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Full waveform inversion: a synthetic test using the PSPI migration

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Summary

Full Waveform Inversion (FWI) is a tool for the estimation of P-wave velocity. We demonstrate, for a 2D acoustic marine synthetic case, that the methodology can be applied with reasonable results, using the PSPI migration with a deconvolution imaging condition and a line search to calculate the step length to invert for P-wave velocity. We analyze the importance of the initial velocity model for convergence and we obtain good responses for velocity models with smoothed flat layers based on simulated wells in the real velocity model. Our results suggest that we can obtain a high resolution inversion if an initial model is created by an interpolation of two or more wells. The test in the Marmousi 2D model shows encouraging results but more tests are necessary.

Introduction

Full waveform inversion, or FWI, is an inverse method that uses the least-square technique to minimize the objective function (the difference between a real seismic data and a synthetic seismic data) by updating the initial parameters model until convergence (Margrave et al., 2011). The update is obtained from a reverse-time migration of the data residual.

The full waveform inversion was proposed in the early 80's (Pratt et al., 1998)) but the technique was considered too computationally expensive. Lailly (1983) and Tarantola (1984) simplified the methodology by using the steepest-descent method (or gradient method) in the time domain to minimize the objective function without explicitly calculating explicitly the partial derivatives. They computed the gradient of the misfit between real and synthetic data by pre-stack migration of the data residuals with reverse-time migration (RTM). Pratt et al. (1998) develop a matrix formulation for the full waveform inversion in the frequency domain and show how to compute the inverse of the Hessian matrix (the step length for convergence in the FWI) by calculating the full-Hessian (Gauss-Newton method) or an approximation of the Hessian (quasi-Newton). The Hessian computation is one of the study areas for the FWI improvement, as it is a large matrix and its computation requires lots of computer power. An overview of the FWI theory and studies are compiled by Virieux and Operto (2009). Margrave et al. (2010) use well control to calibrate the model update (known as step length) and pre-stack wave-equation migration (Ferguson and Margrave, 2005) instead of the RTM, so the iterations are done in time domain but only selected frequency bands are migrated, using a deconvolution imaging condition (Margrave et al., 2011; Wenyong et al., 2013) as a better reflectivity estimation. Warner and Guasch (2014) use the deviation of the Wiener filters of the real and estimated data as the object function with great results.

In this work we investigate the FWI in a similar methodology proposed by Margrave et al. (2010), but not using well control as a model calibrator. The velocity residual is calculated using the wave-equation migration with a deconvolution image condition, so the inversion is done in time domain and only selected frequency bands are migrated. The selection of the frequency bands to migrate is also studied in this paper, with the objective to find a better procedure.

Theory and Methodology

FWI is a least square methodology (Tarantola, 1984) that aims to find a velocity model that minimizes the objective function:

$$\Phi\,(\boldsymbol{m}) \equiv \|\Delta \boldsymbol{d}\|^2 = \|\boldsymbol{u} - \boldsymbol{d}\|^2 \quad (1)$$

where *m* is the velocity model, *u* is a synthetic data generated by a forward modeling in model *m*, *d* is the acquired data and Δd is the data residuals. Minimization of equation 1 is done by a derivation of the objective function around a velocity disturbation, i.e., deriving $\Phi\,(m_0 + \Delta m)$ and equalizing to zero (Margrave et. al., 2011: Virieux and Operto, 2009). The solution leads to a model update in the form:

$$\boldsymbol{m}_{n+1} = \boldsymbol{m}_n + \alpha_n \boldsymbol{H}_n^{-1} \boldsymbol{g}_n \quad (2)$$

where **g** is the gradient, or the backpropagated time-reversed of the data residuals, **H** is the Hessian matrix (the second derivative of the misfit function), *α* is a scale factor determined by using a line search (searching for a scale factor that minimazes the objective function) and *n* is the iteration number. Equation 2 suggests that the correct velocity model can be determined iteratively by updating the model with the migrated data residuals with a proper gain. For each iteration we must do a forward modeling at each shot point using the current velocity model and repeat the process until convergence. Margrave et al. (2011) suggest that the Hessian calculation can be avoided by applying a deconvolution imaging condition in the migrated data residuals. This gives a better estimation of the reflections. Equation 2 can be reduced to:

$$\boldsymbol{m}_{n+1} = \boldsymbol{m}_n + \alpha_n \boldsymbol{g}_n \quad (3)$$

where **g** is now the the backpropagated time-reversed of the data residuals with a deconvolution imaging condition applied. Usually the gradient is calculated with a reverse-time migration (RTM) but in this work we use a PSPI migration (Margrave et al., 2010: Guarido et al., 2014).

Examples

The tests are done using synthetic data. We created a velocity (figure 1a) to be used in the forward modeling and use the synthetic shots as real shots. The simulation is done using 96 shots with 100m spacing and 30m depth with 401 receivers at the surface with 10m spacing. The shot gather has a split-spread geometry. The dominant frequency of the wavelet is 10Hz.

For the first test, the initial velocity model (figure 1b) used is a smoothed version with a Gaussian window of the real velocity model. The initial velocity model is used in the forward modeling of 96 synthetic shots (one for each shot point) and are compared to the real shots (the difference) to have the data residuals. Each of the data residuals are migrated with the PSPI migration with a deconvolution imaging condition. The migration is done on a selected frequency band, starting from low frequencies to avoid a local minimum (Pratt et al., 1998; Plessix et al., 2010) and increasing to higher frequencies for higher iterations. A mute is applied to avoid far offset artifacts. All the migrated residuals are stacked to generate the gradient but it has small values. A scale factor is determined by a line search, which consists on trying different scale factors values to create a temporary velocity model, do a forward modeling (we use only the shot point in the center of the model) to calculate the difference with the real shot and chose the scale factor that provides the lowest difference, or lowest error. The first update is shown on figure 1c and the layers interface started to show up and after 60 iterations (figure 1d) the velocity model is inverted with high resolution. This was expected as we started with an initial guess very close to the global minimum.

An initial velocity model (figure 2b) based on a sonic log simulation (dashed line on figure 2a) is used in the second test. In the first iteration is already possible to note the some interfaces showing up, as the high velocity body in the bottom. 55 iterations resulted also in a high resolution velocity model, inverting correctly the layers interfaces. However, even inverting the borders of the high velocity body, we could not “fill” it with the correct velocity.

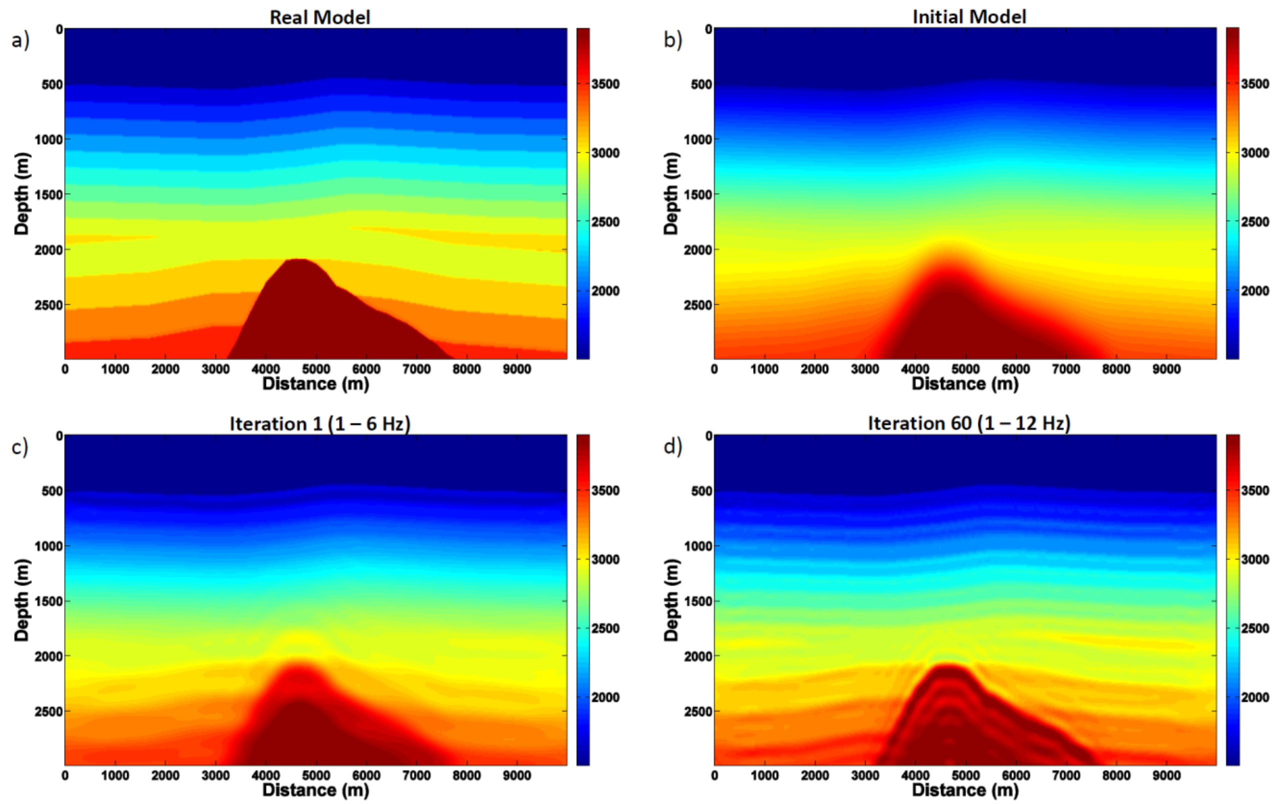

Figure 1: a) Real velocity model, b) initial model, c) first update and d) iteration 60.

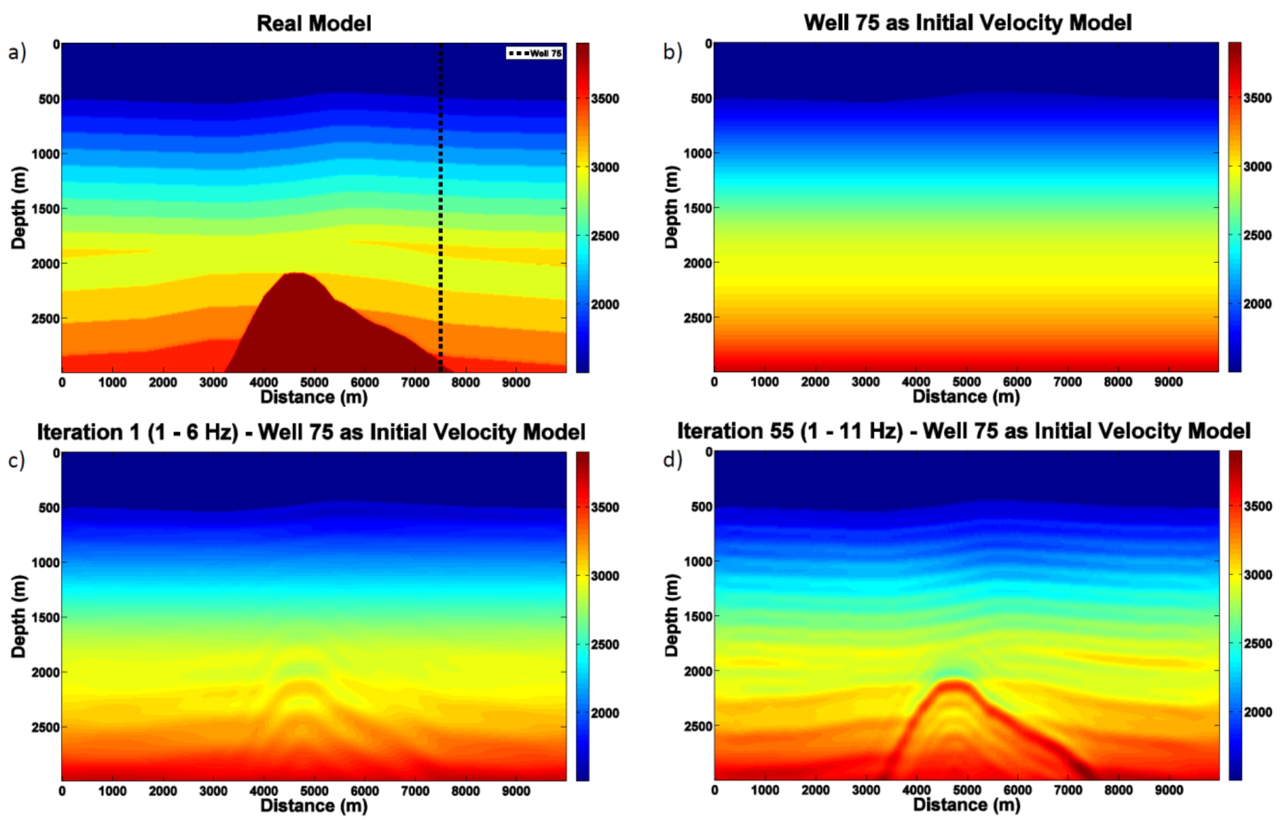

Figure 2: a) Real velocity model, b) initial model (based in a well log simulation), c) first update and d) iteration 55

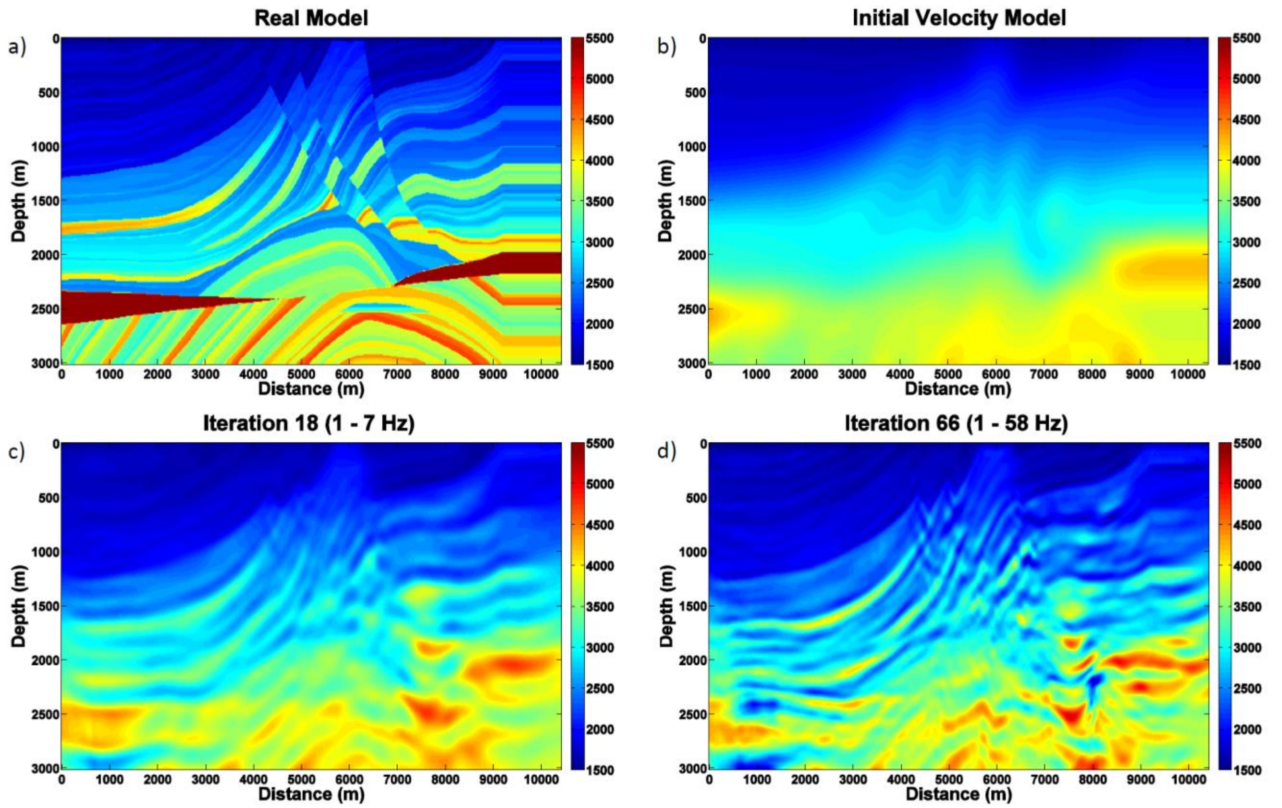

Figure 3: a) Marmousi velocity model, b) initial model, c) first update and d) iteration 66.

The Marmousi model (figure 3a) is used in the third test. Now there are used 102 shots placed in the surface. For the processing the only difference is that the gradient is smoothed at each iteration. The guessing model (figure 3b) is a smoothed version of the Marmousi model model. With 18 iterations (figure 3c) the inverted model has a good resolution showing some of the major structures in the model. After 66 iterations (figure 3c) the resolution of the shallow part (above 2000m) is increased but the deep part loses resolution. This can be explained that for higher iterations are inverting higher frequencies that are not in the seismic data. However, the results are encouraging.

Conclusions

A full waveform inversion using the PSPI migration with a deconvolution imaging condition applied on synthetic data resulted on high resolution velocity models when a simple velocity model is used. The result is better when the initial guess is close to the real answer, but we obtained very good results using a starting model based on a single sonic log simulation. This suggests that an initial model resulting from an interpolation of information from two or more wells can be used to have a starting model closer to the real answer.

The resulting velocity model in the Marmousi model is impressive but is constrained of the absence of higher frequencies in the simulated data to invert properly thinner layers.

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