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Robust principle component analysis (RPCA) for seismic data denoising

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Summary

Principal component analysis (PCA) is an effective tool for random noise attenuation. It has been widely used in seismic data processing for the enhancement of the signal-to-noise ratio of seismic data. However, PCA lacks robustness to outliers and, therefore, its applications to random noise suppression has limitations. We present a robust PCA (RPCA) method that can be utilized in the frequency-space domain to eliminate erratic noise. The method adopts a nuclear norm constraint that exploits the low rank property of the desired data while using an ℓ_1 norm constraint to properly estimate erratic (sparse) noise. The problem is then tackled via the first-order gradient iteration with two steps of soft-thresholding. We use synthetic examples to illustrate the effectiveness of the algorithm.

Introduction

Seismic data are always contaminated with noise. Therefore, signal-to-noise ratio enhancement plays an important role in seismic data processing. A variety of methods that exploit the differences between signal and noise have been developed. Noise removal techniques based on principal component analysis (PCA) are of special interest to this article. Ulrych et al. (1999) illustrated how matrix rank reduction methods can be utilized to eliminate incoherent noise from seismic records in time domain. A related family of methods, the Karhunen-Loeve transform, has been introduced for the enhancement of signal-to-noise ratio (Al-Yahya, 1991) as well. To handle dipping events, Trickett (2003) proposed the $f-x-y$ eigenimage filtering that applies matrix rank reduction to frequency slices. Cadzow de-noising (Trickett and Burroughs, 2009), or singular spectrum analysis (SSA) (Sacchi, 2009; Oropeza and Sacchi, 2011) also operates in frequency space domain. However, Cadzow denoising does not operate on the $f-x$ data themselves but on Hankel matrices that are formed from data in $f-x$ domain.

Although the aforedescribed rank reduction methods are very effective for attenuating the random Gaussian noise, their application to real data problems can be limited because they lack robustness to erratic noise. In seismic data processing, erratic noise includes swell noise, power line noise and artifacts caused by glitches in the recording instrument. Outliers tend to manifest as high-amplitude isolated signals that do not obey the Gaussian distribution. Therefore, the conventional least squares error criterion utilized by PCA will perform poorly (Golub and van Loan, 1996; Trickett et al., 2012; Chen and Sacchi, 2014).

In this article, we present an algorithm named robust PCA (RPCA) to suppress erratic noise and blending noise in simultaneous sources acquisition. We consider the minimization of a cost function that combines a weighted nuclear norm and an ℓ_1. The problem is then tackled via the gradient iteration with two steps of soft-thresholding.

Theory

We consider to apply the proposed algorithm on a 3D seismic volume $D(t,x,y)$. The Fourier transform can be adopted to map data from $t-x-y$ domain to $\omega-x-y$ domain. At a given frequency ω_0, the spatial data, $D(\omega_0-x-y)$, can be denoted via a matrix $D(x,y)$. We can remove ω_0 from the analysis knowing that the algorithm operates for all frequency slices. As the desired signal is coherent along two spatial directions, it can be estimated via a low rank matrix $\mathbf{X}$. The observed data can be expressed via

$$\mathbf{D} = \mathbf{X} + \mathbf{E}, \tag{1}$$

where $\mathbf{E}$ is a sparse matrix corresponding to the additive erratic noise. The RPCA method suggests the following problem. Find $\mathbf{X}$ such is of low rank and $||\mathbf{E}||_0$ is minimum. Where $||\mathbf{E}||_0$ is the ℓ_0 norm of $\mathbf{E}$ means the number of non-zero element in $\mathbf{E}$. The latter is an NP-hard problem. In order to make the problem tractable, we consider to the ℓ_1 norm, which is defined by the summation of absolute values of the matrix elements, to replace the ℓ_0 norm and to use the nuclear norm which is defined as the summation of singular values of the matrix, to replace the low rank constraint. The ℓ_1 norm is the tightest convex relaxation of the ℓ_0 norm and the nuclear norm is the tightest convex relaxation to the low rank constraint, respectively. We also introduce further relaxation with a *Frobenius* norm constraint, $||\mathbf{D}-\mathbf{X}-\mathbf{E}||_F^2$, to tolerate the inclusion of Gaussian noise in the seismic data. The resulting cost function is as follows

$$\min J = \frac{1}{2\mu}||\mathbf{D}-\mathbf{X}-\mathbf{E}||_F^2 + \lambda||\mathbf{E}||_1 + ||\mathbf{X}||_*, \tag{2}$$

where λ is a trade-off parameter that balances the sparsity and low rank constraints. The scalar μ is a small constant that controls the inclusion of Gaussian noise.

The alternating first-order gradient method is utilized to estimate the low-rank data $\mathbf{X}$ as well as the sparse matrix that represents the erratic noise $\mathbf{E}$. We consider to minimize the cost function via an iterative scheme. For this purpose we adopt a sub-gradient optimization technique (Zhou et al., 2010)

$$\begin{aligned} \mathbf{X}^{k+1} &= \min ||\mathbf{X}||_* + ||\mathbf{X}-\hat{\mathbf{X}}^k||_F^2 \\ \mathbf{E}^{k+1} &= \min \lambda||\mathbf{E}||_1 + ||\mathbf{E}-\hat{\mathbf{E}}^k||_F^2 \,, \end{aligned} \tag{3}$$

where $\hat{\mathbf{X}}^k$ and $\hat{\mathbf{E}}^k$ are given by the gradient of the separable quadratic system

$$\begin{aligned} \hat{\mathbf{X}}^k &= \mathbf{X}^k - \frac{1}{2\mu}(\mathbf{X}^k + \mathbf{E}^k - \mathbf{D}) \\ \hat{\mathbf{E}}^k &= \mathbf{E}^k - \frac{1}{2\mu}(\mathbf{X}^k + \mathbf{E}^k - \mathbf{D}) \,. \end{aligned} \tag{4}$$

The two sub-problems defined by equation (3) are commonly seen in the field of compressive sensing and matrix completion, respectively. The matrix $\mathbf{E}^{k+1}$ is typically given by soft-thresholding the entries of $\hat{\mathbf{E}}^k$ as follows

$$\mathbf{E}^{k+1}(i,j) = \max(|\hat{\mathbf{E}}_k(i,j)| - \frac{\lambda\mu}{2}, 0)\,, \tag{5}$$

where $\hat{\mathbf{E}}_k(i,j)$ denotes the element in matrix $\hat{\mathbf{E}}_k$. Similarly, $\mathbf{X}^{k+1}$ can be computed by applying soft-thresholding to the singular values of $\hat{\mathbf{X}}^k$ (Fazel, 2002; Recht et al., 2010) as follows

$$\hat{\Sigma}(i,i) = \max(|\Sigma(i,i)| - \frac{\mu}{2}, 0)\,, \tag{6}$$

where we assume $\hat{\mathbf{X}}^k = \mathbf{U}\Sigma\mathbf{V}^*$ and $\mathbf{X}^{k+1}$ are then recovered by the new set of singular values $\hat{\Sigma}(i,j)$. The resulting algorithm is summarized in Algorithm (1). In each iteration, we modify the current estimate of the low-rank data and erratic noise in the opposite direction to the gradient of the quadratic term. Then we apply two steps of soft-thresholding to the modified estimators . The convergence of this algorithm is very similar to the convergence of the FISTA algorithm utilized for $l_1 - l_2$ inverse problems (Zhou et al., 2010; Beck and Teboulle, 2009).

Algorithm 1 RPCA

Inputs:
Spectral matrix $\mathbf{D}$, trade-off parameter λ and stopping criterion ε
Initialize:
$\mathbf{X}^0 = 0$; $\mathbf{E}^0 = 0$; $k = 1$
repeat
$\hat{\mathbf{X}}^k = \mathbf{X}^k - \frac{1}{2\mu}(\mathbf{X}^k + \mathbf{E}^k - \mathbf{D})$
$\hat{\mathbf{E}}^k = \mathbf{E}^k - \frac{1}{2\mu}(\mathbf{X}^k + \mathbf{E}^k - \mathbf{D})$
$[\mathbf{U}, \Sigma, \mathbf{V}] = \text{svd}[\hat{\mathbf{X}}^k]$
$\hat{\Sigma}(i,i) = \max(|\Sigma(i,i)| - \frac{\mu}{2}, 0)$
$\mathbf{X}^{k+1} = \mathbf{U}\hat{\Sigma}\mathbf{V}^*$
$\mathbf{E}^{k+1}(i,j) = \max(|\hat{\mathbf{E}}_k(i,j)| - \frac{\lambda\mu}{2}, 0)$
$k = k+1$
until $\|\mathbf{D} - \mathbf{X}^k - \mathbf{E}^k\|_F^2 < \varepsilon$

Examples

To test the effectiveness of the proposed algorithm, we first created a synthetic seismic section with 3D linear events (Figure 1a). We then added random, spike-like noise to mimic blending noise generated by simultaneous source shooting (Figure 1b). Figure (1c) shows the de-noising result and Figure (1d) shows the error of the estimator. The proposed method effectively suppressed the incoherent noise. We improve the the signal-to-noise ratio of data from -1.2 dB to a factor of 11.9 dB.

We also tested the algorithm on one common receiver gather of a synthetic 3D VSP data set. Figure (2a) shows the centre shot line of one common receiver gather without noise. Figure (2b) shows the same shot line contaminated with noisy observations. The traces are corrupted with erratic noise (the amplitude of the erratic noise are about 3 times the maximum amplitude of the reflections). Figure (2c) shows the resulting de-noised shot line with the proposed method. As a result, both Gaussian and erratic noise were effectively removed. Through RPCA de-noising, we improve the quality of data from -8.2 dB to a factor of 12 dB.

Conclusions

We presented an algorithm for suppressing erratic noise via a rank-reduction method. The algorithm relies on the low rank approximation of the spatial data at a given monochromatic frequency in the $f-x$ domain domain. A nuclear norm constraint for the data, as well as an l_1 norm constraint for the sparse erratic noise have been utilized to design the cost function of the problem. Through tests with synthetic examples, we show that the proposed algorithm can be utilized for deblending and for suppressing the erratic noise.

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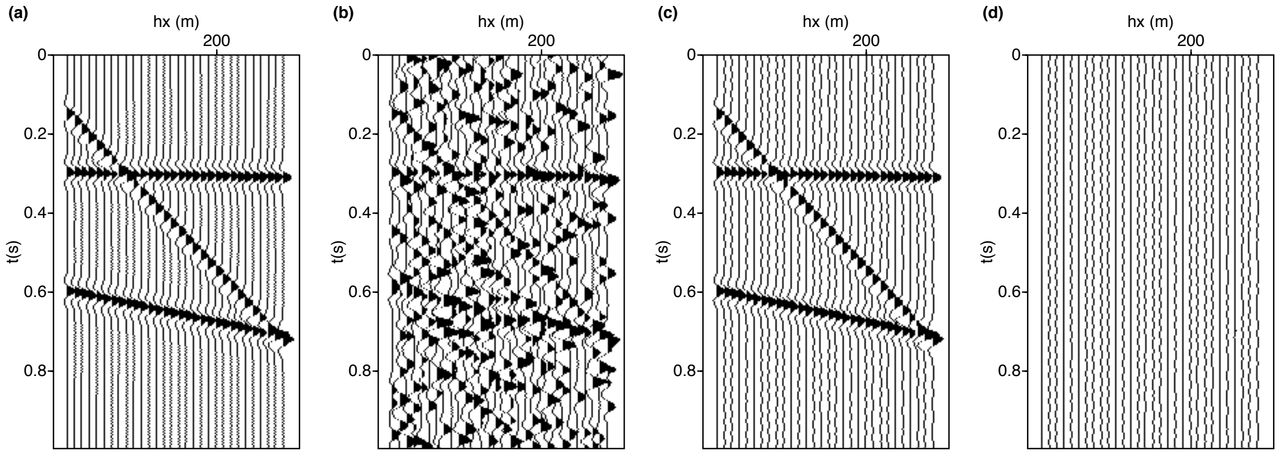

Figure 1 *(a)Synthetic data with linear events in CMPx gather 15. (b) Synthetic data with 3 linear events contaminated with randon, spike-like noise to mimic the pseudo-deblended CMP gather. (c) CMPx gather 15 after RPCA de-noising, the SNR has been improved by 11.9 dBs. (d) The estimation error between section (a) and (c).*

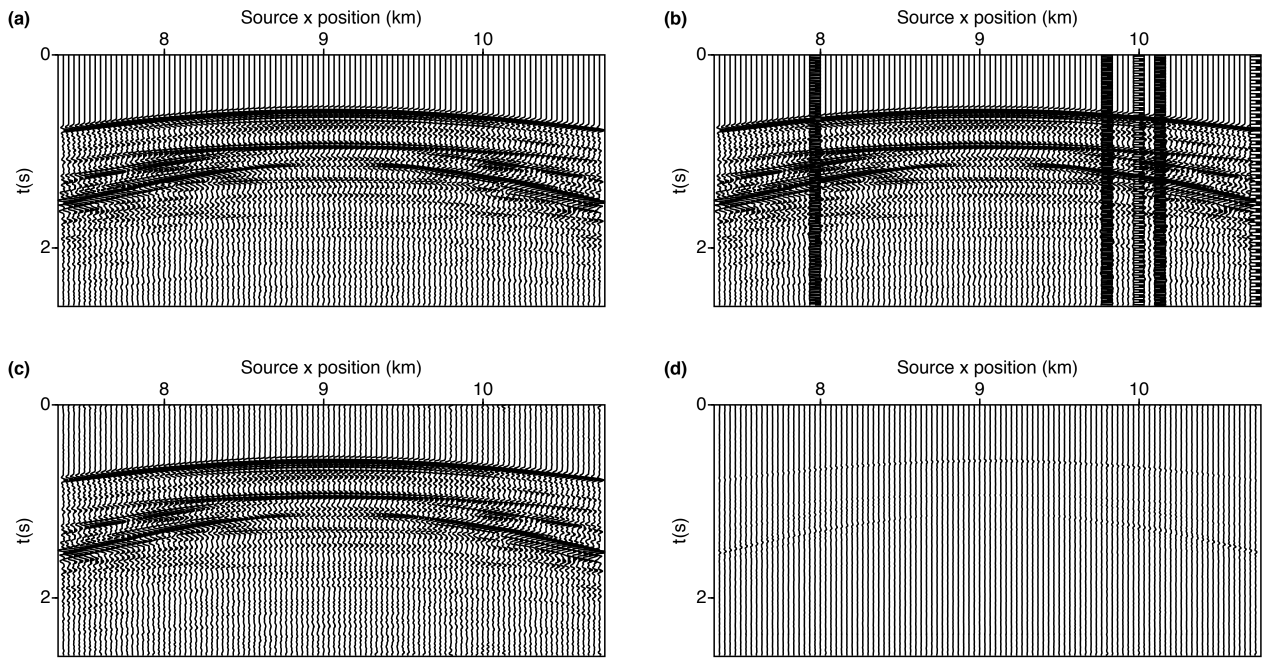

Figure 2 *(a) Centre shot line of the ideal clean common receiver gather of the 3D VSP data set. (b) The centre shot line of the common receiver gather cotaminated with erratic noise. (c) The centre shot line of the common receiver gather after RPCA de-noising. (d) The difference between section (a) and (c).*

References

Al-Yahya, K., 1991, Application of the partial Karhunen-Loéve transform to suppress random noise in seismic sections: Geophysical Prospecting, **39**, 77–93.

Beck, A. and M. Teboulle, 2009, A fast iterative shrinkage-thresholding algorithm for linear inverse problems: SIAM J. Imaging Sciences, **2**, 183–202.

Chen, K. and M. D. Sacchi, 2014, Robust reduced-rank filtering for erratic seismic noise attenuation: Geophysics, **80**, V1–V11.

Fazel, M., 2002, Matrix rank minimization with applications: Master's thesis, Elec. Eng. Dept, Stanford University.

Golub, G. and C. van Loan, 1996, Matrix computations: London: The Johns Hopkins University Press, third edition edition.

Oropeza, V. and M. Sacchi, 2011, Simultaneous seismic data denoising and reconstruction via multichannel singular spectrum analysis: Geophysics, **76**, V25–V32.

Recht, B., M. Fazel, and P. Parrilo, 2010, Guaranteed minimum-rank solutions of linear matrix equations via nuclear norm minimization: SIAM Review, **52**, 471–501.

Sacchi, M. D., 2009, F-x singular spectrum analysis: CSPG CSEG CWLS Convention, 392–395.

Trickett, S., L. Burroughs, and A. Milton, 2012, Robust rank-reduction filtering for erratic noise: 82nd Annual International Meeting, SEG, Expanded Abstracts, 1–5.

Trickett, S. R., 2003, F-xy eigenimage noise supression: Geophysics, **68**, 751–759.

Trickett, S. R. and L. Burroughs, 2009, Prestack rank-reducing noise suppression: Presented at the SEG Technical Program Expanded Abstracts.

Ulrych, T. J., M. D. Sacchi, and S. L. M. Freire, 1999, Covariance analysis for seismic signal processing, chapter Eigenimage processing of seismic sections, 241–274.

Zhou, Z., X. Li, J. Wright, E. Candes, and Y. Ma, 2010, Stable principal component pursuit: Information Theory Proceedings (ISIT), IEEE International symposium, 1518–1522.