Review of hybrid continuum/discontinuum methods for geomechanical modelling of hydraulic fracture growth

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Summary

This paper presents a comparison between two newly developed methods to describe Hydraulic Fracturing (HF) in petroleum industry. The first method is called Finite-Discrete Element Method (FDEM) where a combination of the FEM and DEM is considered. The second approach is based only on the FEM where a fully coupled system consisting of the standard equations of motion to model the rock matrix, and a parallel plate flow simplification (i.e. lubrication theory) to model the flow in the fracture is discussed. The evolution of fractures is tracked by using remeshing. This paper discusses, as well, the issues involved in the development of the two methods, both from a fundamental theoretical point of view along with some related algorithmic considerations essential for the efficient numerical solution of large-scale industrial problems.

Introduction

Considering the discontinuous nature of rock, there is a set of rock engineering problems that aims to maximize the formation of new discontinuities. Examples of these problems include block-caving mining and blasting. In contrast, another set of rock mechanics problems aims to limit failure of rock and the formation of discontinuities. Examples of this set include stability assessment of open pit and natural slopes, dams, and the stability of underground openings. In these cases, the existing rock fabric and discontinuities should be considered as part of the stability assessment (Mahabadi et al., 2012).

Numerical modeling techniques have been used to investigate these problems. The most commonly used techniques are continuum approaches, including finite-difference methods (FDM), finite-element methods (FEM), and boundary-element methods (BEM). These methods have been successfully applied to the assessment of global behavior of rock masses and the analysis of stress and deformation. However, explicit representation of fractures and fracture growth is not straightforward in these methods, mainly because of their continuum assumptions. For instance, in FDM, these assumptions require the functions to be continuous across neighboring cells or grid points. For FEM, the continuum assumptions permit the elements to undergo only small strains. Therefore, even when fracturing is allowed, large-scale opening, sliding, or complete detachment of elements is not possible. Also, a large number of fractures may cause the FEM stiffness matrix to be ill-conditioned. Fracture analysis using BEM has also been limited to isolated, non-interacting cracks. Furthermore, BEM is restricted in handling material heterogeneity and nonlinearity (i.e., plasticity). The limitations of continuum approaches motivated the development of discrete element methods (DEMs). Although continuum models are based on constitutive laws, DEMs are based on interaction laws. Also, unlike continuum techniques, the contact patterns of the DEM system can continuously change as the system deforms (Mahabadi et al., 2012).
Theory

The FDEM/ Y-Geo code

The numerical methodology adopted for this study, namely FDEM, uses continuum mechanics principles and DEM techniques to describe the elastic deformation and the material failure process, respectively. Starting from a continuum representation by finite elements of the solid region in question, progressive fracturing is allowed to take place according to some fracturing criterion, thereby forming discrete elements, which may be composed of one or more deformable finite elements. Subsequent motion of these discrete elements and further fracturing of both remaining continuum and previously created discrete elements is then modeled. This evolution process is continued until either the system comes to rest or up to the time of interest. A combined FDEM simulation comprises a large number of interacting bodies, each with a separate finite-element mesh. To ensure that no bodies overlap at any time, an efficient treatment of contact mechanics is required. From an algorithmic point of view, contact is treated by two processes: contact detection and contact interaction (Munjiza [2004]).

By discretizing discrete elements into finite elements, FDEM is able to model both continuum and discontinuum behavior, thus capturing the whole loading and failure path and the progressive damage process of fractured rocks. A unique feature of FDEM is the ability to model the transition from continuum to discontinuum by means of fracture and fragmentation processes (Munjiza et al. [1999]). The key processes in FDEM include contact detection, interaction, and friction between discrete elements, elastic deformation of finite elements, and fracture of finite elements.

![Figure 1. Material failure modelling in FDEM. (a) Conceptual model of a tensile crack in a heterogeneous rock material. (b) Theoretical FPZ model. (c) FDEM implementation of the FPZ using triangular elastic elements and four-noded crack elements to represent the bulk material and the fracture, respectively. (d) FDEM representation of a fracturable body with continuum triangular elements and embedded crack elements indicated in grey and pink, respectively.](image)

Y-Geo is a new numerical code for geomechanical applications based on the combined finite- discrete element method (FDEM). Several algorithmic developments have been implemented in Y-Geo to specifically address a broad range of rock mechanics problems. These features include (1) a quasi-static...
friction law, (2) the Mohr-Coulomb failure criterion, (3) a rock joint shear strength criterion, (4) a dissipative impact model, (5) an in situ stress initialization routine, (6) a material mapping function (for an exact representation of heterogeneous models), and (7) a tool to incorporate material heterogeneity and transverse isotropy (Mahabadi et al., [2012]).

The FEM with remeshing/GEOS code

The FEM with remeshing is considering realistic simulations of hydraulic fracturing where several phenomena are taken into consideration. Such phenomena include: 1. fracture interaction in the presence of three-dimensional heterogeneous properties, 2. competing paths for existing flowpaths, and, 3. effects of changes to the stress field resulting from the development of fluid pressure field on fracture propagation (Settgast et al., [2014]).

To account for such phenomena, this method provides a fully coupled system consisting of the standard equations of motion to model the rock matrix, and a parallel plate flow simplification (i.e. lubrication theory) to model the flow in the fracture. The evolution of a fracture proceeds along element interfaces, and fluid elements are inserted at the newly formed crack faces. Thus, as the fracture grows, the mesh topology for both the fluid and solid meshes evolves.

Though not totally appropriate, an explicit method is chosen to run the time integration scheme. A standard Newmark method is applied to the finite element equations, while a standard forward Euler method is applied to the finite volume equation to describe fluid diffusion. A rupture criterion is first specified and evaluated on every internal face. Upon completion of a closed path of surfaces where either the rupture criterion is satisfied, or the surface is external to the body, a new set of nodes, edges, and faces are generated, and the mesh topology is updated. To account for geologic materials high non-linearities, the stress intensity factor is calculated at the crack tip for Mode I and Mode II of fracture propagation. The energy release rate is estimated by multiplying the field of displacement by the nodal forces at the crack tip. The energy release rate is then converted to the stress intensity factor.

Examples

The FDEM/ Y-Geo code (Lisjak et al., [2014])

![Figure 2](image)

*Figure 2.* Developed injection pressure as function of time for the Schla and BDS models. Schla and BDS models refer to different values of the far field stresses; see (Lisjak et al., [2014]) for more details.
The FEM with remeshing/GEOS code (Settgast et al., [2014])

Figure 3. Comparison of GEOS against asymptotic solutions for viscosity dominated case, (Savitski and Detournay, [2002]). Fracture radius and wellbore aperture are plotted versus time, and the aperture and pressure profiles at the end of the simulation are plotted against the radial coordinate.

Conclusions

A preliminary evaluation of a hybrid continuum-discontinuum numerical code to simulate hydraulic fracture propagation has been carried out. The FDEM code was enhanced with a computational module dedicated to the simulation of fluid-pressure-driven fracturing. The approach was validated by reproducing injection pressure responses in homogeneous and isotropic conditions.

A fully coupled Finite Element/Finite Volume approach to modeling the evolution of hydraulic fractures using the GEOS code is presented as well. This approach has been verified against accepted solutions for the viscosity and toughness dominated regimes for radial fracture propagation and for the viscosity dominated regime for lateral fracture propagation.

Future studies are ongoing to verify and apply both Y-Geo and GEOS for modeling networks of intersecting fractures, variations in the in-situ stress field, variations in material properties, and non-linear as well as anisotropic material response. The effect of possible thermal strains, due to temperature change during the injection of cold fluid, as well as fluid leak off from the fracture surfaces are to be considered.

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References


