A PML absorbing boundary condition for 2D viscoacoustic wave equation in time domain: modeling and imaging

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Summary

The constant-Q wave propagation by series of standard linear solid mechanisms using perfectly matched layers absorbing boundary condition (PML) are investigated. An PML with an unsplit-field is derived for the viscoacoustic wave equation by introducing the auxiliary variables and their associated partial differential equations. The unsplit PML are tested on a homogeneous velocity and Marmousi velocity models by applying the 2-4 staggered grid finite-difference scheme. When the wave propagating in the subsurface the amplitude and phase of seismic wave distort due to attenuation. The acoustic reverse time migration (RTM) can not explain this distortion, so we used an unsplit viscoacoustic wave equation with constant Q-model. Comparing the numerical tests on synthetic data for unsplit viscoacoustic reverse time migration and acoustic reverse time migration show the advantages of our approach over acoustic RTM when the recorded data had strong attenuation effects.

Introduction

Absorbing boundary conditions (ABCs) are used in numerical simulations of wave propagation in unbounded problems. To absorb outgoing waves, the reflections from the outer boundary of the computational domain, the ABCs are in finite difference method. There are a number of ABCs for use in finite-difference modeling of acoustic and elastic wave propagation. The ABCs of Clayton and Engquist (Clayton and Engquist, 1977) introduce an approach based on a paraxial approximation of the elastic wave equation. Although these ABCs successful in many applications, but they adsorb wave imperfectly and the artificial reflections occur at the edges of the computational domain. The perfectly matched layer (PML)introduced by Berenger (Berenger, 1994) for numerical simulation of electromagnetic wave propagation. Instead of finding an absorbing boundary condition, Berenger found an absorbing boundary layer. An absorbing boundary layer is a layer of artificial absorbing material that is placed to the edges of the grid. The wave is attenuated by the absorption and decays exponentially when it enters the absorbing layer. The PML is considered because of its highly effective, excellent absorption over a wide range of angles and insensitivity to frequency. The PML has been developed for elasticity (Chew and Liu, 1996), poroelasticity (Zeng and Liu, 2001) and anisotropic media(Becache and Joly, 2001). Berenger's original formulation is called a split-field PML, because it splits the variables into two independent parts in the PML region. In this paper, we consider a 2-D viscoacoustic medium unsplit-field PML formulation. We describe the PML for viscoacoustic medium and give numerical results using test simulations.

Unsplit PML formulation for viscoacoustic wave equation

In order to introduce the PML for viscoacoustic wave, the first-order linear differential equations will be modified using a complex coordinate stretching approach. In the frequency domain, the PML formulations can be derived as

$$\partial x \to (1+\frac{id(x)}{\omega})\partial x \quad (1)$$

$$\partial z \to (1+\frac{id(z)}{\omega})\partial z \quad (2)$$

By applying the complex coordinate stretching expressed to the linearized equation of motion and equation of deformation the frequency domain (for one memory variable, L = 1) and transformed back to time domain the unsplit-field PML formulations obtain as

$$\frac{\partial u_x}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} - d(x)u_x \quad (3)$$

$$\frac{\partial u_z}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - d(z)u_z \quad (4)$$

$$\frac{\partial p}{\partial t} = -\rho c_p^2\left(\frac{\partial(u_x + d(z)\boldsymbol{u_x})}{\partial x} + \frac{\partial(u_z + d(x)\boldsymbol{u_z})}{\partial z}\right)\left(\frac{\tau_\varepsilon}{\tau_\sigma}\right) - \big(d(x)+d(z)\big)p - d(x)d(z)\boldsymbol{p} - \boldsymbol{r} \quad (5)$$

$$\frac{\partial r}{\partial t} = -\frac{1}{\tau_\sigma}r + \rho c_p^2\left(\frac{\partial(u_x + d(z)\boldsymbol{u_x})}{\partial x} + \frac{\partial(u_z + d(x)\boldsymbol{u_z})}{\partial z}\right)\frac{1}{\tau_\sigma}\left(1-\frac{\tau_\varepsilon}{\tau_\sigma}\right) - \big(d(x)+d(z)\big)r - d(x)d(z)\boldsymbol{r} \quad (6)$$

where the auxiliary variables $\boldsymbol{u_x}$, $\boldsymbol{u_z}$, $\boldsymbol{p}$ and $\boldsymbol{r}$ are the time-integrated components for velocity, pressure and memory variable fields. They are defined as

$$\boldsymbol{u_x}(X,t) = \int_{-\infty}^{t} u_x(X,t')\,dt', \qquad \boldsymbol{p}(X,t) = \int_{-\infty}^{t} p(X,t')\,dt'$$
$$\boldsymbol{u_z}(X,t) = \int_{-\infty}^{t} u_z(X,t')\,dt', \qquad \boldsymbol{r}(X,t) = \int_{-\infty}^{t} r(X,t')\,dt' \quad (7)$$

The relaxation parameters τ_σ and τ_ε are defined as (Robertsson et al., 1994):

$$\tau_\sigma = \frac{\sqrt{1+1/Q^2} - 1/Q}{f_0} \quad (8)$$

$$\tau_\varepsilon = \frac{1}{f_0^2\tau_\sigma} \quad (9)$$

where f_0 is the central frequency wavelet.

There are various unsplit-field PML formulations (Abarbanel and Gottieb, 1997; Sacks et al., 1995; Turkel and Yefet, 1988; Fan and Liu, 2001; Zhou, 2005) as well-posed which are based on the uniaxial PML and the complex coordinate stretching methods. A simple and systematic method to derive the well-posed PML formulations are useful to apply the PML methods to more complex media. Fan and Liu (Fan and Liu, 2001) proposed the unsplit-field PML formulations, well-posed PML in Cartesian coordinate, for the acoustic wave equations in a lossy medium. In this work, we propose the unsplit-field for viscoacoustic media in Cartesian coordinate.

Numerical results

To investigate the accuracy of solution of the constant-Q wave equation using the PML absorbing boundary condition we consider the constant velocity model. The viscoacoustic medium considered here is characterized by the constant velocity model. The size of the grid is 251×251,and the source is located at point (500 m, 12 m), which is a zero-phase Ricker wavelet with a centre frequency of 30 Hz.

The grid spacing in the x and z directions is 4 m. To reduce artificial reflections that are introduced by the edge of the computational grid, a PML absorbing boundary condition is applied to the sides and bottom of the model. Figures 1a and 1b shows the effect of attenuation on amplitude and phase of a propagating seismic wave in a homogeneous medium with a background velocity of 2500 m/s and for different values of quality factor (Q=20 , Q=50 and Q=100). The viscoacoustic wavefield has the reduced amplitude and advanced phase in comparison with the acoustic wavefield.
The RTM images are computed by the time-space domain FD methods for a layered and Marmousi models with attenuation. The source wavelet is a zero-phase Ricker wavelet with a center frequency of 25 Hz. The synthetic data are migrated by using acoustic RTM and viscoacoustic RTM. Perfectly matched layer (PML) absorbing boundary conditions are used to attenuate the reflections of an artificial boundary. A layered model is shown in Figure 2a with the Q anomaly (see Figure 2b). The model grid measures are 301×401, the grid size is $4m \times 4m$, the quality factors for background and anomaly (red) are 100 and 20 respectively. The sampling interval is 0.4 ms and the recording length is 2 s.

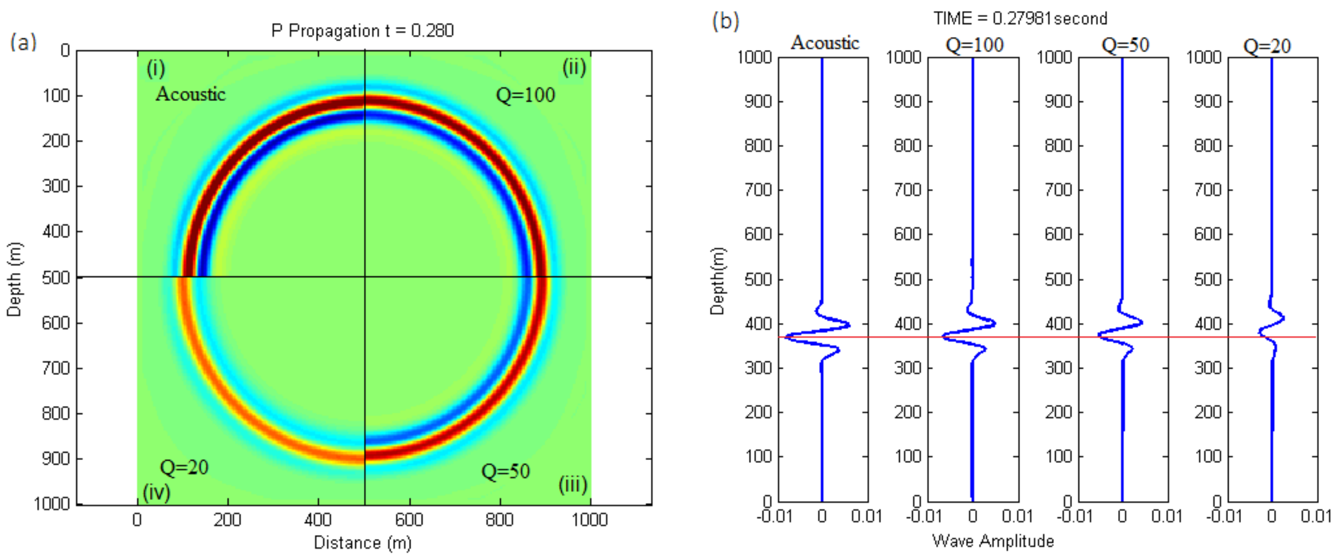

Figure 1: Illustrate the separation of velocity dispersion (a) and amplitude loss effects(b) at the same simulation configuration. i) Acoustic wavefield, ii) viscoacoustic wavefiled (Q=100), iii) viscoacoustic wavefiled (Q=50), and iv) viscoacoustic wavefiled (Q=20). The value of $v = 2500\, m/s$ and $\rho = 1200\, g/m^3$.

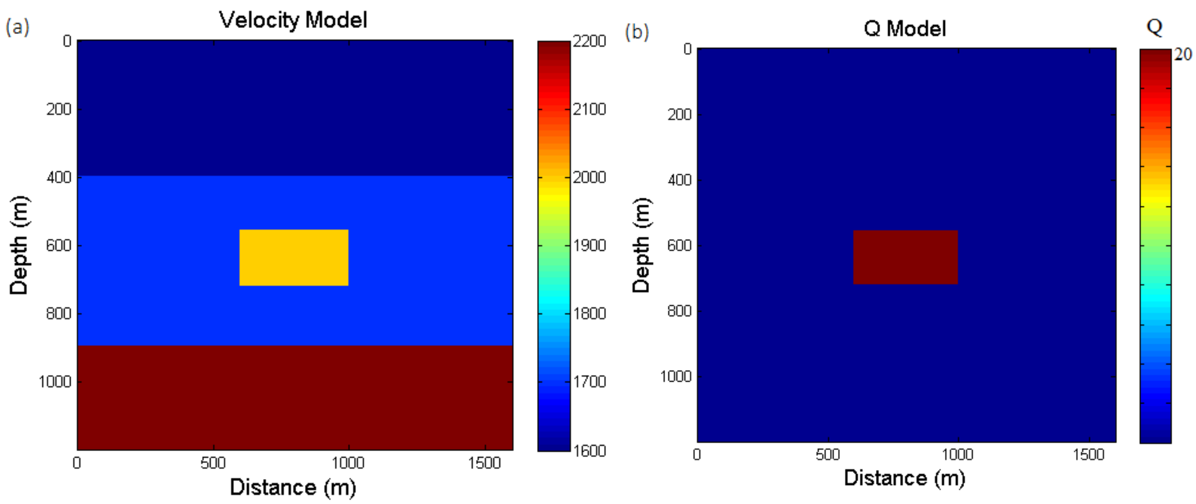

Figure 2: Illustrate the layered velocity model (a) and Q model (b).

Figures 3a and 3b compares the RTM images for acoustic and viscoacoustic approximations with the attenuation. The acoustic RTM image has similar artifacts and amplitudes in the shallow layers compared with the viscoacoustic RTM image, while the acoustic RTM image has very weak amplitudes at the deeper layers specially blew the layers with the strong attenuation (blue arrows in Figure 3).This is because strong attenuation affects the amplitudes and the phases of the propagating waves. However, for viscoacoustic RTM the migration amplitudes of layers are more accurate than the acoustic RTM and the reflectors are imaged at the correct locations. To eliminate source signatures and low-frequency noises we used the imaging condition and highpass filter respectively (see Figure 4). Imaging conditions are used to correlate the source and receiver wavefield snapshots to get the subsurface images.

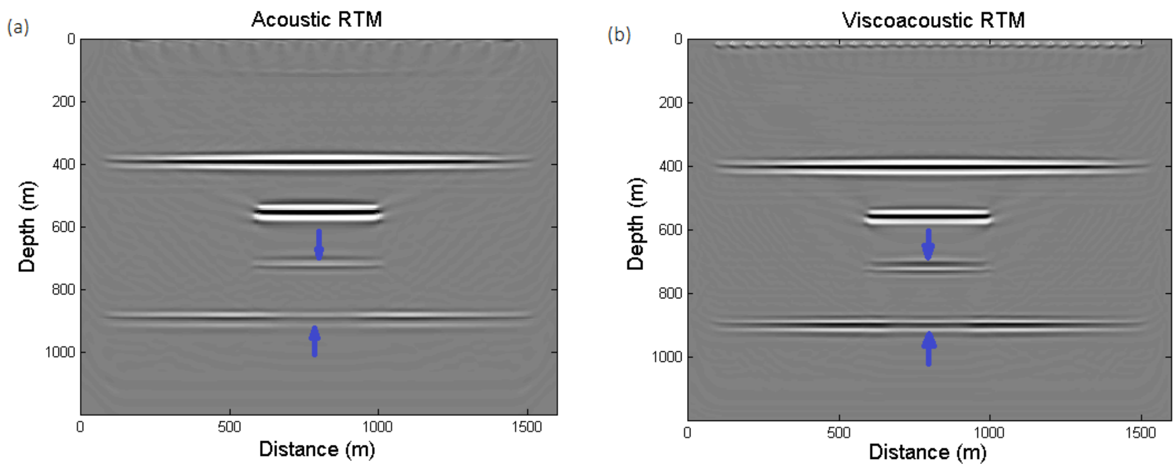

Figure 3: The RTM images of the layered model (a) acoustic RTM, and (b)viscoacoustic RTM. The blue arrows refer to the reflectors below the attenuation layer where we can see that RTM image is clear and the position is accurate from viscoacoustic RTM, compared with the acoustic RTM image.

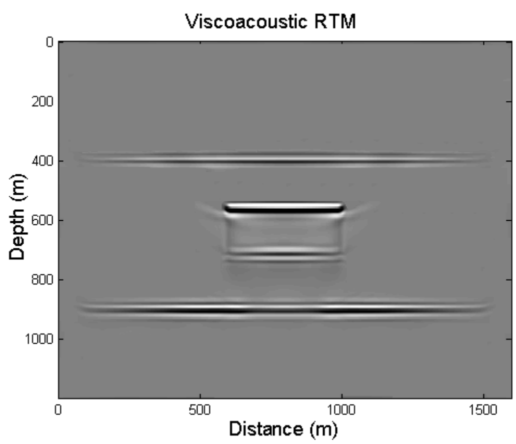

Figure 4: The viscoacoustic RTM images of the layered model applying the imaging condition and highpass filter.

Conclusion

A time-domain constant-Q wave propagation by a series of standard linear solid mechanisms is investigated to compensate for the distortion in amplitudes and phases of seismic waves propagating in strong attenuative layers. Numerical synthetic data illustrated for strong attenuation, the acoustic RTM cannot correct for the attenuation loss, while the unsplit PML viscoacoustic wave equations can compensate the attenuation loss during the iterations. Comparing the synthetic data results for unsplit viscoacoustic and acoustic RTMs show that the migration amplitudes of layers are more accurate than the acoustic RTM and the reflectors are imaged at the correct locations in strong attenuative media.

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