

## Analysis of azimuthal AVO curvature

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### Summary

Azimuthal AVO can be used to determine anisotropic elastic parameters in the subsurface. AVO methods often separate the effects of elastic parameters on the reflection coefficient into three terms: intercept, gradient, and curvature. In this paper we show that the gradient is characterized by multiple independent terms that change with azimuth, leading to nonuniqueness when used to characterize anisotropy, while the curvature term is only influenced by a single term proportional to the change in horizontal P-wave velocity across the interface along an azimuth. This gives the curvature the capability of determining anisotropy without ambiguity and makes it a useful quantity to estimate, especially when it is combined with the gradient. Additionally, we analyze the nonlinearity present in the precritical region of large-contrast reflections and demonstrate the advantage of using exact reflection coefficients for nonlinear inversion by employing a Markov chain Monte Carlo algorithm.

### Introduction

Estimation of the azimuthal variation of elastic parameters is useful for a number of reasons. For example, it can be used to improve imaging (see Tsvankin et al., 2001) and can provide information about natural fracture density and orientation (Hudson, 1981; Schoenberg and Sayers, 1995), preferred stress orientation (Prioul et al., 2004), and brittleness (Parney et al., 2010).

Methods for azimuthal AVO used in industry traditionally assume a single set of vertical fractures and HTI symmetry (e.g. Rüger, 1998), whereas most reservoirs typically contain multiple fracture sets (e.g. Gillespie et al., 1993). It has been suggested that one of the reasons these methods often fail is due to this assumption (Sayers, 2009), and accurate estimation of more general anisotropic parameters could lead to better results.

Another problem in azimuthal AVO inversions is an ambiguity in the fracture direction. The equation often used to determine the symmetry axis of an HTI medium from Rüger (1998) is nonlinear and produces two possible solutions, with two mutually perpendicular fracture orientations possible. The inversion can be reduced to a single solution if the sign of the AVO gradient is known, but as shown by Goodway et al. (2010) the sign can be negative or positive and is difficult to constrain.

In this paper we use a formulation for general anisotropy provided by Vavryčuk and Pšenčík (1998) to write the effect of stiffness coefficients along a single azimuth on the reflection coefficient along that same azimuth. Using this formulation, we show that the simplicity of the curvature allows for unique inversions, while the complexity of the gradient (it has two terms that vary with azimuth) is what causes the nonuniqueness of its solutions.

### Resolving anisotropy orientation ambiguity with AVO curvature

Following the framework in the derivation of generally anisotropic reflection coefficients in Vavryčuk and Pšenčík (1998), we analyze the PP reflection coefficient along a single vertical plane containing the source and receiver. Within this parameterization the AVO curvature can be expressed as being influenced by a single azimuthally changing stiffness contrast:

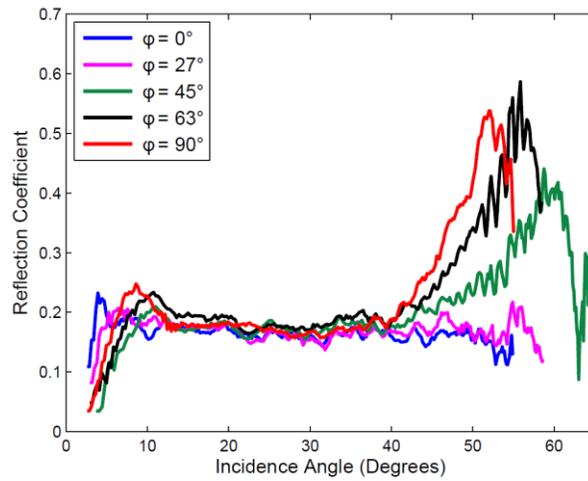
$$AVO \text{ Curvature} = \frac{\Delta A'_{11}}{4\alpha^2} \quad (1)$$

while the AVO gradient is affected by two independent azimuthally changing stiffness contrasts,  $\Delta A'_{13}$  and  $\Delta A'_{55}$  ( $\Delta\rho$  and  $\Delta A'_{33}$  don't change with azimuth):

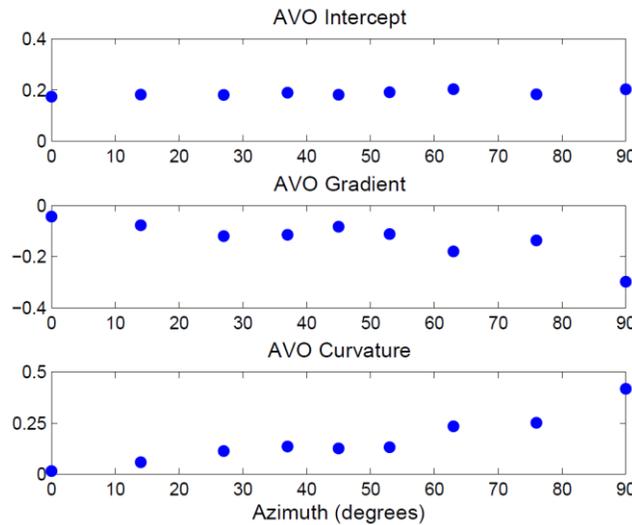
$$AVO \text{ Gradient} = \frac{1}{2\alpha^2} \left[ \Delta A'_{13} - 2\Delta A'_{55} - 4\beta^2 \frac{\Delta\rho}{\rho} - \frac{\Delta A'_{33}}{2} \right], \quad (2)$$

leading to an ambiguity in the estimated anisotropy orientation. The ' symbol denotes that the parameters are in the coordinate system which is rotated to be aligned with the vertical plane containing the source and receiver.

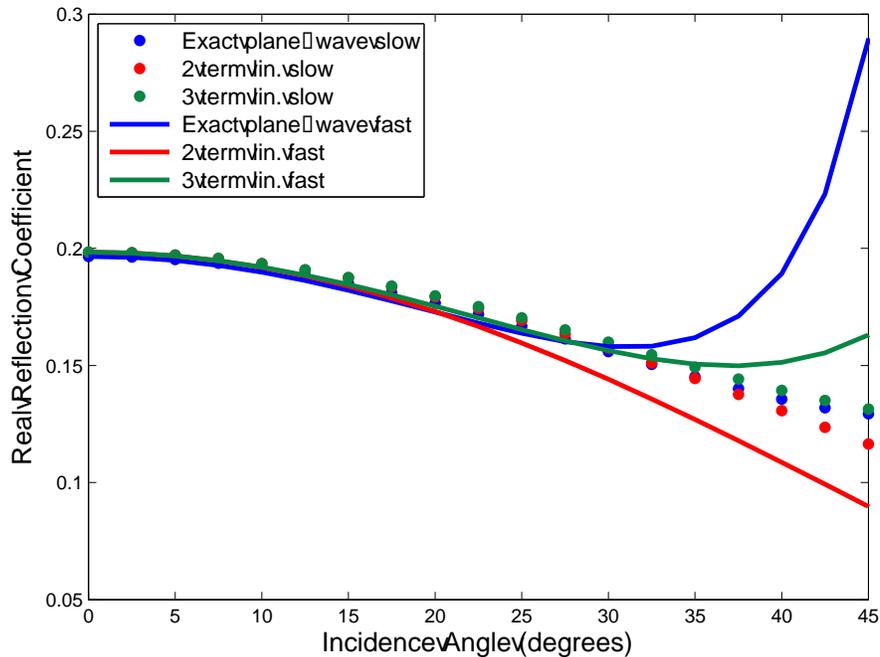
Using azimuthal reflection physical modeling data from Mahmoudian (2013), which is shown in Figure 1, the azimuthally changing AVO intercept, gradient, and curvature were calculated and are displayed in Figure 2. The AVO curvature can constrain the anisotropy orientation, but as shown in Figure 3, can be inaccurate when a linearization is used and there is a large velocity contrast across the interface.



**Figure 1. Azimuthal reflection data from Mahmoudian (2013) for a physical model with an isotropic top layer and an approximately HTI lower layer.**



**Figure 2. Azimuthal variations in the AVO intercept, gradient and curvature calculated from azimuthal reflection data shown in Figure 1.**

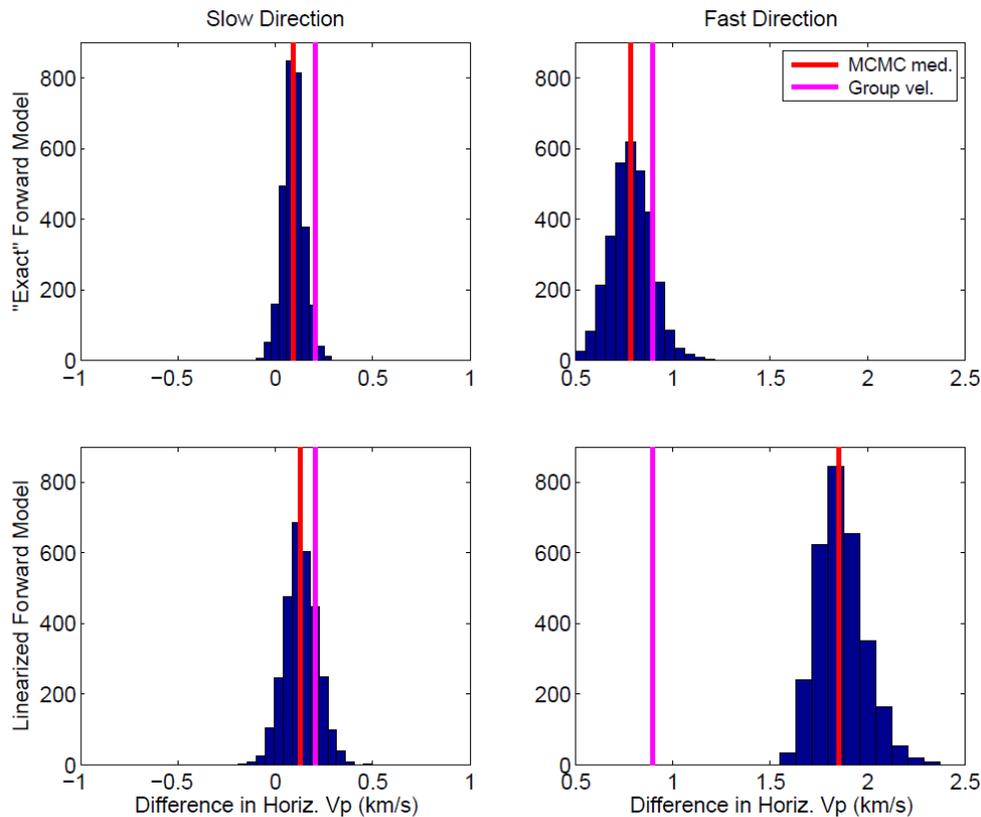


**Figure 3. Comparison of anisotropic reflection coefficient linearization to exact plane-wave coefficient for the physical model used in Mahmoudian (2013). In blue are the exact plane-wave anisotropic reflection coefficients. In green are the linearizations from Vavryčuk and Pšenčík (1998), and in red are the linearizations using only 2 angle terms (AVO intercept and gradient). The solid lines are for the fast direction of the lower medium, representing the direction parallel to a fractured medium. The dotted lines are for the slow direction of the lower medium, representing the direction perpendicular to a fractured medium. Using all 3 angle terms, including the AVO curvature, more closely approximates the exact plane-wave reflection coefficient than just using 2 terms and in the fast direction in which there is a larger contrast in velocities, the 3 term linearization diverges around 30 degrees rather than at 20 degrees. For the fast direction, in which the contrast of elastic parameters across the layer is greater, the linearizations do not approximate the exact plane-wave equation well at large angles in the precritical region.**

### Markov chain Monte Carlo analysis

Because of the nonlinearity in the reflection coefficients and the inaccuracy it causes in the estimated stiffnesses, we decided to perform an inversion using exact reflection coefficients. We used a Markov chain Monte Carlo (MCMC) algorithm to analyze data from the slow and fast directions of the physical model.

Figure 4 shows histograms of the difference in estimated horizontal P-wave velocities,  $\Delta V_{PH}$ , across the interface for the slow and fast directions of the physical model, which are proportional to the posterior distributions (Metropolis et al., 1953). The MCMC algorithm was run using the exact (plane-wave) forward model (top) as well as using the linearized forward model (bottom). Comparing the median result to the velocity change using the results obtained by Mahmoudian (2013) using group velocity measurements shows that for the case of a large velocity contrast (right), using exact equations results in much more accurate estimates.



**Figure 4. Histograms showing posterior probability densities of the change in velocities across the interface in the slow and fast directions of the physical model. Red lines indicate the medians of the Markov chain values and magenta lines indicate the values estimated from group velocities by Mahmoudian (2013). (left) Slow direction of lower medium. (right) Fast direction of lower medium.**

## Conclusions

Formulating azimuthal AVO as a series of AVO inversions along different azimuths allows for an alternative representation of the theory. Typical inversion schemes assume symmetry of some sort and use it to provide a relation between the AVO gradient terms along different azimuths in order to simplify the problem. By allowing for arbitrary anisotropy, the media may be more realistic in some areas. Although it is not possible to invert for all the stiffness coefficients of a triclinic medium using PP reflection data, it is possible that allowing for arbitrary anisotropy and calculating stiffnesses along different azimuths may tell us which directions are more or less stiff, which could be useful information. Additionally, the AVO curvature is only dependent on one elastic stiffness, normalized by the background velocity, and this simplicity allows for it to estimate a change in stiffness uniquely, which could be useful on its own if the data quality is good, or as a constraint to alternative anisotropic methods such as using the AVO gradient. Furthermore, we showed that the curvature cannot accurately estimate horizontal P-velocities for a large contrast and is more useful as a constraint in these situations. Nonlinear inversion methods, such as the MCMC algorithm presented here, can be used if the horizontal P-velocities need to be estimated more accurately.

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