Testing the Reliability of Moment Tensor Inversion Using Surface and Borehole Monitoring

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Summary

Source mechanisms of microseismic events, obtained using moment tensor inversion, can provide valuable information for monitoring hydraulic fracturing treatments. In hydraulic fracturing monitoring, data are usually obtained using either surface or borehole arrays of sensors. Surface arrays appear to obtain mechanisms with high shear components whereas borehole arrays tend to constrain more variable mechanisms with higher tensile components. Synthetic tests are conducted to compare the reliability of moment tensor solutions from surface and borehole arrays based purely on geometry. The results demonstrate that for the surface array, all inversions are able to constrain reliable results (with negligible bias and low variance), whereas borehole geometries with either two or three wells produce reliable results only when including both P- and S-wave amplitudes recorded on three components in the inversion, as is usual. Surface array inversion results are also more stable in that they show less bias and variance (spread) for noisy data compared with borehole results, most likely due to a greater sampling of the focal sphere. Adding an extra well to the borehole geometry greatly improves the results by lowering the biases and estimation variances. The larger biases and estimation variances resulting from inverting noisy data on downhole monitoring arrays, especially in those with a strong shear component, may explain in part why downhole moment tensor inversion studies tend to resolve more variable mechanisms (usually with stronger tensile components) than surface arrays. However as both geometries are capable of accurately resolving both tensile and shear mechanisms, a lack in detected tensile components on surface arrays may also be due to smaller recorded amplitudes. Nonetheless as well as geometry, other influences (e.g. signal-to-noise ratio) will likely influence the reliability of the solutions in real settings, which should also be taken into account in the design of microseismic monitoring arrays. Tests such as those in this study could be useful to help design appropriate monitoring geometries and/or understand possible sources of bias and uncertainty.

Introduction

Moment tensor inversion (MTI) is the common method for calculating seismic source mechanisms (Baig & Urbancic, 2010; Eyre and van der Baan, 2015). MTI in a hydraulic fracturing environment is complicated by the possibility/likelihood of volumetric contributions to the source due to the presence of high pressure fluids (Baig & Urbancic, 2010), which greatly increases the possible solution space and therefore non-uniqueness of the solutions. In hydraulic fracturing environments, microseismic data are generally recorded on either 2D surface arrays or borehole arrays of sensors (van der Baan et al., 2013). Each has their own advantages and disadvantages with respect to financial costs, practicality and data signal-to-noise ratio. However, this study aims to investigate the relative reliability of the results for both tensile and shear source mechanisms based solely on the two acquisition geometries. An investigation into the reliability of each setup for MTI is interesting as surface arrays appear to constrain mechanisms with high shear components (Snelling et al., 2013; Neuhaus et al., 2012), whereas borehole arrays tend to constrain more variable mechanisms with higher tensile components (Baig & Urbancic, 2010). A comparison to investigate any substantial advantages and/or biases in the results based purely on geometry therefore forms the basis of this study. MTI is carried out on synthetic amplitude data generated for examples of each of these cases. We also examine the effects of these different input parameters in the inversion, namely P-wave amplitudes and combined P- and S-wave amplitudes, and
also recordings on vertical component and 3- component sensors. The sensitivity of the results to other factors such as noise, source location and fracture plane orientation are also investigated.

![Figure 1](Image)

**Figure 1.** 3D view of the setup of the synthetic tests: (a) 2D star-shaped surface array, and (b) two vertical borehole arrays (also shown is the third borehole array included in some tests (light circles)). Circles show seismic stations and stars show locations of vertical SW - NE striking fracture planes.

**Method**

In order to demonstrate the challenges and merits of moment tensor inversion carried out using different observables, the P- and S-wave polarities and ray amplitudes are synthetically calculated for simple radiation from a point source in a homogeneous elastic medium using the equations of Aki & Richards (2002) for simple, yet realistic, setups based on real surface and borehole monitoring examples (Figure 1). The fracture orientation for all monitoring geometries is set as SW - NE striking vertical strike-slip surface since monitoring wells are usually designed to be at an angle to the regional principle horizontal stress directions to best sample the focal sphere, and strike-slip failure is commonly observed (Snelling et al., 2013; Neuhaus et al. 2012). This study focuses solely on the relative geometry of the source and receivers; results remain identical for different spatial scales, since the signal-to-noise ratio is assumed to be independent of distance. We investigate both shear and tensile sources as well as combined tensile-shear mechanisms. The angle between the slip vector and the fracture plane can be described by the angle α (Vavryčuk, 2001), where α = 0° in the case of pure shear and α = 90° for pure tensile events. Four different possible mechanisms are studied: α = 90°, α = 45°, α = 20° and α = 0°.

MTI is carried out using a simple amplitude-based least squares method detailed in Eaton & Forouhideh (2011), with possibilities to include P- and/or S-wave amplitudes, and one or three component recordings. Green’s functions (the modelled propagation effects) are calculated using identical values to the forward model so that the only errors in the inversion are caused by the source-receiver geometry. To test the stability of the moment tensor solutions, a Monte Carlo technique is employed, where random noise is added to each arrival for each trace included in the inversions before the data is inverted. This is repeated 50 times to give a range of solutions, thus revealing both accuracy and precision in the inversion results. From the obtained moment tensors, source-type plots can be constructed using the methods of Hudson et al. (1989), and the mechanism orientations can be calculated from the principal axes (i.e. eigenvalues and eigenvectors) of the moment tensor (Gasperini & Vannucci, 2003; Vavryčuk, 2011).

A Hudson et al. (1989) plot uses two parameters $T$ and $k$ to characterize the type of deviatoric component in the source ($T$) and the proportion of volumetric change ($k$), respectively. Both parameters are computed from the eigenvalues of the moment tensor. A mechanism plots in different regions of the diagram depending on the relative proportion of double-couple, Compensated Linear Vector Dipole (CLVD), and isotropic energy, and also distinguishes the polarity. Double-couple mechanisms plot in the centre, explosive and implosive events at the top and bottom of the diagram, respectively, and opening and closing tensile crack mechanisms on the top-left and bottom-right edges, respectively. In order to construct an equal area plot, parameters $T$ and $k$ are transformed to new parameters $u(T,k)$ and $v(T,k)$ such that the joint a priori probability density of $u$ and $v$ is uniform (Hudson et al., 1989). In this way, source types are therefore represented in Hudson et al. (1989) plots using the parameters $T$ and $k$ plotted in the $u$-$v$ plane, which gives a uniform distribution of source types throughout the plot.
Figure 2. Hudson et al. (1989) source-type plots showing the results of moment tensor inversion of the synthetic amplitude data, for (a) the surface array and (b) the borehole arrays, for $\alpha = 90^\circ$ and $\alpha = 0^\circ$. “True” corresponds to true source, “P Z” = inversion using P-wave vertical component, “P 3” = inversion using 3-components of P-wave, “SP Z” = inversion using P- and S-wave vertical components only, and “SP 3” = inversion using 3-components of P- and S-waves. Random noise of up to 50% of the amplitudes is added to the data, and repeated 50 times for each inversion method, to investigate the stability of the results.

**Examples**

Here we present examples of some of the results. These are discussed in terms of the systematic bias (i.e. accuracy) and the variance (i.e. precision) in the results due to the added random noise. Figure 2 shows the results of the MTI for the pure tensile and the pure shear mechanisms with 50% additional random noise. Inversions are carried out for P-wave amplitudes for vertical component and for three component data, and also for both P- and S-wave amplitudes, again for both vertical and three component data. Results suggest that the use of only P-waves or P- and S-waves on one component in the inversion for the borehole data results in strongly biased solutions. Using S-wave amplitudes as well greatly improves the results when using three-component data, with little bias although high variance due to noise. The latter approach is commonly used in practice. The surface array performs substantially better, both for single and three component inversions, with lower variance (i.e. low sensitivity to noise) and low bias. Similar results are observed for the mechanism orientations.

Next we investigate variations in results due to relative source-receiver locations. To do this, the synthetic tests are carried out for a horizontal grid of possible source locations, and bias and variance in the results are estimated. Here the bias is quantified as the distance between the true and calculated mechanism on the Hudson et al. (1989) source-type plot, i.e.:

$$E = \sqrt{\Delta u^2 + \Delta v^2},$$

(1)

where $u$ and $v$ are the plotting parameters. This is a reasonable method to calculate the bias due to the uniform distribution of source types in the $u$-$v$ plane. The mean values of $u$ and $v$ for the 50 iterations are calculated separately and then the bias is calculated by inputting the mean values into equation 1, so that $E$ equals zero when there is no bias in the results (Figure 3). The mean bias is very low for the surface array (of the order of $10^{-3}$), however for the two borehole array the calculated results are up to two orders of magnitude higher, and also show a narrow linear region passing through the two boreholes with high bias. This is unsurprising as in this region the two arrays sample the same plane through the focal sphere. It is
noticeable that the mean bias is generally higher for the pure tensile mechanism than the pure shear mechanism. The variance in results is estimated through the standard deviations of $E$ from Equation 1 for the same 50 realisations for each grid point. The variance results have a similar magnitude to the biases, and similar patterns are observed.

Figure 3. Maps showing the bias for noisy data with a 10% random noise level, when calculating for a 2D grid of different possible source locations, and for a SW-NE oriented fracture plane. (a) shows the results for the surface array and (b) for the borehole arrays, and plots are shown for $\alpha = 90^\circ$ and $\alpha = 0^\circ$. Red stars indicate receiver locations. Note different colour scales for surface and borehole results: for the surface array the results are approximately two orders of magnitude smaller.

Conclusions

For both surface and borehole geometries it has been demonstrated that a reliable solution can be obtained. In both cases, including both P- and S-wave amplitudes in the inversions provides more reliable results, likely due to the increased number of observations and reduced number of potential sources of error. The effects of noise and errors in the inversion on the solutions are therefore greatly diminished. It also appears to be advisable to use three component recordings, and this has the added benefit of helping with the ability to pick both P- and S-wave arrivals. For acquisition arrays of two boreholes, both P- and S-wave amplitudes on three components must be used in order to gain reliable solutions, otherwise large biases are obtained.

From the surface and borehole array geometries investigated it is apparent that surface arrays provide more accurate constraints on the true mechanisms based solely on geometries. This may explain why downhole monitoring MTI studies tend to resolve more variable mechanisms than surface arrays which often determine high double-couple components, due to higher bias and variance in the results, especially in those with a strong shear component: however it may also be caused due to a lack of detection of tensile events on surface arrays which could be caused by smaller recorded amplitudes (possible due to the different source processes or differences in radiation patterns). The reliability of the borehole acquisition solutions is also much more dependent on the fracture orientation and relative locations of the boreholes and source location region, and therefore more care must be taken when designing borehole monitoring acquisitions and interpreting the results. However, many other factors also affect the reliability of surface versus borehole monitoring, including signal quality due to different source-receiver distances.

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References