Practical regularization approach to simultaneous estimation of seismic source wavelet and reflectivity

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Summary

An alternative least-squares algorithm for simultaneous estimation of the seismic source wavelet and reflectivity, i.e., blind deconvolution, requires a solution of the constrained optimization problem with constraints imposed on both reflectivity and wavelet. Herein, a smoothness constraint on the wavelet in the form of a wavelet base function is proposed. The reason behind such a proposal is that the physical seismic wavelet can be presented as a bandlimited, smooth function. Numerical experiments show that wavelet smoothness constraints can improve the deconvolution robustness when dealing with data contaminated by noise.

Introduction

In seismic exploration, a short-duration seismic source wavelet (pulse) is transmitted from the surface, reflected from boundaries between subsurface earth layers, and detected by an array of sensors on the surface. With the assumption that this wavelet is not distorted during its propagation (perfectly elastic media), the recorded seismic trace $\boldsymbol{x}$ can be presented as a result of the convolution of the reflectivity sequence (e.g., layered earth model) with a source wavelet:

$$\boldsymbol{x} = \boldsymbol{w} * \boldsymbol{r} + added\ noise \qquad (1)$$

where $\boldsymbol{r}$ refers to a time series of reflection coefficients and $\boldsymbol{w}$ refers to a source wavelet. The matrix presentation of equation 1 can be written as

$$\boldsymbol{x} = \boldsymbol{W} * \boldsymbol{r} + added\ noise \qquad (2)$$

or

$$\boldsymbol{x} = \boldsymbol{R} * \boldsymbol{w} + added\ noise \qquad (3)$$

where $\boldsymbol{W}$ and $\boldsymbol{R}$ represent convolution matrices for wavelet and reflectivity. Conveniently, equation 2 can be used to estimate reflectivity with a fixed wavelet, and equation 3 to estimate the wavelet with fixed reflectivity. Because the source wavelet is frequency band-limited, the convolution procedure destroys low and high frequencies outside the wavelet bandwidth. This effect causes sharp thin layered structures to be smeared in the receiving traces. Removing the effect of the source wavelet's impact on the seismic traces is of great significance in detecting thin structures.

The deconvolution process of recovering reflectivity from the observed trace $\boldsymbol{x}$, without knowledge of both reflectivity and wavelet, is known as a blind deconvolution. This type of inverse problem has various applications such as: communications (equalization or channel estimation), nondestructive testing, geophysics, image processing, medical imaging, and remote sensing; therefore, seeking a robust algorithm becomes a subject of much research.

Utilizing the property of cross relationships among multichannel records, equation 1 can be transformed into a homogenous matrix equation with respect to the reflectivity sequences. Solving $\boldsymbol{r}$ involves finding an eigenvector located in null space of the cross-relation matrix. To limit non-uniqueness, a sparseness constraint is employed (Šroubek and Milanfar, 2012; Yu et al., 2012; Kazemi and Sacchi, 2015). The convenient feature of this homogeneous equation is that only the reflectivity is involved and, therefore, the blind deconvolution can be divided into two steps. First, estimate reflectivity and then estimate the wavelet using equation 3. However, the cross-relation matrix is not one rank deficient, which can lead to minimization and can be easily trapped into a local minimum due to more than one eigenvector in the null space. The non-uniqueness makes this approach strongly dependent on the quality of the initial guess of the true reflectivity. Moreover, due to the ambiguous amplitude of the reflectivity estimate from the homogeneous equation (scale uncertainty), the whole data set must be involved at the same time. This leads to a very large equation system that becomes computationally infeasible. Therefore, applying this approach may not be practical.

A more acceptable method of solving blind deconvolution is to split the optimization of equation 1 into two sub-optimizations of equations 2 and 3 and solve them alternatively (Repetti et al., 2015; Šroubek and Milanfar, 2012, Krishnan et al., 2011). Even if the convergence of this approach has not been deeply investigated, they appear to be quite efficient in practice.

At publication, many authors also provided their algorithms; this gave us an opportunity to test and compare different methods. The results of our tests show that these algorithms are very sensitive to the signal-to-noise ratio, i.e., satisfactory results can only be obtained when the ratio is very high. An interesting fact is that the failed tests always end up with a non-smooth wavelet estimate. The reason, we believe, is that all the published approaches focus mostly on obtaining sparse solutions to equation 1 and paid less attention to the wavelet estimate in equation 2. Moreover, the sparseness constraint to reflectivity is also applied to the wavelet estimate, e.g., Krishnan et al. (2011). The blind deconvolution is an ill-posed problem and any reasonable a priori information is of great value. Because we know that the seismic wavelet is a bandlimited smooth function, we should incorporate it into the deconvolution procedure.

It is our understanding that the smoothness constraint to the wavelet (blur function) has not been well-addressed in blind deconvolution. Wang et al. (2016) proposed an approach that uses equation 2 to estimate the wavelet where the convolution matrix is set as a Toeplitz matrix. However, when solving the equation, the Toeplitz matrix is expanded to the vector that represents the wavelet and, therefore, Wang et al. (2016) actually ended up with the same equation as equation 3. Because the wavelet length is now equal to the length of the reflectivity, the corresponding matrix ***R*** becomes very large, making computation inefficient. Considering that the effective length of the wavelet is small, the fused least-absolute shrinkage and selection operator (LASSO) regularization (Tibshirani and Saunders, 2005) was employed to expect a smooth wavelet estimate. However, fused LASSO regularization is designed to control sparsity with continuity instead of smooth regularization and, therefore, the fused LASSO regularization may not be optimal for wavelet estimation. Bhuiyan et al. (2013) used damped least-squares method to solve equation 3, but simple damped least squares may not necessarily produce a desired smooth output and, instead, a differential regularization operator is often used in geophysical inversion (Lizarralde and Swift, 1999, Reichel and Ye, 2009).

Here, we apply an alternative optimization for seismic blind deconvolution. In addition to the sparseness constraint to the reflectivity estimation, we apply a wavelet base function as a smoothness constraint on the wavelet update. Numerical experiments show that the smoothness constraint can improve the deconvolution robustness of the noise-contaminated data and, therefore, can be an important tool for practical applications.

Method description

Blind deconvolution can be formulated as solving an optimization problem:

$$\min_{r,w} ||x - r * w||_2 + \lambda\boldsymbol{\varphi}(r) + \beta\boldsymbol{\psi}(w),$$

where the first term is the likelihood that takes into account equation 1; the second and third terms are the regularizations for reflectivity and wavelet, respectively; λ and β are the parameters that control the strength of constraint regularization functions, $\boldsymbol{\varphi}$ and $\boldsymbol{\psi}$. A formula such as equation 3 is nonconvex and the standard approach to optimization of this problem is to split the equation into two sub-optimization equations and alternating minimization of the squared Euclidean distance expressed by the Frobenius norm regarding ***r*** and ***w*** that are subject to constraint.

Update reflectivity

The reflectivity is obtained by minimizing the objection function:

$$\min_r ||x - Wr||_2 + \lambda||r||_1. \quad (4)$$

We investigated available algorithms and came to the conclusion that, when ***W*** is well defined, e.g., the wavelet is known and the signal-to-noise ratio is high, all the algorithms perform very well in reconstructing a reflectivity series; otherwise, the re-weighted algorithm performs better than others. When ***W*** is poorly defined, the LASSO algorithm (Tibshirani, 1996) can perform relatively better. Based on this observation, we choose an adaptive LASSO algorithm (Zou, 2006) to update the reflectivity.

Update wavelet

The wavelet is updated by minimizing the objection function:

$$\min_v ||x - R\boldsymbol{\phi}v||_2 + \lambda||v||_1, \quad (5)$$

where $\boldsymbol{\phi}$, a base function that consists of rotated Ricker wavelets with different phase, and ***v*** is a vector that makes wavelet estimation equal to $w = \boldsymbol{\phi}v$.

The same regularization form can be found in literature, e.g., using B-spline base (Yu et al., 2012). Our choice of phase-rotated Ricker wavelets is based on the observation that, in real seismic data, the wavelet is usually close to this type of function.

The process of alternative minimization of the described objection functions, i.e., equations 4 and 5, is repeated iteratively until desired convergence of the reflectivity and wavelet solutions is achieved.

Examples

The first example shows the comparison of source wavelet estimation with the damped least-squares method and wavelet base function regularization. The data are generated by convolving a random series of spikes with a 40° Ricker wavelet. Random noise was added to contaminate the model data. Equation 3 is used to estimate a wavelet with known reflectivity. The results show that estimation with wavelet base function regularization produces a noticeably smoother wavelet.

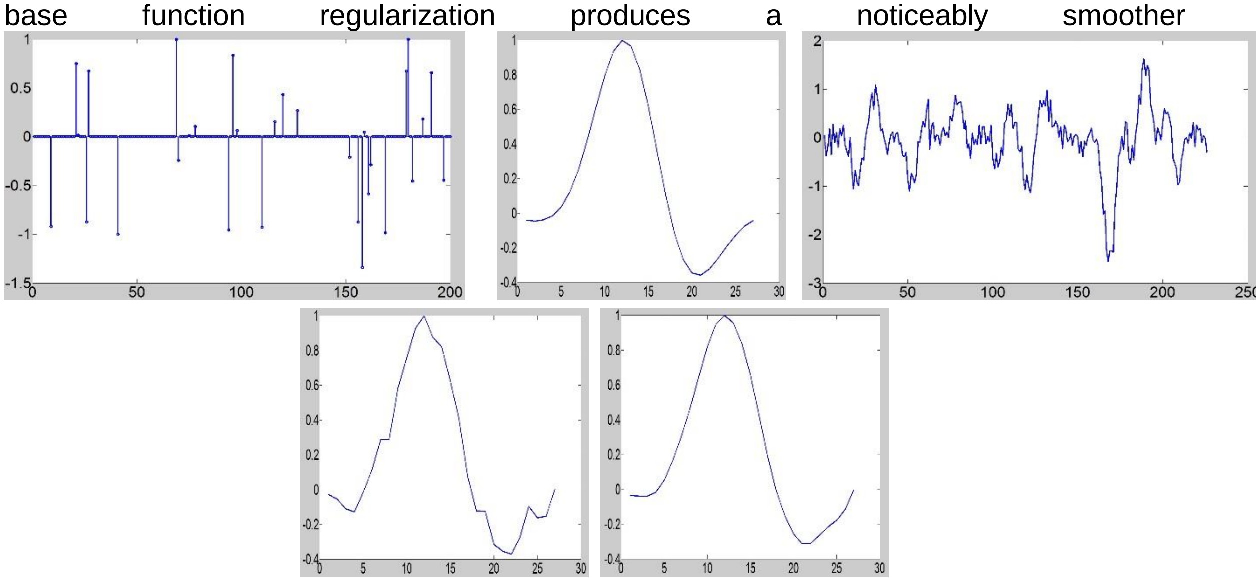

Figure 1. From top left to bottom right: reflectivity model; source wavelet; synthetic data with noise; wavelet estimated from first-order differential operator regularization, and wavelet base regularization.

The second example shows a 'wedge' reflectivity model that consists of 15 traces with layer thicknesses in time varying from 1 ms to 15 ms. The data are generated by convolving the model with a 60° rotated Ricker wavelet. White noise is added to the model with a ratio to signal of 10%. The result in Figure 2 shows that both the model reflectivity and source wavelet are well estimated.

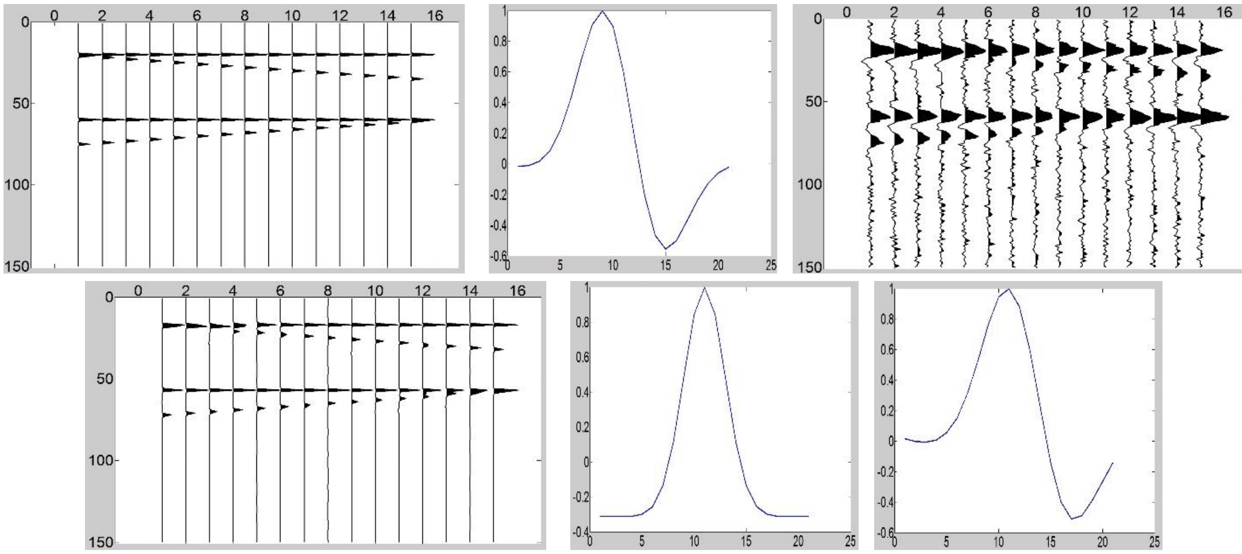

Figure 2. From top left to bottom right: reflectivity model, source wavelet, noised data, estimated reflectivity, initial wavelet for deconvolution, and estimated model and wavelet.

Conclusions

We applied the smoothness constraint on wavelet estimation in blind deconvolution; this constraint reflects the natural property of the source wavelet and, therefore, using it as a priori information is physically meaningful. Numerical tests show that the smoothness constraint on the wavelet improves the result of deconvolution of noisy data, which can make application of the proposed deconvolution approach particularly useful for real data.

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