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A Simple Method for Computing Conversion Point Coordinates: Application to 3D CCP Binning

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Summary

The conventional method for finding common conversion point (CCP) coordinates computes a CCP offset as a function of depth using a quartic equation solution. However, it is possible to derive a simpler formula for determining the CCP coordinates using an expression for CCP depth as a function of offset. This approach ultimately solves the same CPP binning problem, and determines all locations of CCPs by running through the range of all possible offsets and finding depth, but with fewer computations and a more explicit formula.

Introduction

The calculation of CCP coordinates for converted wave processing, even for a single homogeneous layer and horizontal reflector, is always cumbersome because it requires a quartic equation solution. To simplify this calculation, Schneider (2002) proposes an analytical solution for finding CCP coordinates that involves solving a cubic equation and computing trigonometric functions. This paper demonstrates that it is possible to determine the CCP coordinates with fewer computations.

A conventional approach to determining CCP coordinates (Tessmer and Behle 1988) uses an analytical solution of a quartic equation to find the shot-CCP offset h_s as a function of reflector depth z, shot-receiver offset h, and P and S-wave velocity ratio$\gamma = V_P / V_S$, as shown in Eq. 1:

$$h_s = F(z, h, \gamma) \ . \qquad (1)$$

However, if we reformulate the problem to look for the CCP coordinates as a function of offset h_s rather than depth (Eq. 2),

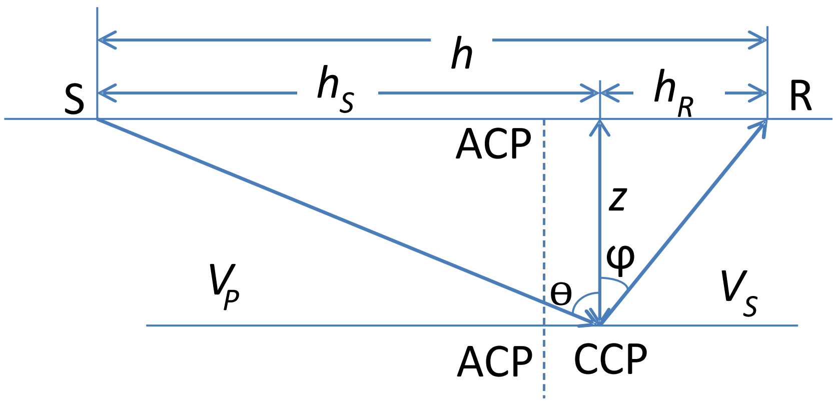
$$z = F^{-1}(h_S, h, \gamma), \qquad (2)$$

then it is possible to derive a simple formula for computing the depth z. This approach is justified by the fact that the ultimate goal is CCP binning, and Eq. 2 enables the reflector depth to be determined for each of the bins located on the line between the source and the receiver.

To estimate z using Eq. 2, only a limited number of h_S values must be checked, which corresponds to the bins between the asymptotic conversion point (ACP) and the receiver; the conventional method, however, requires incrementing through all of the trace samples to determine the corresponding CCP offsets and bins.

Method

Fig. 1 shows shot-receiver geometry for a P-SV converted wave. Fig. 2 shows an example of CCP depth vs. of source-receiver offset function.



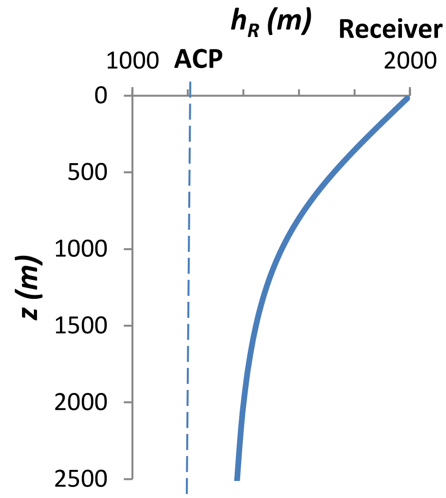


Figure 1. Ray diagram for P-SV wave conversion. S is shot, R is receiver, ACP is asymptotic conversion point, CCP is common conversion point, V_P is p-wave velocity, V_S is s-wave velocity, z is conversion point depth, θ is P-wave angle of incidence, φ is S-wave angle of reflection, h is shot-receiver offset, h_S is shot-conversion point offset, and h_R is receiver-conversion point offset.

Figure 2. An example of *z* vs. $\boldsymbol{h_R}$ function (CCP offset vs. depth).

From Snell's law and the assumption of a homogeneous, flat-earth model and a horizontal reflector (Eq. 3),

$$\sin\vartheta = \gamma \sin\varphi \tag{3}$$

where $= \frac{V_p}{V_S}$.

The depth of the conversion point z can be obtained in two ways, as shown in Eq. 4:

$$z = h_S \cot\theta = h_R \cot\varphi. \tag{4}$$

Squaring Eq. 4, using Eq. 3 and the identity $cos^2\varphi = 1 - sin^2\varphi$, obtains

$$h_S^2 \frac{1 - \gamma^2 sin^2\varphi}{\gamma^2 sin^2\varphi} = h_R^2 \frac{1 - sin^2\varphi}{\gamma^2 sin^2\varphi} \tag{5}$$

which is sufficient to obtain an expression for $sin^2\varphi$ as a function of h_S, h_R and γ (Eq. 6):

$$sin^2\varphi = \frac{\gamma^2 h_R^2 - h_S^2}{\gamma^2 (h_R^2 - h_S^2)} \tag{6}$$

From this point, it is easy to determine $cos^2\varphi$ and (Eq. 7):

$$cot^2\varphi = \frac{cos^2\varphi}{sin^2\varphi} = \frac{h_S^2 - \gamma^2 h_S^2}{\gamma^2 h_R^2 - h_S^2} \tag{7}$$

Using Eq. 4 and 7, and substituting $h_R = h - h_S$, a simple formula for z^2 as a function of a conversion point offset h_S is obtained (Eq. 8):

$$z^2 = \frac{\gamma^2 - 1}{\frac{1}{(h - h_S)^2} - \frac{\gamma^2}{h_S^2}} \tag{8}$$

If we had a goal to derive from Eq. 8 an expression for a conversion point offset h_S as a function of z, we would come again to the fourth degree equation. Eq. 8 is an equivalent to the result of Tessmer and Behle (1988), but in more explicit form. Because their goal was to derive a formula for finding h_S as a function of z, it was impossible to use any simpler method other than solving a quartic equation.

In Eq. 8, $z \geq 0$ for all values of h_S. If $\gamma > 1$ (when $V_P > V_S$), then $\gamma^2 - 1 > 0$. In this case, the fraction will be positive if and only if the denominator is positive:

$\frac{1}{(h - h_S)^2} - \frac{\gamma^2}{h_S^2} > 0$. The denominator is positive when $h_S > \frac{\gamma}{1+\gamma}\ h$. Together with the condition $h_S \leq h$, we get the expected range of admissible values h_S (Eq. 9):

$$\frac{\gamma}{1+\gamma}\ h < h_S \leq h\ . \tag{9}$$

Note, when $\frac{1}{h_R^2} - \frac{\gamma^2}{h_S^2} \to 0$, $z \to \infty$ in Eq. 8, we get the asymptotic solution at $\frac{1}{h_R^2} - \frac{\gamma^2}{h_S^2} = 0$ or $h_S = \frac{\gamma}{1+\gamma}\ h$, which is the same as the Tessmer and Behle (1988) equation for the asymptotic conversion point.

Eq. 8 and 9 define the relationship between z and the range of all admissible h_S. However, it is not necessary to check all of these offsets; rather, it is sufficient to compute z only for those h_S, which are defined by the points on the line between the shot and receiver that are closest to the bin centers.

For each depth z, corresponding travel time t_{PS} for PS trace can be found as (Eq. 10):

$$t_{PS} = \frac{\sqrt{h_R^2 + z^2}}{V_{P\,RMS}(z)} + \frac{\sqrt{h_S^2 + z^2}}{V_{S\,RMS}(z)} \tag{10}$$

supposing that $V_{P\,RMS}(z)$ and $V_{S\,RMS}(z)$ are known. $V_{P\,RMS}(z)$ can be computed from $V_{P\,RMS}(t)$, and $V_{S\,RMS}(z)$ from $V_{P\,RMS}(z)$, assuming a known constant γ or $\gamma(z)$. This formula is similar to the double square-root equation for migration in Bancroft (2014).

After correspondence between bin centers and times on PS trace were found, the trace can be segmented into pieces to be placed into their bins.

Cost of Computing

The cost of computing the CCP binning using the Tessmer and Behle method and quartic equations is *O(n)*, where *n* is the number of trace samples because all trace samples must be checked.

If Eq. 8 and 9 are solved for h_S, rather than for z, then the cost of CCP binning is *O(m)*, where *m* is the number of bins on a line between ACP and the receiver points, which is much smaller than *n*. In this case, *m* will never be greater than $\frac{1}{1+\gamma}\ \frac{h}{b} \approx \frac{1}{3}\ \frac{h}{b}$, where h is shot-receiver offset, and b is bin interval.

Thus, *m* is approximately one third of a number of bins on the line between the shot and the receiver, and for an average survey $m \ll n$.

3D CCP Binning Example

Fig. 3 shows an example of this method applied to the 3C-3D Blackfoot dataset acquired by CREWES (Lu and Margrave 1998). The purple squares on the lines between the shot and the receivers show bins must be checked. In this example, to perform the CCP binning by this method for the trace recorded at

R_3, only 11 offset values must be checked for 11 bins on a line between ACP_3 and R_3 to determine the reflector depth and two-way time on the trace. With the conventional method for a complete calculation, all 1501 samples would have to be checked for a 3-second trace recorded at 2 ms intervals to determine the CCP offsets to identify the bins. If, as with the conventional method, only each tenth sample will be checked, then this would require 150 steps, as compared to 11 steps for this method.

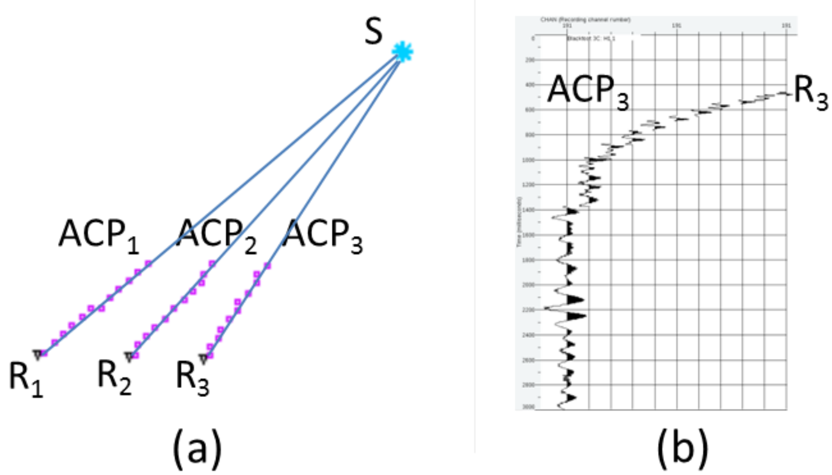


Figure 3. CCP binning example on CREWES 3C-3D data. (a) Shot-receiver geometry for three receivers. (b) Trace split into CCP bins for receiver R3.

Conclusions

This paper proposes a new approach for computing CCP coordinates for binning. This approach requires approximately 1/10 the number of steps as compared to the conventional approach, and and it seems to be more intuitive. Because the ultimate goal is the CCP binning, this method performs the search from the bin perspective, rather than from the trace perspective, and goes through the bin locations to determine the corresponding times on the PS trace. By doing this, only a limited number of bins must be checked on the line between ACP point and receiver, whereas the standard method would require that all times be checked (possibly through some increment) on the PS trace to ensure that no bins are missed.

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