# Bayesian Markov Chain Monte Carlo inversion for fluid term and dry fracture weaknesses

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## Summary

Fracture weaknesses and fluid factor are important parameters to identify the location of underground fractures and the type of fluids. We demonstrate a direct method to estimate Lamé constants, fluid term, and dry fracture weaknesses from partially incident- angle- stack seismic data, based on azimuthal elastic impedance (EI) inversion. Combining stiffness parameter perturbations and scattering function, we derive a linearized PP- wave reflection coefficient and azimuthal EI for the case of an interface separating two horizontal transverse isotropic (HTI) media. The estimation of fluid term and fracture weaknesses is implemented as a two- step inversion which includes the inversion of partially- incident- stack seismic data for EI at different azimuths, and the prediction of fluid term and fracture weaknesses from the inversion results of azimuthal EI using a Bayesian Markov- chain Monte Carlo (MCMC) method. Tests on synthetic and real data can confirm the stability of the proposed inversion method, and the inversion method appears to be useful for fracture detection and fluid discrimination.

## Introduction

Fracture detection and filling discrimination are two important tasks for the unconventional reservoir (shale, tight sand, etc.) characterization. Hudson cracked model (Hudson 1980) and the linear slip model (Schoenberg and Douma, 1988; Schoenberg and Sayers, 1995) are defined to describe effects of cracks and fractures on stiffness and compliance matrices. Bakulin et al. (2000) present relationships between fracture properties (fracture density, fracture aspect ratio, and fillings) and fracture weaknesses. Combining these relationships, Chen et al. (2014) propose an indirect method to estimate the normal and tangential fracture weaknesses first and then to calculate the fluid factor which is proposed by Schoenberg and Sayers (1995).

A rock, containing a set of vertical or sub-vertical fractures, is considered to be a horizontal transverse isotropic (HTI) medium. Rüger (1997) derives an approximate expression of PP- wave reflection coefficient in terms of anisotropic parameters for HTI media. Shaw and Sen (2004) present a different method to combine scattering function and stiffness parameter perturbations to derive linearized reflection coefficients for weak anisotropic media. Based on the Rüger equation, amplitude variation with offset and azimuth (AVOA) data are employed to estimate anisotropic parameters (Downton and Roure, 2010). However, AVOA data are usually influenced by random noises. Elastic impedance (EI) is first introduced by Connolly (1998). For a vertically fractured medium, EI changes with the angle of incidence and azimuth (Martins, 2006).

In the present study, under the assumption of small fracture weaknesses and low-moduli fillings, we derive the linearized PP- wave reflection coefficient and azimuthal EI in terms of Lamé constants, density, Gassmann fluid term, and dry fracture weaknesses. Based on the derived azimuthal EI, we demonstrate a method to estimate Lamé constants, density, fluid term, and fracture weaknesses from azimuthal seismic data. Azimuthal EI inversion from partially incident-angle-stack seismic data is implemented by using a least- square algorithm, and Bayesian Markov- chain Monte Carlo (MCMC) method is utilized to extrac parameters from the inversion results of azimuthal EI. Synthetic tests indicate that the unknown parameters can be estimated reasonably while seismic traces contain a moderate noise. A test on real data demonstrates that the proposed inversion method is efficient for fracture prediction and fluid discrimination.

## Linearized PP- wave reflection coefficient and azimuthal EI

Effects of fluids on stiffness parameters of an anisotropic rock is first presented by Gassmann (1951), and Huang et al. (2015) propose a set of equations for fluid substitution in HTI media. Under the assumption of small fracture weaknesses and low moduli fillings, we present expressions of perturbations in stiffness parameters across the interface which separates two HTI media

$$\begin{aligned}
\Delta C_{11}^{\mathrm{sat}} &\approx \Delta(\lambda+2\mu)-(\lambda+2\mu)\delta_{\Delta_\mathrm{N}}+\Delta f+2\left(\Delta_\mathrm{N}\Delta K_f+\delta_{\Delta_\mathrm{N}}K_f\right),\\
\Delta C_{12}^{\mathrm{sat}} &\approx \Delta\lambda-\lambda\delta_{\Delta_\mathrm{N}}+\Delta f+(\chi+1)\Delta_\mathrm{N}\Delta K_f+(\chi+1)K_f\delta_{\Delta_\mathrm{N}},\\
\Delta C_{23}^{\mathrm{sat}} &\approx \Delta\lambda-\lambda\chi\delta_{\Delta_\mathrm{N}}+\Delta f+2\chi\Delta_\mathrm{N}\Delta K_f+2\chi K_f\delta_{\Delta_\mathrm{N}},\\
\Delta C_{33}^{\mathrm{sat}} &\approx \Delta(\lambda+2\mu)-(\lambda+2\mu)\chi^2\delta_{\Delta_\mathrm{N}}+\Delta f+2\chi\Delta_\mathrm{N}\Delta K_f+2\chi K_f\delta_{\Delta_\mathrm{N}},\\
\Delta C_{44}^{\mathrm{sat}} &\approx \Delta\mu,\quad \Delta C_{55}^{\mathrm{sat}}\approx\Delta\mu-\mu\delta_{\Delta_\mathrm{T}},
\end{aligned} \tag{1}$$

where $\lambda$ and $\mu$ are Lamé constants, $K_f$ is the bulk modulus of fluid, $f$ is the Gassmann fluid term, $\chi=\lambda/(\lambda+2\mu)$, and $\Delta_\mathrm{N}$ and $\Delta_\mathrm{T}$ are the normal and tangential fracture weaknesses.

Following Shaw and Sen (2004), we use the perturbation of stiffness parameter to derive a linearized PP- wave reflection coefficient for the saturated fractured medium.

$$R_{PP}=a_{\lambda_d}(\theta)R_{\lambda_d}+a_\mu(\theta)R_\mu+a_\rho(\theta)R_\rho+a_f(\theta)R_f+a_{\Delta_\mathrm{N}}(\theta,\varphi)\delta_{\Delta_\mathrm{N}}+a_{\Delta_\mathrm{T}}(\theta,\varphi)\delta_{\Delta_\mathrm{T}}, \tag{2}$$

where $\theta$ is P-wave incident angle, $\varphi$ is the azimuth, $a_{\lambda_d}(\theta)=1/\left(4\cos^2\theta\right)$, $a_\mu(\theta)=1/\left(4\cos^2\theta\right)-2g_s\sin^2\theta$, $a_\rho(\theta)=\cos2\theta/\left(4\cos^2\theta\right)$, $a_f(\theta)=1/\left(4\cos^2\theta\right)(1-g_s/g_d)$, $a_{\Delta_\mathrm{N}}(\theta,\varphi)=-1/\left(4\cos^2\theta\right)(g_s/g_d)\left[1-2g_d\left(\sin^2\theta\sin^2\varphi+\cos^2\theta\right)\right]^2$,

and $a_{\Delta_\mathrm{T}}(\theta,\varphi)=-g_s\tan^2\theta\cos^2\varphi\left(\sin^2\theta\sin^2\varphi-\cos^2\theta\right)$.

In addition, $R_{\lambda_d}$ and $R_\mu$ are Lamé constant reflectivities of the dry rock, $R_\rho$ is the density reflectivity, $R_f$ is the Gassmann fluid term reflectivity, $g_s=\mu/M_s$, $g_d=\mu/M_d$, and $M_s$ and $M_d$ are P-wave moduli of the saturated and the dry rock, respectively.

Following Buland and Omre (2003), we express the derived PP-wave reflection coefficient as a time-continuous function

$$\begin{aligned}
R_{PP}(t,\theta,\varphi)&=\frac{\partial}{\partial t}\ln\mathrm{EI}(t,\theta,\varphi)=a_{\lambda_d}(t,\theta)\frac{\partial}{\partial t}\ln\lambda_d(t)+a_\mu(t,\theta)\frac{\partial}{\partial t}\ln\mu(t)\\
&+a_\rho(t,\theta)\frac{\partial}{\partial t}\ln\rho(t)+a_f(t,\theta)\frac{\partial}{\partial t}\ln f(t)+a_{\Delta_\mathrm{N}}(t,\theta,\varphi)\frac{\partial}{\partial t}\Delta_\mathrm{N}(t)+a_{\Delta_\mathrm{T}}(t,\theta,\varphi)\frac{\partial}{\partial t}\Delta_\mathrm{T}(t)
\end{aligned}, \tag{3}$$

where $\lambda_d(t)$ and $\mu(t)$, $\rho(t)$, $f(t)$, $\Delta_\mathrm{N}(t)$, and $\Delta_\mathrm{T}(t)$ are time-dependent Lamé constants, density, Gassmann fluid term, and the normal and tangential fracture weaknesses, respectively.

The expression of azimuthal EI is given by taking an integral operation

$$\mathrm{EI}(t,\theta,\varphi)=\left[\lambda_d(t)\right]^{a_{\lambda_d}(\theta)}\left[\mu(t)\right]^{a_\mu(\theta)}\left[\rho(t)\right]^{a_\rho(\theta)}\left[f(t)\right]^{a_f(\theta)}\exp\left[a_{\Delta_\mathrm{N}}(\theta,\varphi)\Delta_\mathrm{N}(t)+a_{\Delta_\mathrm{T}}(\theta,\varphi)\Delta_\mathrm{T}(t)\right]. \tag{4}$$

## Nonlinear Inversion of azimuthal EI for fluid term and fracture weaknesses

In order to predict Gassmann fluid term and fracture weaknesses, we demonstrate a method to first estimate EI from partially incident-angle-stack seismic data at different azimuths, and then extract Gassmann fluid term and dry fracture weaknesses from the estimated EI.

The PP- wave reflection coefficient, $R_{PP}(t,\theta,\varphi)$, can be calculated by the azimuthal EI

$$R_{PP}(t,\theta,\varphi)=0.5\Delta\mathrm{EI}(t,\theta,\varphi)/\ \overline{\mathrm{EI}}(t,\theta,\varphi)\approx 0.5d\ln\left[\mathrm{EI}(t,\theta,\varphi)\right], \quad (5)$$

where $\Delta\mathrm{EI}$ and $\overline{\mathrm{EI}}$ are difference and mean values between the upper and lower layers, respectively.

The prediction of azimuthal EI results is an inversion of stack seismic data, which is implemented by using partially incident-angle-stack seismic data and wavelets at different azimuths. The least- square method is employed to solve the inverse problem to obtain the logarithm results of azimuthal EI.

Combining equation (5), we may extract Lamé constants, density, fluid term, and fracture weaknesses from the estimated logarithm results of azimuthal EI. In the present study, following a Bayesian framework, we develop a method to predict the elastic parameters (Lamé constants and density), Gassmann fluid term, and dry fracture weaknesses, based on Markov- chain Monte Carlo (MCMC) algorithm.

## Examples

Azimuthally synthetic seismic data, which are generated by using well- logs, a 40HZ Ricker wavelet, and the convolutional model, are used to verify the proposed inversion method. A random noise is added to synthetic seismic data to test the robustness of the inversion method. Figure 1 shows comparisons between true values (blue) and inverted results (red) of well- log data, which indicates that the proposed inversion method can obtain a reasonable result when seismic data contain a moderate noise.

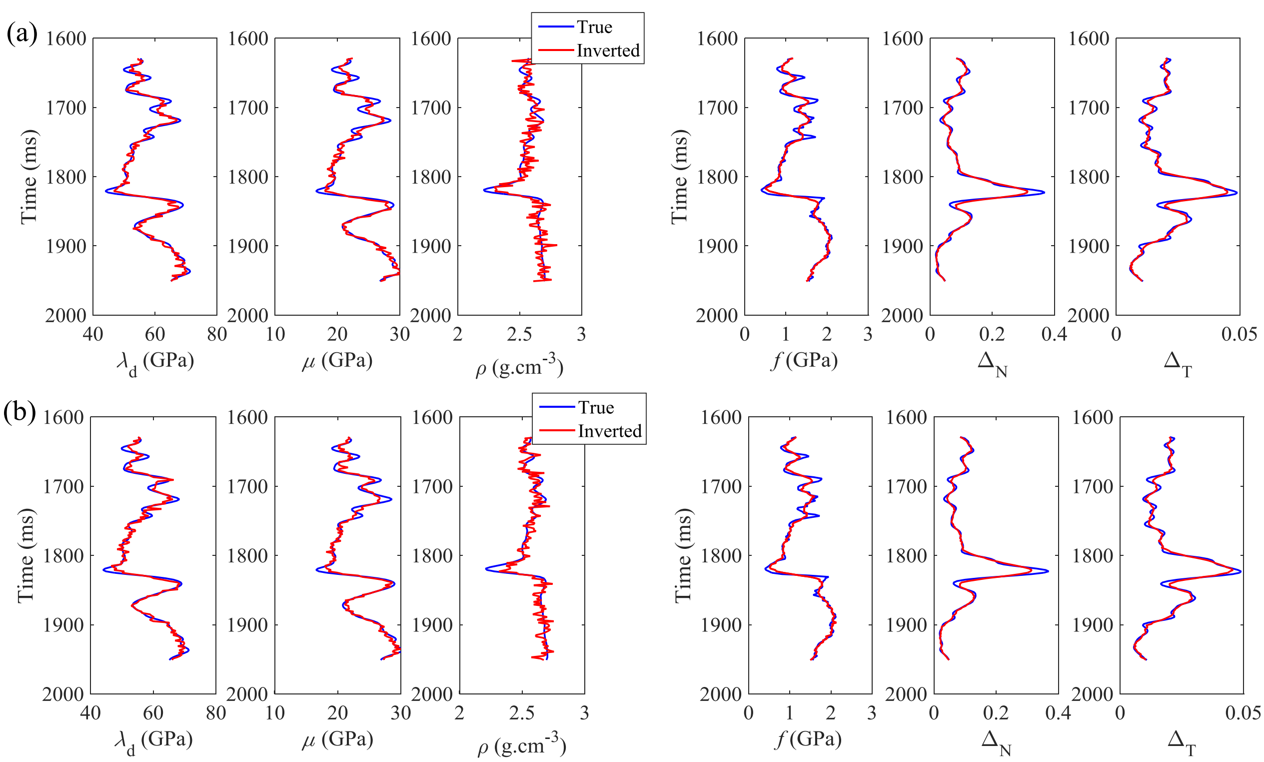


Figure 1. Comparisons between true values (blue) and inverted results (red) of well- log data.

Partially incident-angle-stack real data, which are from a fractured shale reservoir, are utilized to test the reliability of our nonlinear inversion method. After the inversion of azimuthal EI, we may implement the estimation of Gassmann fluid term and dry fracture weaknesses with the proposed Bayesian MCMC inversion method. Figure 2 shows the inversions results of Lamé constants, density, Gassmann fluid term, and dry fracture weaknesses, and the black circle in each figure means the location of the target reservoir. We can see that the inverted Lamé constants and Gassmann fluid term show low values and fracture weaknesses show high values in the location of the reservoir.

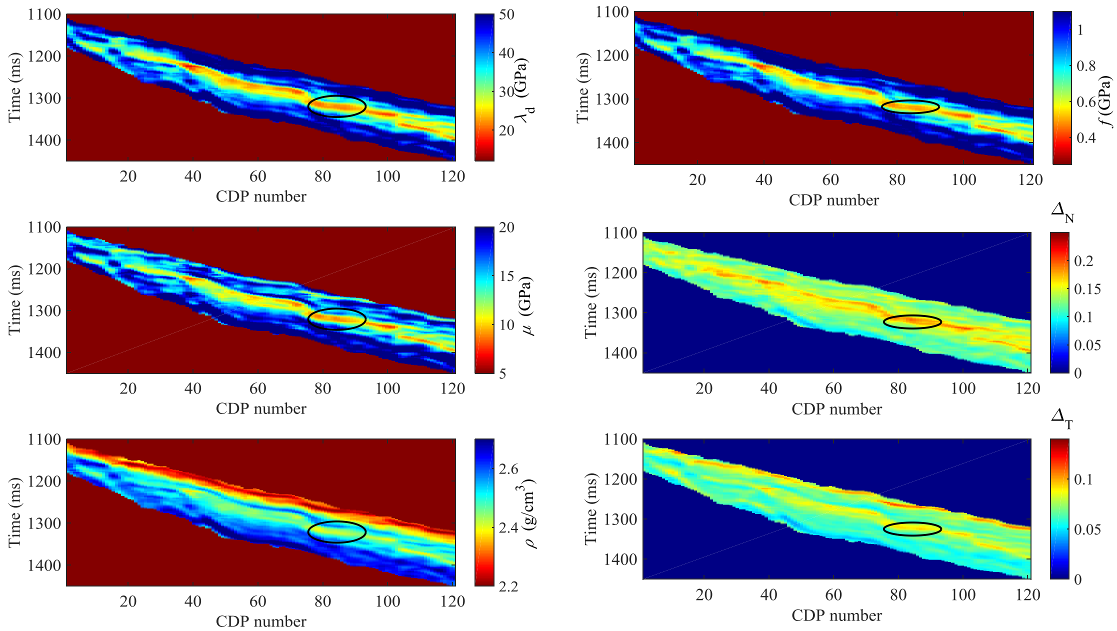


Figure 2. Inversion results of Lamé constants, density, Gassmann fluid term, and dry fracture weaknesses.

## Conclusions

We demonstrate a method to estimate Gassmann fluid term and dry fracture weaknesses from azimuthally seismic data, based on azimuthal elastic impedance (EI) inversion. We first propose perturbations in stiffness parameters for an interface separating two HTI media. Then we derive the linearized expression of PP- wave reflection coefficient and azimuthal EI in terms of Lamé constants, density, Gassmann fluid term, and dry fracture weaknesses, which can isolate effects of the isotropic dry rock framework, fluid, and fractures. Based on the azimuthal EI, we demonstrate a method to predict Gassmann fluid term and dry fracture weaknesses from azimuthally seismic data. The prediction is implemented as a two-step inversion, which includes partially incident-angle-stack seismic data inversion for azimuthal EI using a least- square method, and the extraction of Lamé constants, density, Gassmann fluid term, and dry fracture weaknesses from the inverted azimuthal EI with a Bayesian MCMC inversion algorithm. Synthetic seismic traces (SNRs are 5, and 2, respectively) and real data are utilized to verify the stability of our inversion method. Synthetic tests indicates that our method may obtain a reasonable result when seismic data contain a moderate noise. The real data test shows that the inverted Lamé constants and Gassmann fluid term show low values and fracture weaknesses show high values in the location of a gas-bearing fractured shale reservoir, which confirms that our inversion is useful for fluid identification and fracture detection.

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