first-arrival traveltime tomography by stochastic optimization

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Summary

Near-surface imaging plays a significant role in producing high quality data from land or shallow marine environments. First-arrival traveltime tomography is a common approach for solving the near-surface imaging problem from which results are needed to calculate robust and accurate static solutions. However, traveltime tomography involves inverting massive first-arrival picks. We believe that there is redundant information in traveltime picks and we propose to explore randomization methods to decrease the number of first-arrival picks that are used to compute near surface models. The latter leads to an improvement in terms of the efficiency of traveltime tomography leading to faster turnaround times. An efficient inversion also permits to explore solutions in terms of the trade-off parameters that control the smoothness of tomographic inversion. We adopt a Stochastic Approximation (SA) strategy to use only a small part of the data to retrieve high quality near surface models. A synthetic and a real data example are used to test the validity of the method. We also use these tests to understand the amount of data reduction that the method can tolerate before degrading the solution. For this purpose, a detailed statistical analysis was carried out. Relatively large deviations from the inverted data using all the data and the data that uses only a small percentage of picks occurs in areas of poor ray coverage. However, this effect is negligible in terms of the statics solutions that one can obtain by the tomographic inversion. For the real data example, the long-wavelength statics calculated from the velocity model derived by the new method is identical to model obtained by classical non-linear optimization using all the picks. However, a saving of 96% in terms of computational cost was achieved using the SA strategy.

Introduction

Motivated by the field of Stochastic Approximation (SA) (Nemirovskii et al., 2009), we intend to enhance the efficiency of the first-arrival traveltime tomography (Zhang and Toksöz, 1998; Zhu et al., 2008). This research is especially meaningful for dealing with large 3D datasets. However, our initial efforts will pertain 2D non-linear traveltime inversion. SA has been adopted in conjunction with compressing sensing to alleviate the computational Full Waveform Inversion (FWI)(Herrmann et al., 2011). Our focus is on reducing the computational cost of the traveltime tomography by applying SA to the solution of the non-linear tomographic inverse problem. Additionally, we further analyze the relationship between the stability of the inversion result and the efficiency of the SA method.

Theory

Equation (1) shows the objective function of first-arrival traveltime tomography

$$\phi(m) = \min \left\{ \sum_{s,r} (d_{s,r} - G(m, q_{s,r}))^2 + \alpha \| L(m) \|^2 \right\}. \quad (1)$$

The vector $m$ is the model slowness, $d_{s,r}$ represents the observed traveltime corresponding to the $s^{th}$ shot and $r^{th}$ receiver. Similarly, $G(m, q_{s,r})$ describes the calculated traveltime, $q_{s,r}$ is the raypath from the $s^{th}$ shot to the $r^{th}$ receiver. The scalar $\alpha$ is a constant parameter for balancing the data misfit and model regularization term. In our work, $L$ is the Laplacian operator that is used to minimize
roughness in the solution of the tomographic images. SA is a method for solving large optimization problems of the form

\[ \min_{x \in \Theta} f(x), \]  

where the real-valued function cannot be computed exactly, but it can be estimated through a stochastic simulation. We assume that the function \( f \) cannot be observed or computed directly and we also assume that \( f(x) = E[Y(x, \xi)] \), where \( \xi \) is a random variable with a distribution that does not depend on \( x \). We also consider \( Y(\cdot, \cdot) \) a real-valued function. In SA, one selects \( x_1, x_2, \ldots, x_n \), all having the same distribution as \( x \), and set

\[ f_n(x) = \frac{1}{n} \sum_{i=1}^{n} Y(x, \xi_i). \]  

If the sample \( \xi_1, \xi_2, \ldots, \xi_n \) are renewed in each iteration during every computation of \( Y(x, \xi) \) then one can solve the problem \( \min_{x \in \Theta} f(x) \) via

\[ \min_{x \in \Theta} f_n(x). \]  

In our problem, this is equivalent to randomly extract a part of picks in each iteration of the non-linear optimization process. We call our algorithm SAtomo.

**Synthetic test**

We test the new method on a synthetic model which is shown in Figure 1a. The acquisition geometry includes 120 shots and 100 receivers which are evenly distributed on the two sides of each shot. The shot spacing is 40 m and the receiver interval is 20 m, respectively. We extract the traveltime data by the way of the Monte Carlo Method. Both shots and receivers are extracted randomly. For example, we extract 10% shots and 10% receivers, which means we use 1% of the traveltime data in each iteration of the inversion process. Similarly, we test SAtomo with different percentage of traveltime data, including 4% (20% shots, 20% receivers), 9% (30% shots, 30% receivers), ... , 81% (90% shots, 90% receivers). For each test, we perform 100 inversions to investigate the relationship between the percentage traveltime picks and the deviation of the result from the ideal solution that uses all the picks.

Figure 1a shows the true model. Figure 1b is the result using all the traveltime picks. Figure 1c is one model of the 100 solutions computed using in each iteration of the non-linear optimization 4% of the data. Figure 1d shows the ray paths. Similarly, Figures 1e and 1f portray the standard error of the velocity estimate for each grid point for 1% and 4% data, respectively. Figures 2a shows the RMS error of the difference between the SAtomo solution and the solution using all the traveltime picks versus percentage of data used by SAtomo. The 100 realizations run for each test permitted us to also estimate error bars. According to the raypaths shown in Figure 1d, we found that large deviations occurs in areas with poor ray coverage. This phenomenon is reasonable because even for the standard result, which is obtained using all traveltime data, the reliability of the area with low ray coverage should not be high. However, for the area with high ray density, there the deviation is minimal. We stress that there is no large difference between the solutions with all the data and 4% of the data in each iteration. However, SAtomo has a saving in computational cost of 96%.

**Real data application**

We further apply the SAtomo method to a real dataset. This is a 2 – D land seismic data acquired in the middle east. The acquisition system is a roll-over geometry, which includes 558 shots and 1124 receivers with 60 m shot spacing and 30 m receiver interval. We pick the first breaks and generate a model by applying the Generalized Linear Inversion (GLI) method, which will be employed as the initial model for the subsequent traveltime tomography after smoothing (Figure 3a). For this case, we also perform 100 inversions and 20 iterations for each inversion. Figure 2b shows RMS error curve of this real data application. We observe that the curve is similar property to the synthetic one. We also
display the error of each grid to investigate the area with low stability (Figures 3e and 3f). According to the raypaths shown in Figure 3d, the large deviation also occurs in the area with poor ray coverage, which is consistent with the synthetic test. Figures 3b and 3c show the standard result and one solution of the 100 inversions when the percentage of data is 4%, respectively. We found that they are almost identical. Since the ultimate deliverables from the near-surface modeling are shot-receiver statics, not the near-surface model itself, which should be treated as an intermediate product, we also show the long-wavelength statics in Figure 2c. The latter were calculated by standard inversion (Figure 3b) and via SAtomo (Figure 3c), respectively. The red dots and blue dots represent the shot statics and receiver statics calculated by standard inversion that uses all traveltime picks. On the other hand, the pink and green dots are employed to represent the statics calculated via SAtomo. Figure 2d shows the difference between the two sets of long-wavelength statics, in which the red and yellow dots are for shots and receivers, respectively. We observe that the largest deviation of long-wavelength statics derived by the two near-surface models is approximately 3 ms; such a small deviation is considered to be negligible for subsequent data processing. Again, a saving of 96% in computational cost was achieved by using SAtomo.

Figure 1: (a) The true model. (b) The standard tomographic inversion obtained using all the data. (c) The SAtomo result with 4% of data in each iteration of the non-linear optimization algorithm. (d) The raypaths of the true model. (e) The standard deviation of the model which corresponding to adopting 1% of data in SAtomo. (f) The standard deviation the model corresponding to adopting 4% of data in SAtomo.

Conclusions

An efficient traveltome tomography method named SAtomo is proposed for reducing the computational cost of traveltome tomography. We verify that the SA method can be employed for the problem of first-arrival tomography. A synthetic test and a real data application verify the efficiency of the new method. Our future plans entail applying SAtomo to 3D traveltome tomography.

References


Figure 2: (a) Error curve for the synthetic mode. The error measures departure of the SAtomo solution from the standard solution. Error bars were obtained by running 100 realization per inversion. (b) Error curve for the real data example, (c) The long-wavelength statics of the real data example, which is calculated by different near-surface models generated by standard inversion and SAtomo, respectively. (d) The difference between the statics derived from standard and SAtomo inversions.

Figure 3: (a) The initial model of the real data example. (b) The standard tomographic inversion obtained using all the data. (c) The SAtomo result with 4% of data in each iteration of the non-linear optimization algorithm. (d) The raypaths of the standard result. (e) The standard deviation of the model which corresponding to adopting 1% of data in SAtomo. (f) The standard deviation the model corresponding to adopting 4% of data in SAtomo.