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Towards seismic moment tensor inversion for source mechanism

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Summary

The objective of this work is to obtain seismic moment tensor, M_{pq}, from amplitude inversion of multi-component microseismic data. M_{pq} describes source mechanism and can be decomposed into double-couple, isotropic and compensated-linear-vector-dipole (related to tensile fracturing) components - the tensile components might show a correlation to hydrocarbon production rate. The retrieved M_{pq} are sensitive to the accuracy of the source location, the accuracy of the velocity model, and the receiver array geometry. Excluding the first two factors, the determination of the proper observation geometry for which the full moment tensor is resolvable was sought. Having two vertical or surface receiver arrays, or a combination of them, M_{pq} may be fully determined using the inversion of P- and S-wave first arrival amplitudes. This configuration, which is sufficient for this purpose, includes the receiver arrays located in the vertical and horizontal part of a single deviated well. To avoid the cumbersome task of picking first-arrival amplitudes, we adopted a waveform inversion scheme based on the method proposed by Vavrycük and Kühn (2012). This method combines inversions in both the time and frequency domains. The first requirement was the inversion for the source-time function in the frequency domain. Second, a time domain inversion for M_{pq} using the source-time function calculated in the first step was initiated. For this waveform inversion, we have not yet achieved an accurate retrieval of M_{pq}, but we have been able to obtain an accurate source-time function estimate.

Introduction

Determination of seismic moment tensor is a routine procedure in earthquake seismology (e.g., Dziewonski et al., 1981; Jost and Hermann, 1996; Šílený, 1998; Shearer, 1999). The seismic moment tensor provides a general representation of the seismic source and can be determined from inversion of seismic amplitudes detected on surface/downhole receiver arrays. M_{pq} is commonly decomposed into a double-couple tensor, produced by shear faulting, and a non-double couple tensor, produced by the opening or closing of faults (tensile fracturing) (Vavryčuk, 2001). Hence M_{pq} provides information related to the physical processes at the source (e.g., Ross et al., 1999; Maxwell and Urbancic, 2001; Foulger et al., 2004; Vavryčuk, 2007).

A point source with a general radiation pattern can be represented by the seismic moment tensor. In a Cartesian coordinate system, the seismic moment tensor is symmetric 3×3 matrix of six independent force-couples M_{pq} $(M_{11}, M_{22}, M_{33}, M_{23}, M_{13}, M_{12})$. A force-couple M_{pq} is defined as a pair of opposing forces pointing in the p direction, separated in the q direction. The 3-components of the displacement wavefield observed at a geophone located at $\vec{x}_r$ due to a source located at $\vec{x}_s$ (which described by its moment tensor), are expressed as (Aki and Richards, 2002, equation 3.23)

$$u_n(\vec{x},t) = g_{np,q}(\vec{x},t) * m_{pq}(t), \tag{1}$$

where $*$ is convolution symbol, $n = 1, 2, 3$, summation convention is applied, and $\vec{x} = \vec{x}_r - \vec{x}_s$ is the vector of source-receiver offset. $m_{pq}(t)$ is the time-dependent seismic moment tensor. $g_{np,q}(\vec{x},t)$ is the spatial derivative of the Green's function which describes the properties of the medium. For a point source, the time-dependent moment tensor can be factorized as $m_{pq}(t) = M_{pq} S(t)$, where *S(t)* is the source-time function and M_{pq} is the seismic moment tensor which are trying to invert for.

Determination of the M_{pq} is a demanding procedure influenced by several factors. The most important of these are: 1) Observations from many stations (good station coverage of the focal zone), which here is termed receiver geometry effect. 2) Deriving the representative source-time function. 3) An accurate knowledge of the velocity model and focal location, in order to calculate the Green's functions. 4) The availability of quality data, to enable event picking in the source location step. We report the effect of receiver geometry on retrieval of the seismic moment tensor, for vertical or surface arrays while ignoring the other three factors listed above. As was established by others (e.g., Vavryčuk, 2007; Rodriguez et al., 2011), we found that the full moment tensor may be determined using P- and S-wave first-arrival amplitudes from two receiver arrays (vertical or surface) observing the source from two distinctive azimuths. Using data from a single vertical or surface array, five terms may be resolved. Having receivers in both vertical and horizontal part of a deviated well, we found, regardless of the azimuth of the well, the full moment tensor is resolvable when using P- and S-wave amplitude data.

Another key factor in recovering seismic moment tensor is the determination of the time variation of the source, *S(t)*. The source-time function is often assumed to have a simple form, a single pulse appearing in all elements of the moment tensor. While this is an oversimplified assumption for earthquake sources, it is a reasonable assumption for a point source directly applicable to microseismic events. With this assumption, the determination of M_{pq} and the source-time function is still a non-linear problem (from equations 2) and requires non-linear inversions of the seismic waveform (Šílený, 1998). Avoiding a non-linear inversion, we follow Vavryčuk and Kühn (2012) and apply a two-step linear waveform inversion first to estimate the source-time function in frequency domain, and then resolving the M_{pq} in time domain.

Moment tensor inversion of P- and S- wave amplitudes to investigate receiver geometry effect

The 3-components of the total far-field displacement wavefield (equation 2) can be written as the sum of the far-field P- and S-wave displacements, $u_n(\vec{x},t) = u_n^P(\vec{x},t) + u_n^S(\vec{x},t)$ which for a homogeneous medium are expressed as (Aki and Richards, 2002)

$$u_{\boldsymbol{n}}^P(\vec{x},t) = \sum_p \sum_q \frac{\gamma_n \gamma_p \gamma_q}{4\pi\alpha^3 r} M_{pq} \dot{S}(t - \frac{r}{\alpha}) \ , \tag{2}$$

$$u_{\boldsymbol{n}}^S(\vec{x},t) = \sum_p \sum_q \frac{(\delta_{np} - \gamma_n \gamma_p)\gamma_q}{4\pi\beta^3 r} M_{pq} \dot{S}(t - \frac{r}{\beta}). \tag{3}$$

r , a, and b are the density, P-velocity, and S-velocity respectively. $r = |\vec{x}|$ is the source-receiver distance. Comparing equations 2-3 to equation 1, the Green's function can be defined as

$$g_{np,q}(\vec{x},t) = \begin{cases} \gamma_n \gamma_p \gamma_q / 4\pi\alpha^3 r, & \text{for } \ t = r/\alpha \\ (\delta_{np} - \gamma_n \gamma_p)\gamma_q / 4\pi\beta^3 r, & \text{for } \ t = r/\beta \\ 0, & \text{for other } \ t \end{cases} \ . \tag{4}$$

Equation (3) can be written as $u_{\boldsymbol{n}}^P(\vec{x},t) = A\gamma_n \sum_p \gamma_p \sum_q M_{pq}\gamma_q$, which for simplicity we took $A = \dfrac{1}{4pr\,a^3}$ and assumed that the source-time function is delta function with unit amplitude. This has the form of matrix multiplication; hence equation (2) can be written as

$$\begin{bmatrix} u_1^P \\ u_2^P \\ u_3^P \end{bmatrix} = A \begin{bmatrix} g_1 \\ g_2 \\ g_3 \end{bmatrix} \begin{bmatrix} g_1 & g_2 & g_3 \end{bmatrix} \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{12} & M_{22} & M_{23} \\ M_{13} & M_{23} & M_{33} \end{bmatrix} \begin{bmatrix} g_1 \\ g_2 \\ g_3 \end{bmatrix}. \tag{5}$$

Doing this matrix multiplication, the P-wave displacement can be written as a linear function of the six independent M_{pq} as

$$\begin{bmatrix} u_1^P \\ u_2^P \\ u_3^P \end{bmatrix} = A \begin{bmatrix} g_1^3 & g_1g_2^2 & g_1g_3^2 & 2g_1g_2g_3 & 2g_1^2g_3 & 2g_1^2g_2 \\ g_2g_1^2 & g_2^3 & g_2g_3 & 2g_2^2g_3 & 2g_1g_2g_3 & 2g_2^2g_1 \\ g_3g_1^2 & g_3g_2^2 & g_3^3 & 2g_3^2g_2 & 2g_3^2g_1 & 2g_1g_2g_3 \end{bmatrix} \begin{bmatrix} M_{11} \\ M_{22} \\ M_{33} \\ M_{23} \\ M_{13} \\ M_{12} \end{bmatrix}. \tag{6}$$

Similarly, from equation 3, the S-wave displacement can be written as

$$\begin{bmatrix} u_1^S \\ u_2^S \\ u_3^S \end{bmatrix} = B \begin{bmatrix} g_1 - g_1^3 & -g_1g_2^2 & -g_1g_3^2 & -2g_1g_2g_3 & g_3 - 2g_1^2g_3 & g_2 - 2g_1^2g_2 \\ -g_2g_1^2 & g_2 - g_2^3 & -g_2g_3 & g_3 - 2g_2^2g_3 & -2g_1g_2g_3 & g_1 - 2g_2^2g_1 \\ -g_3g_1^2 & -g_3g_2^2 & g_3 - g_3^3 & g_3 - 2g_3^2g_2 & g_1 - 2g_3^2g_1 & -2g_1g_2g_3 \end{bmatrix} \begin{bmatrix} M_{11} \\ M_{22} \\ M_{33} \\ M_{23} \\ M_{13} \\ M_{12} \end{bmatrix}, \tag{7}$$

which for simplicity *B* is defined similar to *A*. Using equations 6 and 7, P- and S-wave first arrival amplitudes from multiple receivers can be used to set up a linear system of equations $D = \underline{G}M,$ where *D* is data vector (P- and S-wave first arrival amplitudes), $\underline{G}$ is the geometry matrix, and *M* the model parameters $(M_{11}, M_{22}, M_{33}, M_{23}, M_{13}, M_{12})$; we solved it using damped least-squares method (the damping factor is decided based on the ratio of the max/min singular values of the matrix $\underline{G}$.

We tested the moment tensor inversion on 3C synthetics seismograms for a homogeneous isotropic medium with α = 3000 m/s, β = 2000 m/s, and ρ = 2000 Kg/m^3. We generated the 3C synthetics data by convolving a source-time function with the Green's functions. The synthetics data are recorded at multiple borehole/surface arrays and used in the moment tensor inversion described above. For all of our tests, the moment tensor source of $M = (M_{11}, M_{22}, M_{33}, M_{23}, M_{13}, M_{12}) = (1, -2, 4, -1, 0.5, 6)$ has been used. The resolvability of the full moment tensor, under single and multiple vertical/surface receiver array geometries, is examined. Employing P- and S-wave amplitudes simultaneously in the moment tensor inversion, we found that 1) Having a single vertical or surface array, all elements but the M_{22} will be resolved and there exists only one null singular values. 2) Having two vertical or surface array, the full moment tensor will be resolved. 3) The resolvability of the model parameters is insensitive to the distance between the array of receivers and the source. As most of the microseismic monitoring are based on a single well configuration, the use of receiver arrays in a deviated well which receivers are laid out in both vertical and nearly horizontal part of the well is suggested. Hence a simultaneous amplitude inversion, using data from a deviated well should satisfy the resolvability of full moment tensor. This deviated well configuration was suggested by Rodriguez et al. (2011). Figure 6 shows a deviated well, with receivers in both vertical and horizontal part of the well, that full moment tensor is retrievable.

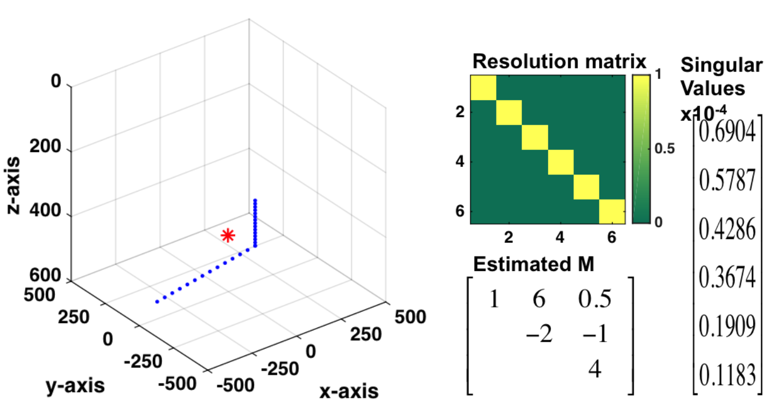

Figure 1: Moment tensor inversion for a deviated well geometry, displaying the resolution matrix and singular values

Presented above, the amplitude data were picked from synthetics noise-free seismograms; we ignored all the complexity in picking the P- and S-wave first arrivals. For real data, first arrival amplitude picking is a cumbersome task and normally performed on data after being rotated into radial and transverse components. Additionally, knowing the source-time function that was used in generating the numerical modeled data, we applied the accurate scalar to remove the effect of source-time function. However, knowledge of the source-time function is never achieved in real data prior to the moment tensor inversion. Next following Vavrycuk and Kuhn (2012), we apply a waveform inversion to first estimate the source-time function and second estimate the full moment tensor in a time-frequency approach. No amplitude picking is required for the following waveform inversion.

Moment tensor inversion of waveforms

Step 1: The pair of indices in the Green's function (equation 4) can be reduced to define the G_{kj} $(k=1:3, j=1:6)$ functions as: $G_{k1}=g_{k1,1}$, $G_{k2}=g_{k2,2}$, $G_{k3}=g_{k3,3}$, $G_{k4}=g_{k2,3}+g_{k3,2}$, $G_{k5}=g_{k1,3}+g_{k1,3}$, and $G_{k6}=g_{k1,2}+g_{k2,1}$. Then in frequency domain equation 1 will transform

$$\hat{u}_n(\vec{x},\omega)=G_{kj}(\vec{x},\omega)m_{pq}(\omega), \tag{8}$$

For each frequency, ω, equation 8 could be used to construct a linear system of equations "$d=Gm$" to invert for the $m(W)=(m_{11}(W),m_{22}(W),m_{33}(W),m_{23}(W),m_{13}(W),m_{12}(W))$, which the data vector is the Fourier transform of the recordings at each receive. We solved this linear system using a least-squares method. Then taking Fourier transform of each column of this "*m(ω)*" matrix will result in the matrix of the six time-dependent moment tensor vectors, $m(t)=(m_{11}(t),m_{22}(t),m_{33}(t),m_{23}(t),m_{13}(t),m_{12}(t))$. We took the singular value decomposition of this "*m(t)*" matrix, the eigenvector associated with the largest singular value is the source-time function, *S(t)* (Vasco,1989).

Step 2: Now that the source-time function is estimated, it will be taken out of the displacement wavefield. In this regard, the elementary seismograms and their spatial derivatives are defined as

$$e_{np}(\vec{x},t)=S(t)*g_{np}(\vec{x},t), \qquad \mathrm{e}_{np,q}(\vec{x},t)=S(t)*g_{np,q}(\vec{x},t). \tag{9}$$

Substituting these elementary functions into equation 1, the displacement components become:

$$u_n(\vec{x},t)=e_{np,q}(\vec{x},t)M_{pq}. \tag{10}$$

With the same fashion as practiced in defining the "G_{kj}" functions, these elementary functions can be treated to obtain E_{kj} functions. By constructing the E_{kj} functions, a linear system of equation can be set up to solve for M_{pq}. The data vector is the recorded trace at each receiver, $u_n(\vec{x},t)$. Again, this time-domain inversion is solved using a damped least-squares method. We applied this time-domain waveform inversion on the synthetics microseismic data (generated using TIGER software from SINTEF Petroleum Research) inverting for the source moment tensor, and obtained promising results but not the exact moment tensor that we used in the modeling. However, we obtained the source-time function that we used in the modeling (Figure 2). Retrieving the exact M_{pq} is expected for such noise free data with accurate source location and velocity model, which requires more research at this point.

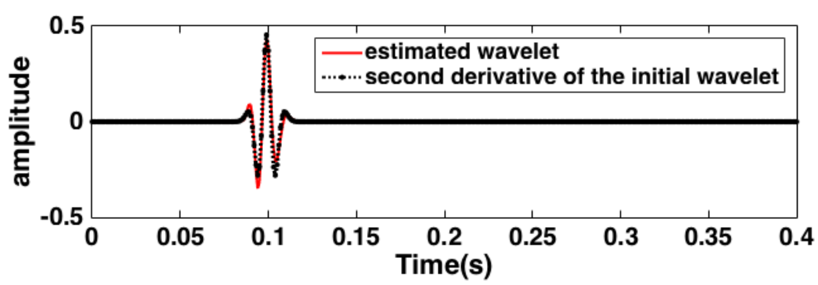

Figure 2: Estimated source-time function from frequency domain inversion in step 1.

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