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Comparing step length calculation using well logs with a line-search method

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Summary

We study incorporating well log information into Full Waveform Inversion (FWI) and compare the result with a line-search optimization scheme. The well information is used to obtain a scalar which multiplies the gradient for the model update. We use a smooth version of the true velocity model as the starting model for this study. The final inverted model from incorporating well information is compared to the final inverted model using a line-search optimization scheme. After 90 iterations, our results show that the final model using well information resolves the top and the sides of the intrusion better than the line-search method. The reflectors above the intrusion are aslo more visible. Furthermore, obtaining a scalar using well logs saves a lot of computational time compared with performing a line search.

Introduction

Full waveform inversion is an optimization technique that seeks to find a model of the subsurface that best matches the recorded field data at every receiver location. The method begins from an estimate of the true model, which is iteratively improved using linearized inversions methods. FWI is formulated as a generalised inverse problem with a numerical solver-a forward modelling code and its adjoint. FWI can be viewed as an iterative cycle involving modelling, pre-stack migration and velocity model updating in each iteration (Margrave et al, 2010).

Despite its success, FWI suffers from convergence problems when the starting model is far from the true model and in the absence of low frequencies. However different approaches have been developed to mitigate the problems with conventional FWI, such as incorporating well information to FWI (Margrave et al, 2011a). Well information can aid in (1) calculating the step length (a scalar which multiplies the gradient for the model update), (2) constraining the line-search calculation used in a steepest descent optimization scheme, and (3) improving the wavelet estimate which is essential for proper updates. Other approaches developed in recent years to mitigate the problems with conventional FWI can be found in Biondi and Almomin (2012), and Warner and Guasch (2014).

In this paper, we test our method of calculating a scalar from well information which multiplies the gradient for the model update, with synthetic examples. We compare the results with a line-search optimization scheme.The starting model for the inversion is a smooth version of the true model.

Method

The theory of FWI has been described in literature by Tarantola (1984), Lailly (1983). Pratt et al, (1998) used a frequency-space modelling formalism for FWI. A full mathematical derivation of the theory of FWI can be found in these papers. FWI compares observed and predicted data by subtracting the two datasets to obtain a residual, for real data we anticipate that this residual should be minimized in a least square sense. The FWI objective function (or functional) is the L_2 norm of the residuals and can be represented mathematically as

$$\phi_k = \sum_{s,r} \left(\psi - \psi_k \right)^2 , \tag{1}$$

where ϕ_k is the objective function to minimize, s, r are the sources and receivers over which the sum is taken, ψ is the observed data, and ψ_k is the predicted data for the k^{th} iteration (Margrave et al, 2010).

The model update can be expressed as the gradient of the objective function multiplied by a scalar expressed mathematically as

$$\delta v_k(x,z) = \lambda \int \sum_{s,r} \omega^2 \hat{\psi}_s(x,z,\omega) \delta \hat{\psi}^*_{r(s),k}(x,z,\omega) d\omega , \tag{2}$$

where λ is a scalar, the hat (^) over a variable indicates its temporal Fourier transform, $\hat{\psi}_s\left(x,z,\omega\right)$ is a model of the source wavefield for source s propagated to all $\left(x,z\right)$, ω is temporal frequency, $\delta\hat{\psi}_{r(s),k}\left(x,z,\omega\right)$ is the kth data residual for source s back propagated to all $\left(x,z\right)$, and $*$ is complex conjugation. Specifically $\delta\hat{\psi}_{r(s),k}\left(x,z,\omega\right) = \hat{\psi}_{r(s)}\left(x,z,\omega\right) - \hat{\psi}_{r(s),k}\left(x,z,\omega\right)$ where $\hat{\psi}_{r(s)}\left(x,z,\omega\right)$ is the real data at receivers $r\left(s\right)$ as back propagated into the medium and $\hat{\psi}_{r(s),k}\left(x,z,\omega\right)$ is the kth data model for the same. (Margrave et al, 2010).

A line-search algorithm can be used to obtain the scalar λ, in Equation 2.

Alternatively, the scalar can be calculated from well logs by comparing the current velocity model to that of the known velocity at the well location. We define an objective function β which is the L_2 norm of the difference between the model update calculated from migrating the data residuals and the known velocity at the well and the background velocity model expressed by,

$$\beta = \left\| \lambda G_k - \left(V_{well} - V_{BG} \right)_k \right\|^2 , \tag{3}$$

where G_k is the migration of the data residuals stacked over all shots at the well location, V_{well} is the known velocity at the well location, V_{BG} is the background velocity (or the migration velocity) at the well location, and the L_2 norm is taken over all the samples in the well. (with real data it is necessary to resample the well information to the same sample density as the velocity model).

The scalar λ is derived by minimizing the objective function β in Equation 3 with respect to λ, and making λ the subject of the expression, gives

$$\lambda = \frac{\sum_j \delta V_j G_j}{\sum_j G_j^{\ 2}}, \tag{4}$$

where $\delta V_j = \left(V_{well} - V_{BG}\right)_j$, and j indicates sample number.

We test the two methods of obtaining λ mentioned above with synthetic examples.

Examples

The velocity model used for the synthetic study has slightly dipping layers and an intrusive structure surrounded by high velocity layers at the bottom of the model. We compare calculating a scalar for the model update using Equation 4 with performing a line search. The well penetrates the side of the intrusion and extends from 1040 to 1565 meters. The starting model for the inversion is a smooth version of the true model. We employed a multi-scale approach suggested by Pratt (1999) in the inversion.

The inverted velocity models after 90 iterations from the two methods are shown in Figure 1. Comparing the velocity models, the inversion strategy incorporating well information has recovered the velocity structure between 500 and 1565 meters more accurately than the line-search optimization scheme. Furthermore, the top with the sides of the intrusion can be mapped, and also the reflectors above the intrusion are more visible.

We believe that the inverted model using a line-search optimization scheme can be greatly improved by running more iterations. However the cost of this optimization scheme compared with incorporating well information prohibited us from running more iterations. It may also be worth mentioning that the well used for this study penetrates 2 sedimentary layers before penetrating the side of the intrusion, we conjecture that the final model from incorporating well information may be improved if the well penetrates a few more sedimentary layers. Nonetheless, we are still able to get a reasonable results using this well.

Conclusions

We present an inversion scheme which incorporates well information in the sense that we evaluate a scalar or step length by minimizing a functional which is the L_2 norm of the difference between the model update calculated from migrating the data residuals and the known velocity at the well and the background velocity model. This scalar evaluated from well logs is used to multiply the gradient for the model update. Our synthetic example shows that this method works well and saves a lot of computational time compared with a line-search optimization scheme. Our method can also be improved with good well coverage.

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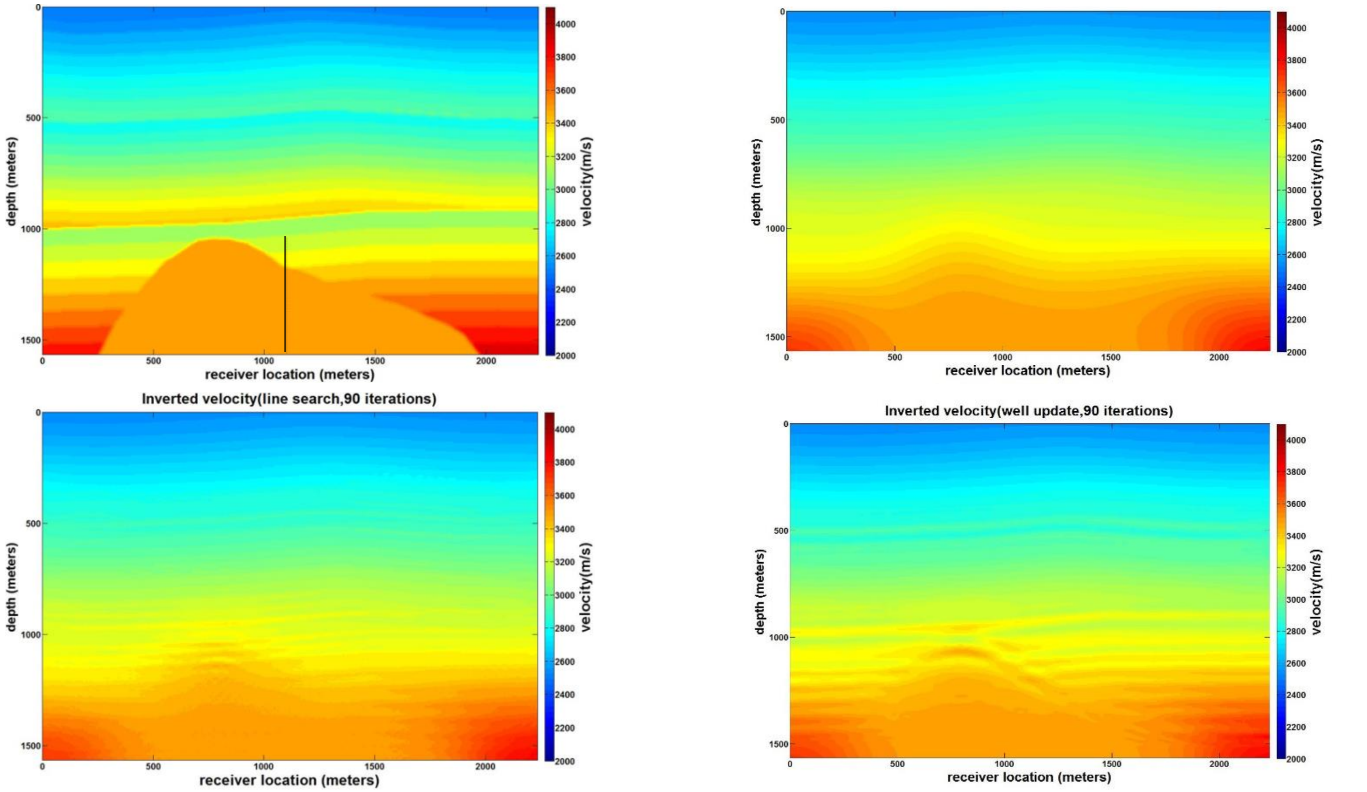

Figure 1. True model, starting model and inverted models after 90 iterations. True velocity model (top left), starting velocity model (top right). Inverted model after 90 iterations using a line-search optimization scheme (bottom left), inverted model using well information (bottom right). The black line on the true model shows the well location (1100 meters).

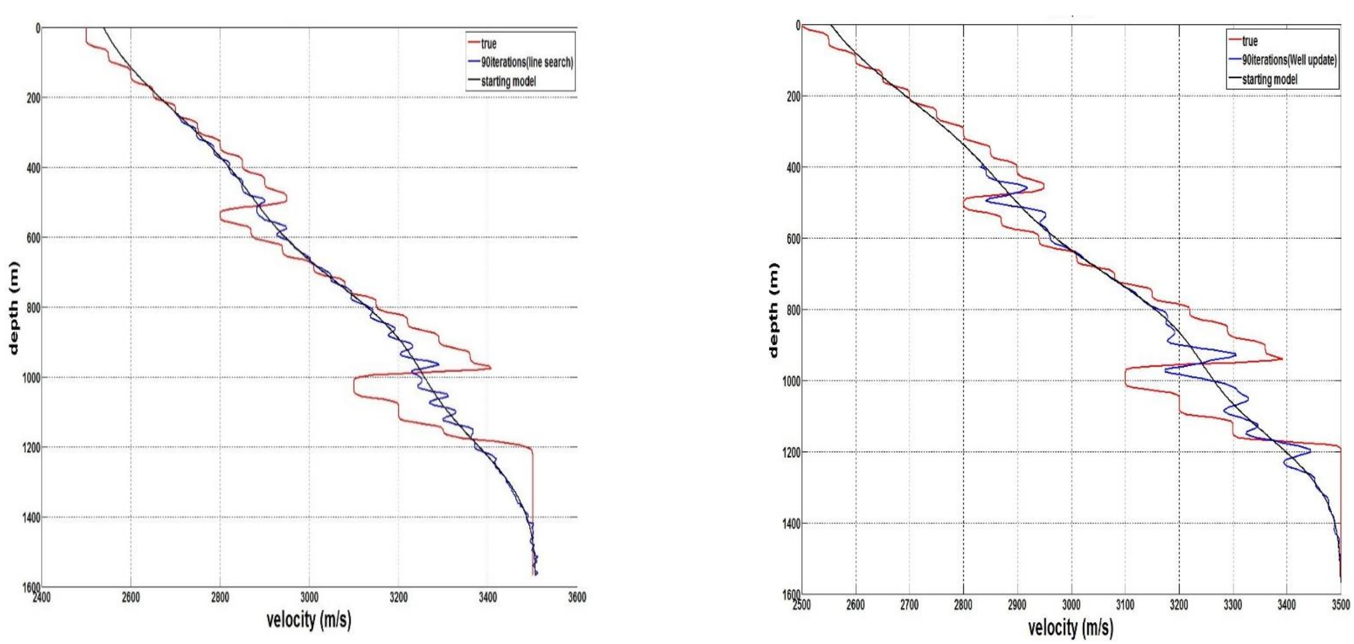

Figure 2. Vertical velocity profile at the well location. Left:True velocity (red), starting velocity (black) and inverted velocity after 90 iterations using a line-search optimization scheme (blue). Right: True velocity (red), starting velocity (black) and inverted velocity after 90 iterations using well information (blue).

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