A new upward and downward wavefield continuation for two-way wave equation migration

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Summary

I describe a new two-way wave-equation migration that combines the efficiency of the one-way wave-equation migration with the imaging capabilities of reverse time migration. The method extrapolates the wavefield in the frequency-wavenumber domain depth slice by depth slice. The key to this method is to perform upward continuation from the bottom depth to compute P_z / P, the ratio of the wavefield and its derivative, using the absorbing boundary condition at the bottom. Then the wavefield is downward-continued from the surface using P_z / P. Thus the image at any subsurface point is dependent on the entire wavefield. Steeply dipping events, turning waves, and prism waves can all be handled. The computational cost is one and a half times that of the one-way wave equation migration, cheaper than the reverse-time migration whose cost is proportional to the fourth power of the frequency. A synthetic data example shows that the result is comparable to the one from the reverse time migration.

Introduction

Over the last decade the wave-equation migration (WEM) has been increasingly preferred over Kirchhoff-type migrations for its ability to handle multiple ray paths. Standard WEM or the one-way WEM splits the two-way wave equation into two one-way wave equations, namely the downgoing wave equation and the upgoing wave equation, which are then solved in either the frequency-space or the frequency-wavenumber domain. For the prestack depth migration, the one-way WEM extrapolates the downgong wavefield (the source wavefield) and the upgoing wavefield (the receiver wavefield) in depth. The operator for extrapolating the upgoing wavefield is the complex conjugate of the downgoing operator so that, in effect, the one-way WEM downward continues both source and receiver wavefields from the surface using the same extrapolation operator. As a result, the image at any subsurface point is only related to the wavefield above this point. This makes the one-way WEM efficient but dip-limited and unable to migrate turning waves.

To overcome these disadvantages, one must directly solve the two-way wave equation instead of decomposing it into two one-way wave equations. A popular approach here is the reverse time migration (RTM), which solves the two-way wave equation forward in time for the source modeling and backward in time for the recorded receiver wavefield for that shot. RTM is preferable to the one-way WEM for imaging complex geological structures, where steep dips, turning waves, and prism waves must be properly handled. But large amounts of memory are required as the entire wavefield has to be calculated at each time step. Further, it is computationally expensive as the cost is proportional to the fourth power of the maximum frequency.

One of the differences between the one-way WEM and RTM is that at every depth step one-way WEM calculates the wavefield only at that depth step and RTM calculates the entire wavefield at every time step. Hence the one-way WEM is more efficient and RTM is more accurate and generates a better image. The natural question is can we solve the two-way wave equation by extrapolating the wavefield along the depth axis as well as frequency-slice-by-frequency-slice as the one-way WEM does, while having the image at any reflector dependent on the entire source and receiver wavefields – in other words, to produce an image comparable to RTM at a cost comparable to the one-way WEM. This is what I describe here.

Theory

My approach is to create an extrapolator to perform the upward continuation from the bottom depth. This first computes the ratio P_z / P using the absorbing boundary condition $P_z / P = i\omega / v$ in Clayton and Engquist (1980). Then source and receiver wavefields are downward continued from the surface similar to the one-way WEM. In effect, the entire wavefield is involved in this downward contination, as it uses P_z / P. Calculation of P_z / P does not involve surface information and can thus be used for calculating both source and receiver wavefields.

For simplicity I consider the 2D two-way acoustic wave equation

$$P_{zz} + P_{xx} = \frac{1}{v^2} P_{tt} \tag{1}$$

where P is the acoustic wavefield and v the velocity. Extending the method to 3D is straightforward.

Assume the velocity is a constant in the interval $(z_0, z_0 + \Delta z)$. Applying a temporal and spatial Fourier transform, the partial differential equation (1) becomes an ordinary differential equation

$$P_{zz}(\omega, k_x, z) = -k_z^2 P(\omega, k_x z), \tag{2}$$

where ω is the angular frequency, k_x is the horizontal wavenumber, and k_z given as $k_z^2 = \frac{\omega^2}{v^2} - k_x^2$ is the vertical wavenumber (we will continue to denote the wavefield as P even though it is now in the frequency-wavenumber domain). The square of the vertical wavenumber k_z^2 is assumed to be *nonn*egative, since a negative value means that the wave is evanescent and can be ignored here. With knowledge of $P(z_0)$, the wavefield at $z_0 + \Delta z$ can be derived from the equation (2) as

$$P(z_0 + \Delta z) = \cos(k_z \Delta z) P(z_0) + \frac{1}{k_z} \sin(k_z \Delta z) P_z(z_0), \tag{3}$$

or equivalently

$$P(z_0 + \Delta z) = P(z_0) \left[\cos(k_z \Delta z) + \frac{1}{k_z} \sin(k_z \Delta z) \frac{P_z(z_0)}{P(z_0)} \right]. \tag{4}$$

Unlike most methods which must calculate $P_z(z_0)$ described in Maji, Gao, Abeykoon, and Kouri (2012) and Sandberg and Beylkin (2009), this approache first calculates $P_z(z_0)/P(z_0)$ for the entire wavefield. We begin at $z_0 = z_{\max}$ with the absorbing boundary condition $P_z(z_{\max})/P(z_{\max}) = i\omega / v$ and then upward continue to the surface. Once P_z / P is known everywhere, equation (4) coupled with the surface boundary condition lets us downward continue the wavefield to any depth.

When velocities vary laterally, a few copies of the wavefield P and the ratio P_z / P in the wavenumber domain with several reference velocities are calculated at each depth step and are then inverse Fourier transformed to the spatial domain. These are then interpolated between two calculated reference wavefields and ratios, similar to the ”phase shift plus interpolation” one-way WEM.

Calculation of P_z / P is the only extra cost over the one-way WEM, and as a result the computational expense is one and half times that of the one-way WEM.

Examples

Figure 1 shows a cross-section of a 2D velocity model meant to simulate the foothills of the Canadian Rockies. Figure 2 shows the RTM taken from in Shragge (2014). Figure 3 shows our new up-and-down continued two-way WEM. Some dipping events can be seen from in the latter that fail to appear in RTM. The image is especially improved near the spatial boundaries.

Final Remarks

I have described a novel two-way WEM that has the computational efficiency of the one-way WEM but the imaging capabilities of RTM. A critical problem with RTM is that the cost goes up as the fourth power of the maximum frequency, which in practice means we are restricted to producing low-frequency images. This new method may overcome this limitation.

Results are encouraging, but they show low-frequency artifacts (although which we might be able to suppress them through postmigration processing.) and linear artifacts near surface Angle gathers and anisotropy will be addressed in the future work, and techniques such as split-step and Fourier finite difference during the upward and downward continuation steps might be other options to handle lateral velocity variations.

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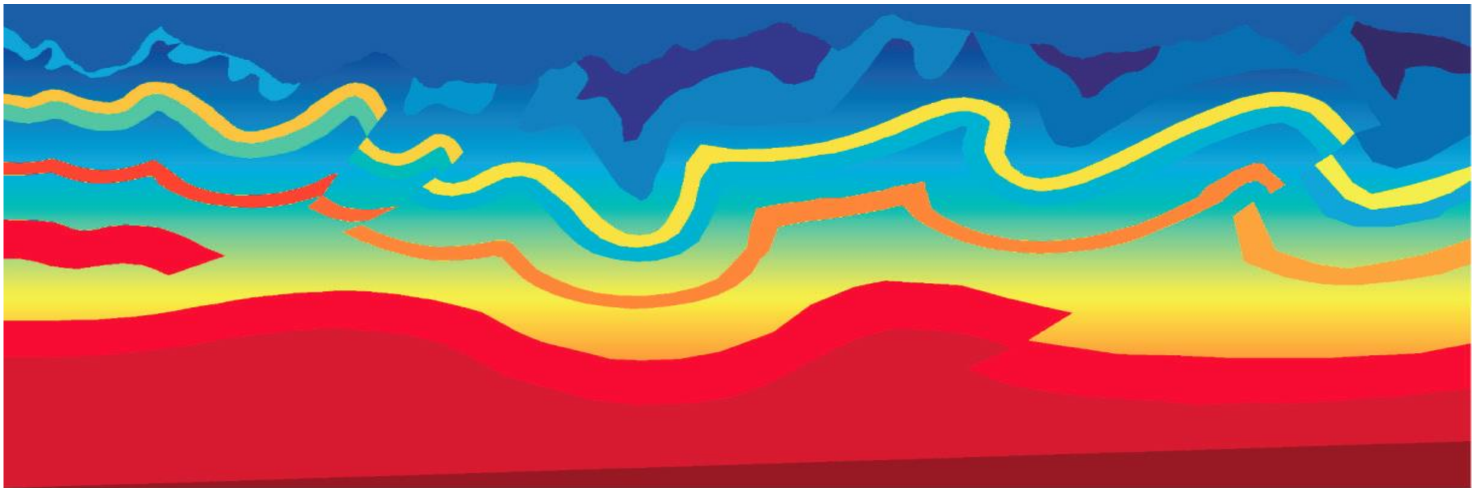

Figue 1. A velocity model simulating the foothills of the Canadian Rocky Mountains.

Figure 2. Reverse-time migration.

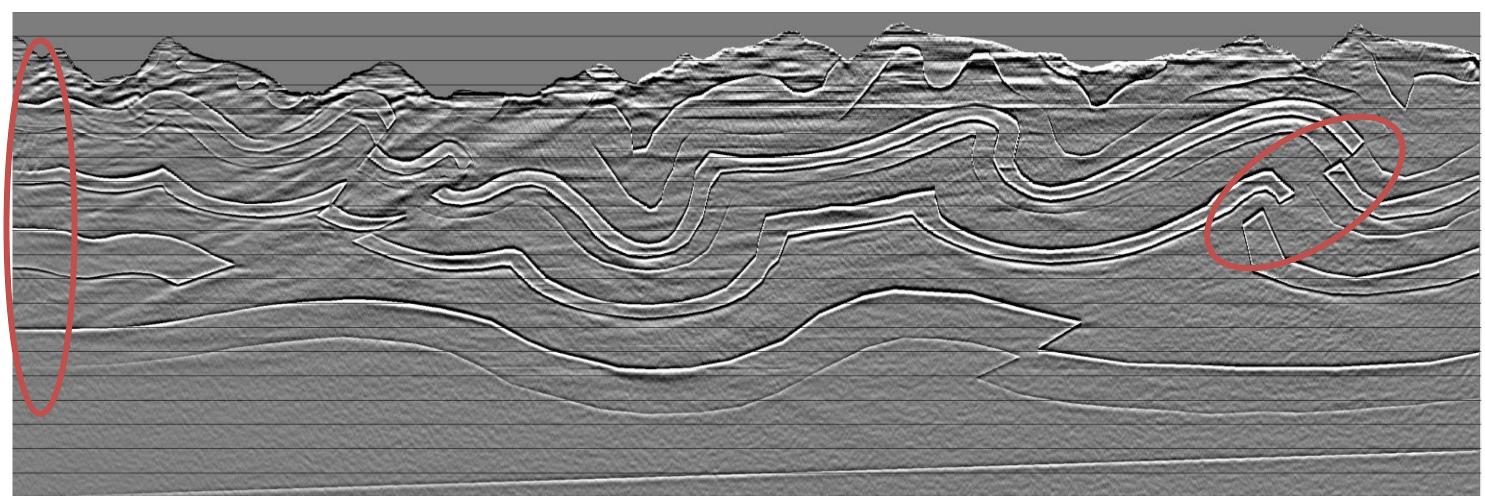

Figure 3. Up-and-down continued wave-equation migration.