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Interpolation using asymptote and apex shifted hyperbolic Radon transform

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Abstract

The asymptote and apex shifted hyperbolic Radon transform is an extension to the apex shifted hyperbolic Radon transform that allows shifts in both the asymptote and apex of hyperbolic events. This extension improves the transform ability to focus seismic diffractions in common shot domain. The transform kernel is implemented using Stolt migration/demigration operators in order to improve its computational speed. Our results show that the new transform can be used to efficiently interpolate data that contain seismic diffraction.

Introduction

In marine streamer acquisition, operational costs and streamer entanglement limits the number of streamers along the cross-line axis. Therefore, seismic data usually require interpolation in order to increase the spatial sampling along the cross-line axis prior to processing and imaging. Most interpolation algorithms use transforms that can focus seismic data using the similarity between the seismic data and the transform dictionary (Terenghi, 2014). Since the travel-times of seismic events can be generally approximated by hyperbolas, Radon transforms that use a hyperbolic dictionary represent a powerful tool for interpolation. The most common of these transforms is the Hyperbolic Radon Transform (HRT). This transform is usually used for processing common midpoint gathers, where the apexes of seismic reflection hyperbolas are usually located at zero offset. However, Trad (2003) proposed interpolating seismic data in the common shot gather domain using an Apex Shifted Hyperbolic Radon Transform (ASHRT), which extends the conventional HRT by scanning for the horizontal location of apexes. The ASHRT has been used to interpolate and/or denoise seismic data in the shot gather domain (Trad, 2003; Ibrahim and Sacchi, 2014c, 2015) or as part of 3D multiple prediction algorithms (van Dedem and Verschuur, 2000, 2005).

In this paper, we present an extension to the ASHRT by scanning for both the apex and asymptote shifts of the hyperbolas. Therefore, the transform dictionary can match both reflection and diffraction hyperbolas more closely. Seismic diffractions can provide important information about subsurface discontinuities such as faults, pinch outs and small size scattering objects that can be used in interpretation or imaging (Khaidukov et al., 2004; Bansal and Imhof, 2005; Klokov and Fomel, 2012). The travel time curve of diffracted seismic waves can be represented by the double square root equation (Landa et al., 1987; Kanasewich and Phadke, 1988)

$$t = \sqrt{t_d^2 + \frac{(x_s - x_d)^2}{v^2}} + \sqrt{t_d^2 + \frac{(x_d - x_r)^2}{v^2}}, \tag{1}$$

where t_d is the one way travel time for the diffraction, x_d is the diffraction location along the horizontal axis, v is the velocity at the diffraction, x_s is the source location and x_r is the receiver location. If we are considering a common shot gather, the value of the first square root is constant for each diffraction hyperbola. Therefore, we can introduce the new parameter, $\tau_0 = \sqrt{t_d^2 + (x_s - x_d)^2/v^2}$, into the previous equation,

$$t = \tau_0 + \sqrt{t_d^2 + \frac{(x_d - x_r)^2}{v^2}}. \tag{2}$$

We use this equation to define the new Asymptote and Apex Shifted Hyperbolic Radon Transform (AASHRT) which scans for the asymptote origin time shift τ_0 (Ibrahim et al., 2015). This equation simplifies to the ASHRT definition when the asymptote origin time shift τ_0 is set to zero and the diffraction time t_d and location x_d are replaced by the apex time τ and location x_a, respectively.

Asymptote and Apex Shifted Hyperbolic Radon Transform

The AASHRT models seismic data by a superposition of asymptote and apex shifted hyperbolas as follows

$$d(t, x_r) = \sum_{\tau_0} \sum_{x_a} \sum_{v} m(\tau = \sqrt{t^2 - \frac{(x_r - x_a)^2}{v^2}} - \tau_0, v, x_a, \tau_0), \tag{3}$$

where $d(t, x_r)$ is the modelled seismic data and $m(\tau, v, x_a, \tau_0)$ is the AASHRT model. A low resolution AASHRT model can be estimated using the adjoint operation as follows

$$\widetilde{m}(\tau, v, x_a, \tau_0) = \sum_{x_r} d(t = \tau_0 + \sqrt{\tau^2 + \frac{(x_r - x_a)^2}{v^2}}, x_r), \tag{4}$$

where $\widetilde{m}(\tau, v, x_a, \tau_0)$ is the estimated AASHRT model.

The time domain implementation of the AASHRT operator is computationally intensive. Fortunately, the AASHRT kernel can be computed efficiently in the $\omega - k$ domain using fast Stolt migration/demigration operators (Ibrahim and Sacchi, 2014a,b; Ibrahim, 2015). These operators perform migration by mapping data in the $\omega - k_x$ domain into $\omega_\tau - k_x$ for a constant subsurface velocity using the dispersion relation (Yilmaz, 2001; Ibrahim and Sacchi, 2015)

$$\omega_\tau = \sqrt{\omega^2 - (v k_x)^2}, \tag{5}$$

where ω_τ is the Fourier dual of the apex time τ, k_x is the horizontal wavenumber and v is the subsurface velocity. Using the exploding reflector principle (Claerbout, 1992) and the constant subsurface velocity assumption, the Stolt migration operator can be used to estimate the subsurface model. Similarly, the Stolt migration operator can be used to estimate the AASHRT model $\widetilde{m}(\tau, v, x_a, \tau_0)$ as follows

$$\begin{aligned} \widetilde{m}(\tau, v, x_a, \tau_0) = \int\int C \exp\left[i\omega_\tau \tau_0\right] d(\omega = \sqrt{\omega_\tau^2 + (v k_x)^2}, k_x) \\ \times \exp\left[-i k_x x - i\omega_\tau(v)\tau\right] \, d\omega_\tau \, dk_x, \end{aligned} \tag{6}$$

where $C = v\ (\omega_\tau/\omega)$ is a scaling factor resulting from the change of variables. There are two points worth emphasizing in deriving the Stolt-based AASHRT operator. First, every horizontal axis location x is treated as a possible receiver and apex location (so $x_a \equiv x_r \equiv x$). Second, the periodicity of $\widetilde{m}$ along x is implied.

The Stolt-based forward modelling can be written as

$$\begin{aligned} d(t, x) = \int\int\int\int \ m(\omega_\tau = \sqrt{\omega^2 - (v k_x)^2}, v, k_x) \\ \times \exp\left[-i\omega_\tau \tau_0\right] \exp[i k_x x + i\omega t] \, d\omega \, dk_x \, dv \, d\tau_0. \end{aligned} \tag{7}$$

These transforms can be rewritten in operator format as

$$\mathbf{d} = \mathbf{L}\mathbf{m} \tag{8}$$

$$\widetilde{\mathbf{m}} = \mathbf{L}^T \mathbf{d}, \tag{9}$$

where $\mathbf{d}$, $\mathbf{m}$ and $\widetilde{\mathbf{m}}$ represent the data, model and estimated model in vector form, respectively. The forward and adjoint AASHRT operators are represented by $\mathbf{L}$ and $\mathbf{L}^T$, respectively.

Sparse inversion

The estimated model $\widetilde{\mathbf{m}}$ and the original model $\mathbf{m}$ are clearly not identical because Radon transforms are not an orthogonal transformations ($\mathbf{L}\mathbf{L}^T \neq 1$). Furthermore, seismic data may be affected by a number of disturbing factors, including significant noise, limited spatial aperture, coarse and irregular spatial sampling, missing traces, etc. The estimation of the Radon model must then be posed as an inversion problem conditioned by a regularization (penalty) term (Sacchi and Ulrych, 1995). The general form of the cost function to be minimized to obtain sparse Radon coefficients is given by

$$J = \|\mathbf{d} - \mathbf{L}\,\mathbf{m}\|_2^2 + \mu \|\mathbf{m}\|_1^1 \tag{10}$$

where μ is the trade-off parameter between the model regularization term and the misfit term. Since we choose a transform dictionary that closely matches the seismic data, the Radon coefficients should be sparse . This cost function can be minimized using the Fast Iterative Shrinkage/Threshold Algorithm (FISTA) (Beck and Teboulle, 2009). The FISTA algorithm requires an approximation for the largest eigenvalue of the $\mathbf{L^T L}$ operator which is calculated using Rayleigh's power method (Larson and Edwards, 2009). For details about the FISTA algorithm refer to Beck and Teboulle (2009).

Example

We tested the new transform on data modeled using the 2004 BP velocity benchmark (Billette and Brandsberg-Dahl, 2005). The un-decimated data is uniformly sampled with traces 12.5m apart. The input to the interpolation algorithm is obtained by uniformly under-sampling the initial data by a factor of 3 (one in every three traces is kept). We recover the data at sampling rate of 12.5 m producing the results shown in Figure 1. In this test, the AASHRT transform scans two velocities (1500 and 2000 m/s) with zero asymptote shift and scan one velocity (1500 m/s) with 5 asymptote shifts (from 0.2 to 1.0 s). The quality of the interpolation is calculated using the formula

$$Q = 10\ log_{10}\left(\frac{\|\mathbf{d}_{original}\|_2^2}{\|\mathbf{d}_{original} - \mathbf{d}_{recovered}\|_2^2}\right).$$

The Q value for the recovered BP/SEG model shot gather is 15.25 dB.

Conclusions

We have implemented an asymptote and apex shifted hyperbolic Radon transform with a Stolt migration/demigration operator as its kernel to speed up computations. The new transform dictionary is designed to closely match both reflections and diffractions. Our tests show that the new transform is a suitable tool for interpolation. Since the new transform is implemented in $\omega - k$ domain, it can be used in combination with the non-uniform Fourier transform to process data with non-uniform spatial sampling. Future work entails generalizing the problem to the 3D shot distribution and application to simultaneous seismic sources separation.

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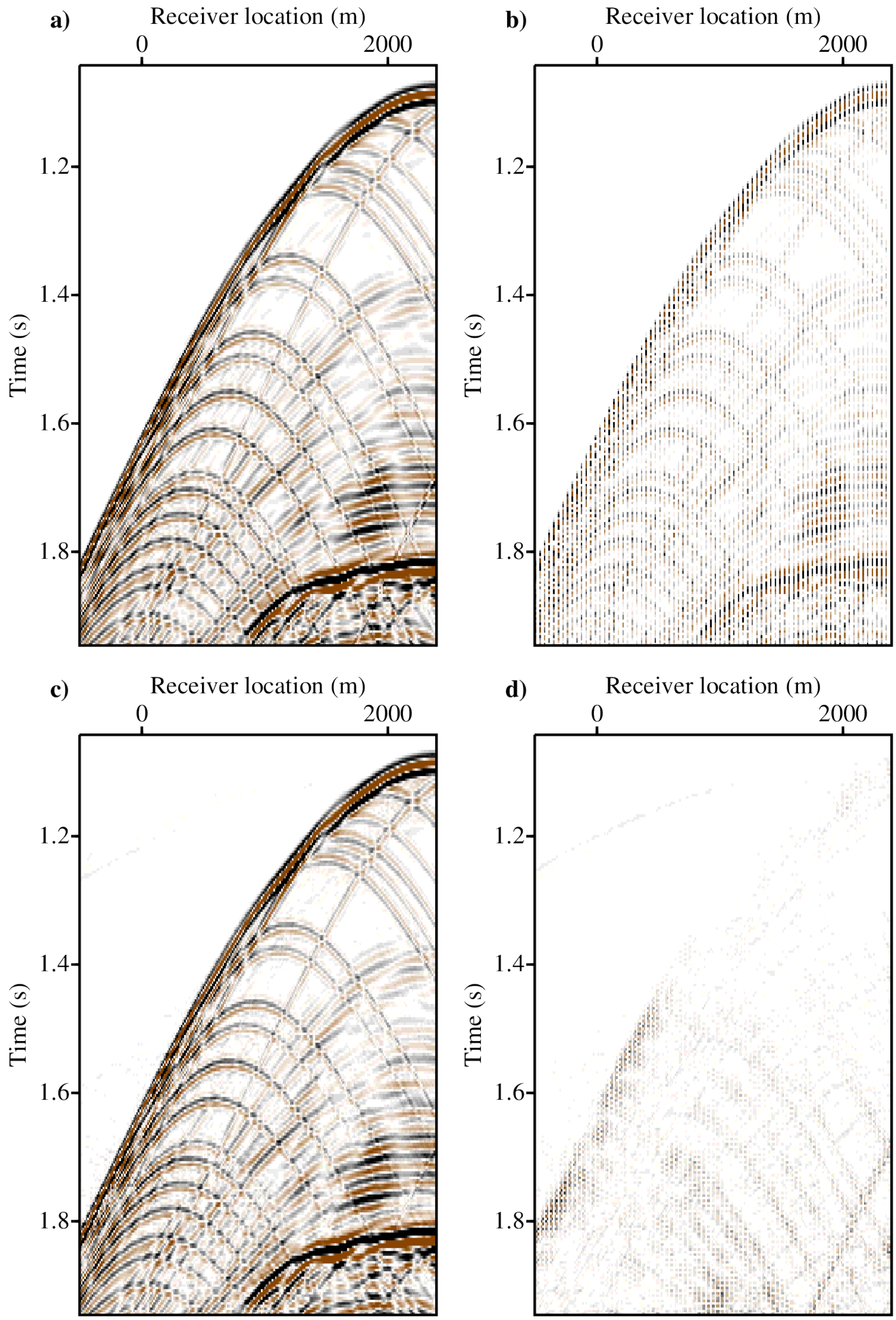

Figure 1 *Common source gather example of the BP/SEG velocity model data. (a) Original gather. (b) Decimated gather. (c) Interpolated gather. (d) Interpolated gather error.*

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