



## Reflection Extraction from Sonic Log Waveforms Using Karhunen-Loeve Transform

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### Summary

Sonic reflection logging, a recently developed borehole geophysical scheme, is in principle capable of providing a clear view of structures up to 40 m away from well site theoretically. Under acoustic well logging conditions, reflected wave signals used in sonic reflection logging are generally lost in the full waveform records, hidden by the dominant direct waves (direct P- and S- waves, and the Stoneley wave). It is critical, therefore, to effectively extract the reflection signals from the acoustic full waveforms in acoustic reflection well logging data processing. The Karhunen-Loeve (KL) transformations combined with a band limiting filter is used to extract reflections of interest out of dominant direct waves. Based on energy difference of each wave component, the direct Stoneley wave, S wave and P wave are to be eliminated separately from high to low energy component. Therefore, the extracted reflections can then be used in migration so as to get a clear image of the structures outside borehole.

### Introduction

Sonic logging was first proposed to measure the speed of sound in a short interval of rock traversed in a wellbore by using a transmitter and two receivers (Marguerite, 1940). However, it was not specifically used to measure P- and S-wave velocities until the 1960s (Kokesh et al., 1965). Usually, a recorded waveform generated by the transmitter consists of not only refracted P- and S-wave, but also the Pseudo Rayleigh wave and the Stoneley wave (Cheng and Toksoz, 1981). Conventionally, sonic logging technology uses direct sonic arrivals (P- and S-waves) to calculate formation P- and S- wave velocities (Kimball and Marzetta, 1984). Recently, secondary arrivals (reflections from structures outside the borehole) have increasingly been used for characterizing geological structures away from a wellbore (Hornby, 1989b; Coates and Schoenberg, 1995; Chabot et al., 2001). However, the reflected waves are difficult to discover amongst the much higher amplitude head waves. Extraction of reflections is a key technological hurdle to practical use of sonic full waveforms. A combination of FK and median filtering techniques has been tested for reflection extraction in single-well imaging with acoustic reflection survey (Li et al., 2002). The median filter is used to remove direct waves, and then the FK filter is applied to separate downgoing and upgoing reflections. Tang (2004) and Zheng (2005) used the parametric prediction method to extract reflection waves from waveforms.

The Karhunen-Loeve (KL) transformation has been widely used in data analysis such as imaging compression (Ahmed and Rao, 2012). The KL transformation has also been used in seismic exploration for signal-to-noise improvement (Hermon and Mace, 1978), diffraction separation from reflections (Yedlin et al., 1987). Hsu (1990) used the KL transformation to sonic logging waveforms to extract direct waves. In his paper, a threshold detection scheme is first applied before using KL transformation to exclude the unwanted signals. However, when extracting the reflection signals, based on the energy difference (the direct waves have higher energy than do the reflections), the unwanted signals can be eliminated by choosing a wide processing window, where a precise threshold is not necessary needed. When fractures exist around the borehole, a Stoneley signal will generate the so-called reflection Stoneley when traveling through fractures (Paillet and White, 1982; Hornby et al., 1989). Therefore, for the Stoneley wave elimination in field data, a band limit filter is instead used, because the reflected Stoneley can not be mitigated by KL transformation.

## Karhunen-Loeve (KL) Transform

The typical sonic log can be considered as a N-dimensional vector  $X = (X_1, X_2, \dots, X_N)$ , each vector  $X_i$  can be treated as a recorded waveform at specific depth.

Therefore, N denotes the total number of recorded waveforms. The mean value of this N-dimensional vector can be described as,  $\mu_x = \frac{1}{N} \sum_{i=1}^N X_i$ . Its covariance matrix thus can be written as,

$$C_X = E\left\{(X_i - \mu_x)(X_i - \mu_x)^T\right\} = \frac{1}{N} \sum_{i=1}^N (X_i - \mu_x)(X_i - \mu_x)^T \quad (1)$$

where, E denotes the mathematical expectation and T denotes the transform of the matrix. Assume  $\lambda_i (i=1, 2, \dots, N)$  being the eigenvalue of the covariance matrix with its correspondent eigenvector  $e_i (i=1, 2, \dots, N)$ , therefore, we have an orthogonal matrix of **A**, which can be described as,

$$A = (e_1^T, e_2^T, \dots, e_N^T)^T, \quad \lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \dots \geq \lambda_N \quad (2)$$

Therefore, we have  $Y = A(X - \mu_x)$ . Because **A** is an orthogonal matrix ( $A^{-1} = A^T$ ), the N-dimensional vector  $X$  can be stated as  $X = A^T Y + \mu_x$ . Now, let's consider the first k largest eigenvectors,

$$A = (e_1^T, e_2^T, \dots, e_k^T)^T, \quad \lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \dots \geq \lambda_k, k < N \quad (3)$$

Then we have an approximate matrix of  $X$ ,  $X = A_k Y + \mu_x$ . In geophysical signal analysis, the received signal is usually considered as a zero mean value (Enders A. Robson, 2000), which means  $\mu_x$  is equal to 0.

The mean square error between  $X$  and  $X$  becomes,  $\varepsilon(k) = \sum_{i=k+1}^N \lambda_i$ , which means if the first k number of dominant eigenvectors are used to reconstruct the original information, the dominant energy can be reconstructed by this method, which is called KL transformation.

## Examples

In order to demonstrate the reflection extraction from sonic logging by KL transformation. A synthetic data simulated by finite difference method is used. The model (shown in Figure 1 (Left)) is a fluid filled borehole model with a fault like interface lies on the one side of the borehole with a dip of 45 degree. Firstly, we move the source point from depth of 2 m to 10.85 m with altogether 60 shots, whose depth interval is 0.15 m. 2D FDM numerical modeling is used to acquire the acoustic array full waveform for each source point. Figure 2 (a) shows the received full waveform when the dip angle is 45 degree. The total recording time is 3 ms. After elimination of direct waves, the reflection signals can be acquired, shown in Figure 2 (b). As shown in Figure 2 (a), because of the slow formation outside the borehole, the first arrived signal is leaky P wave signal; the water wave (whose velocity is about 1500 m/s) signal propagating in the borehole fluid is behind the P wave signal; signal behind the water wave is Stoneley wave. There is no shear wave signal in slow formation. The theoretical reflections can be simply obtained by subtracting the full waveforms with direct arrivals (P wave and Stoneley wave).

If KL transformation is applied at a wide range from 0 ms to 1.5 ms, theoretically, the principal component should be the Stoneley wave. Figure 1 (Right) shows four dominant normalized eigenvectors of covariance matrix calculated from the selected range of full waveforms. Compared the take-off time of each event shown in Figure 2 (a), the first dominant eigenvector consists not only of the Stoneley component, but also of the water wave component. The eigenvalue of the first dominant eigenvector is 0.9949, therefore, the first principal component can be reconstructed by only using the first

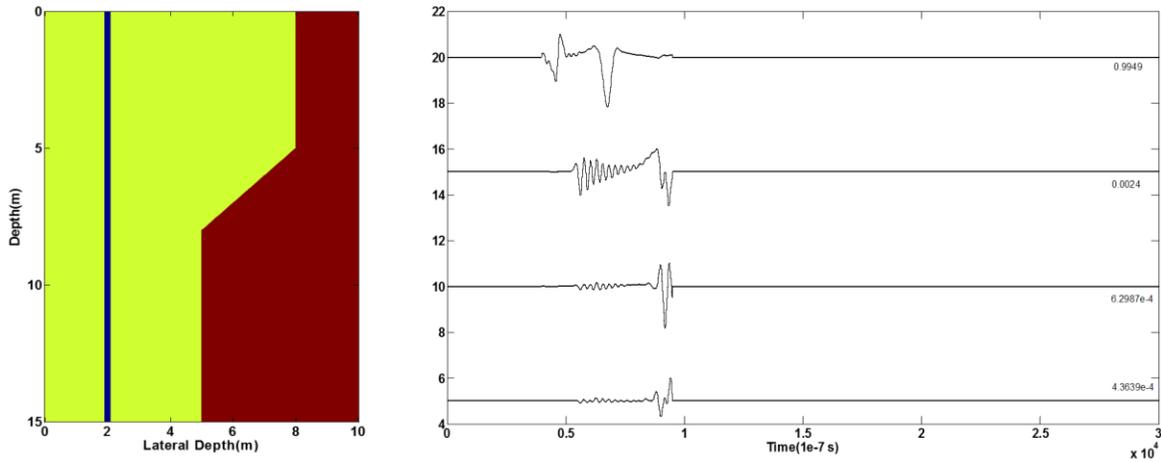


Fig.1: (Left) Synthetic model simulated by finite difference method. (Right) Four dominant normalized eigenvectors of covariance matrix calculated.

dominant eigenvector shown in Figure 2 (c) and Figure 2 (d) shows the covariance residuals after the first principal component is eliminated. The reflected waves emerge as a subsequence following the new principal component in Figure 2 (c). Based on the same recipe, we apply KL to the residuals acquired from the previous step. A new principal component and its new residuals can be obtained shown in Figure 2 (e) and (f). Figure 2 (g) shows the dominant four eigenvectors. Figure 2 (f) turns out to be the final reflections extracted by the KL transformation. Compared with the theoretical reflections shown in

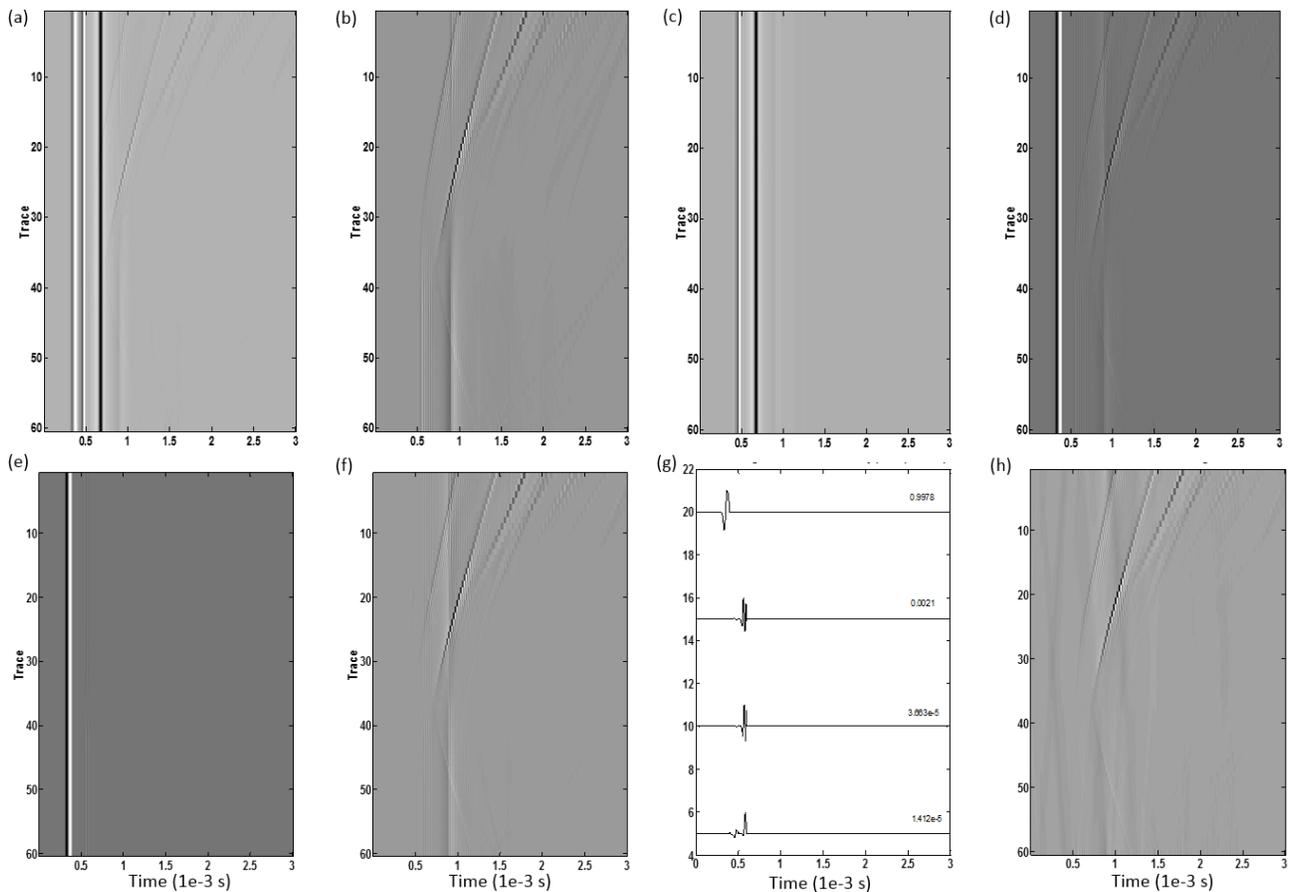


Fig.2: Received full waveform (a) and its real reflections; first principal component (c) and its residuals (d); second principal component (e) and its residuals (f); dominant four eigenvectors (g) reflections extracted using MSTC (h).

Figure 2 (b), almost all the reflection details have been revealed. To make a comparison, reflections extracted using multi-scale slowness-time-coherence (MSTC) (Tao et al., 2008) is also displayed in Figure 2 (h), where some details of the reflection signals are missing and low frequency noises appear.

In the next example, the laboratory data is acquired in a specially designed large water tank. The reflector in the water tank is a steel pad placed at an angle of 20 degrees and distanced 3 m away from the tool. The received full waveforms by the first receiver is shown in Figure 3 (a), where the event at around 6 ms whose speed is about 900 m/s has the strongest energy, which should be the principal component. The P head wave event is at about 4 ms with a velocity of roughly 1400 m/s. Figure 3(b) shows the reflections extracted by KL method, where the shape of the steel pad can be easily delineated. Figure 3 (c) shows the reflection extraction result by using MSTC, where the reflections from the steel pad emerges yet accompanied by some remnants scattering along the depth interval.

The acquired reflections by KL transformations can now be used in the migration step. The borehole reverse time migration (RTM) (Li et al., 2014) is applied, in which the staggered grid finite difference method is used for the forward and backward simulation and the hybrid perfectly matched layer (H-PML) (Zhang et al., 2013) is used for the absorbing layers. The imaging result is shown in Figure 3 (d), the steel pad is clearly shown at an angle of 20 degrees and a distance of 3 m away from the borehole. However, the imaging result using reflections acquired by MSTC in Figure 3(e) suffers inevitably from the influence of the scattered remains.

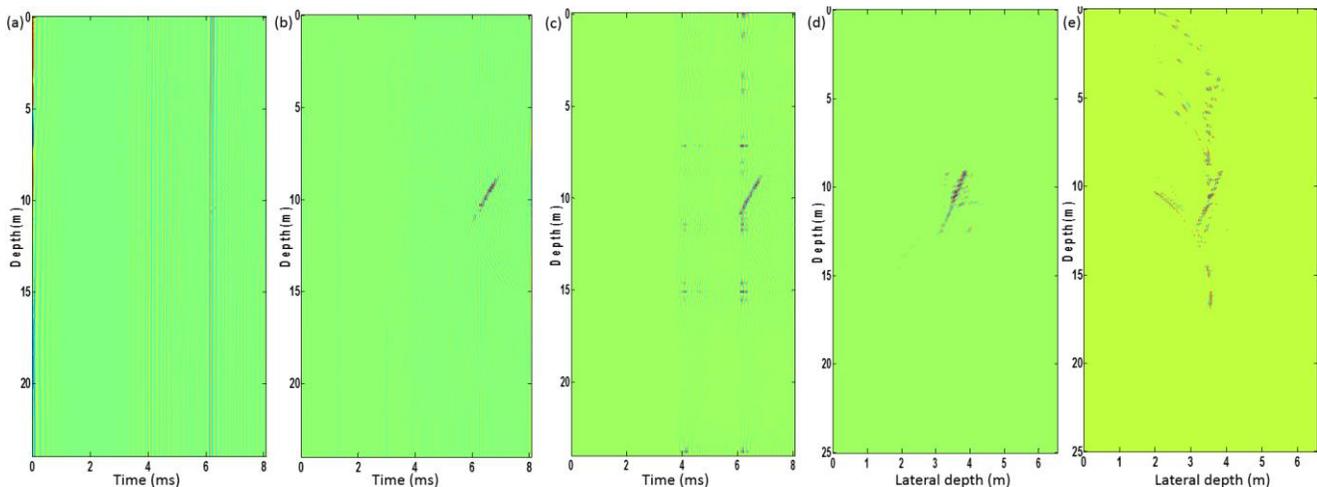


Fig.3: (a) Received full waveform; reflection signals using KL transformation (b) and MSCT (c); imaging result of reflections from KL transformation (d) and result of reflection from MSTC (e).

## Conclusions

The KL transformation is applied in this paper to separate reflections away from direct signals in acoustic reflection well logging data. Based on energy difference of each signal component, the direct P- and S waves as well as the Stoneley wave can be efficiently removed. Comparisons with MSTC method both from synthetic and laboratory data show KL transformation is capable of providing much more precise reflection signals.

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