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Seismic wave scattering in viscoelastic media: Born approximation

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Summary

We consider the problem of scattering of viscoelastic waves from perturbations in five viscoelastic parameters (density, P- and S-wave velocities, and P- and S-wave quality factors), as formulated in the context of the Born approximation. Within this approximation the total wave field is the superposition of an incident plane wave and a scattered wave, the latter being a spherical wave weighted by a function of solid angle called the scattering potential. In elastic media the scattering potential is real, but if dissipation is included through a viscoelastic model, the potential becomes complex and thus impacts the amplitude and phase of the outgoing wave. The isotropic-elastic scattering framework of Stolt and Weglein, extended to admit viscoelastic media, exposes these amplitude and phase phenomena to study, and in particular allows certain well-known layered-medium viscoelastic results due to Borcherdt to be re-considered in an arbitrary heterogeneous Earth. We show that elliptically polarized P- and SI-waves cannot be scattered into linearly polarized SII-waves. Furthermore, the elastic formulation is straightforwardly recovered in the limit as P- and S-wave quality factors tend to infinity.

Introduction

The scattering of seismic waves by purely elastic heterogeneities under the Born approximation has been investigated by many authors (Wu and Aki, 1985; Beylkin and Burridge, 1990; Sato et al., 2012; Stolt and Weglein, 2012). Stolt and Weglein (2012) introduced a formal theory for the description of the multidimensional scattering of seismic waves based on an isotropic-elastic model. We identify as a research priority the adaptation of this approach to incorporate other, more complete pictures of seismic wave propagation. Amongst these, the extension to include anelasticity and/or viscoelasticity (Flugge, 1967), which brings to the wave model the capacity to transform elastic energy into heat, ranks very high. Anelasticity is generally held to be a key contributor to seismic attenuation, or “seismic Q”, which has received several decades worth of careful attention in the literature (e.g., Aki and Richards, 2002; Futterman, 1967). Development of methods for analysis (e.g., Tonn, 1991), processing (Bickel and Natarajan, 1985; Hargreaves and Calvert, 1991; Wang, 2006; Zhang and Ulrych, 2007; Innanen and Lira, 2010), and inversion (Dahl and Ursin, 1992; Ribodetti and Virieux, 1998; Causse et al., 1999; Hicks and Pratt, 2001; Innanen and Weglein, 2007) of wave data exhibiting the attenuation and dispersion of seismic Q remains a very active research area. Borcherdt (2009) has presented a complete theory for seismic waves propagating in layered anelastic media, assuming a viscoelastic model to hold.

In the elastic-isotropic setting, beginning with a plane defined by the incoming wave vector and the outgoing wave vector Stolt and Weglein (2012) develop scattering quantities which in an intuitive manner generalize the layered-medium notions of reflections and conversions of P, SV and SH waves. Generalizing this approach to allow for viscoelastic waves of the type described by Borcherdt has several positive outcomes. First, and foremost, it provides an analytical framework for the examination of processes of scattering of viscoelastic waves from arbitrary three-dimensional heterogeneities, as opposed to layered media. Second, it provides a foundation for direct linear and nonlinear inversion methods for reflection seismic data, which go well beyond existing an-acoustic results (Innanen and Weglein, 2007; Innanen and Lira, 2010). And third, it provides a means to compute and analyze the gradient and Hessian quantities used in iterative seismic inversion (see the review by Virieux and Operto, 2009).

There are three types of waves in a viscoelastic medium: P, Type-I S, and Type-II S. For each wave type there is a corresponding seismic quality factor, Q_P, and Q_S. These quality factors have the standard definitions in terms of ratios of the real and imaginary parts of the complex moduli.
In the case of inhomogeneous waves, the attenuation and propagation vectors are not in the same direction. The wavenumber vector of inhomogeneous waves is represented by

$$\boldsymbol{K} = \boldsymbol{P} - i\boldsymbol{A} \quad (1)$$

Here $\boldsymbol{P}$ is the propagation vector, perpendicular to the constant phase plane $\boldsymbol{P} \cdot \boldsymbol{r} = \boldsymbol{constant},$ and $\boldsymbol{A}$ is the attenuation vector perpendicular to the amplitude constant plane $\boldsymbol{A} \cdot \boldsymbol{r} = \boldsymbol{constant}$. The attenuation vector $\boldsymbol{A}$ is in the direction of maximum amplitude decay. If the attenuation and propagation vectors are parallel, the wave is homogeneous (elastic behaviour is recovered in the limit as $\boldsymbol{A}$ vanishes).

Linearized scattering potential

In the scattering framework, the unperturbed medium is a reference medium and the perturbed medium is associated with the actual medium. The difference between the actual and reference medium wave operators is the perturbation operator or scattering operator. In the elastic-isotropic case, this operator can be expressed in terms of a 3×3 matrix, each element of which corresponds to the scattering of one wave type to another. The diagonal elements refer to scattering which conserves wave type, and off-diagonal elements refer to those which convert wave type.

The scattering potential helps us to identify the effects of physical parameters like density velocities and quality factors on the reflection functions. The seismic scattering formulation, and the resulting scattering operator forms, can be used to generalize "layered medium" wave propagation results, providing expressions describing waves interacting with not 1D media but with arbitrary multidimensional heterogeneities. It can be used in principle to generate exact solutions for such waves, but those solutions are in the form of infinite series, which are subject to often rather thorny questions of convergency. In fact the main application has been in the generation of powerful approximate solutions. The Born approximation is a solution accurate to first order in the scattering operator, and is used as the basis for many types of migration and linearized inversion in seismic applications (Bleistein, 1979; Clayton and Stolt, 1981; Beylkin, 1985, etc.).

Viscoelastic P-P scattering

This element quantifies the potential for a point in a viscoelastic medium to scatter a P-wave into a P-wave. The incident and reflected P-waves can be either inhomogeneous with elliptical motion or homogeneous with linear motion in the direction of propagation, depending on the angle between the propagation and attenuation vectors. The scattering potential for PP mode is

$$ {}^{\mathrm{P}}_{\mathrm{P}}\mathbb{V}_{\mathrm{VE}} = {}^{\mathrm{P}}_{\mathrm{P}}\mathbb{V}_{\mathrm{E}} + i\,{}^{\mathrm{P}}_{\mathrm{P}}\mathbb{V}_{\mathrm{AE}} \quad (2) $$

where elastic scattering potential is given by

$$ {}^{\mathrm{P}}_{\mathrm{P}}\mathbb{V}_{\mathrm{E}} = \left[-1 - \cos\sigma + 2\left(\frac{\beta_{\mathrm{E0}}}{\alpha_{\mathrm{E0}}}\right)^2 \sin^2\sigma\right] A_\rho + 4\left[\left(\frac{\beta_{\mathrm{E0}}}{\alpha_{\mathrm{E0}}}\right)^2 \sin^2\sigma\right] A_\beta - 2A_\alpha \quad (3) $$

and anelastic part of the scattering (Moradi and Inannen, 2014)

$$ {}^{\mathrm{P}}_{\mathrm{P}}\mathbb{V}_{\mathrm{AE}} = {}^{\mathrm{P}}_{\mathrm{P}}\mathbb{V}^{\rho}_{\mathrm{AE}} A_\rho + {}^{\mathrm{P}}_{\mathrm{P}}\mathbb{V}^{\beta}_{\mathrm{AE}} A_\beta + {}^{\mathrm{P}}_{\mathrm{P}}\mathbb{V}^{Q_{\mathrm{S}}}_{\mathrm{AE}} A_{Q_{\mathrm{S}}} + {}^{\mathrm{P}}_{\mathrm{P}}\mathbb{V}^{Q_{\mathrm{P}}}_{\mathrm{AE}} A_{Q_{\mathrm{P}}} \quad (4) $$

where

$$ {}^{\mathrm{P}}_{\mathrm{P}}\mathbb{V}^{\rho}_{\mathrm{AE}} = 2\left(\frac{\beta_{\mathrm{E0}}}{\alpha_{\mathrm{E0}}}\right)^2 \left\{\sin^2\sigma(Q^{-1}_{\mathrm{S0}} - Q^{-1}_{\mathrm{P0}}) + Q^{-1}_{\mathrm{P0}}\left[\sin 2\sigma + \frac{1}{2}\left(\frac{\alpha_{\mathrm{E0}}}{\beta_{\mathrm{E0}}}\right)^2 \sin\sigma\right]\tan\delta_{\mathrm{P}}\right\} $$

$$ {}^{\mathrm{P}}_{\mathrm{P}}\mathbb{V}^{\beta}_{\mathrm{AE}} = 4\left(\frac{\beta_{\mathrm{E0}}}{\alpha_{\mathrm{E0}}}\right)^2 \left\{\sin^2\sigma(Q^{-1}_{\mathrm{S0}} - Q^{-1}_{\mathrm{P0}}) + Q^{-1}_{\mathrm{P0}}\sin 2\sigma\tan\delta_{\mathrm{P}}\right\} $$

$$ {}^{\mathrm{P}}_{\mathrm{P}}\mathbb{V}^{Q_{\mathrm{S}}}_{\mathrm{AE}} = -2Q^{-1}_{\mathrm{S0}}\left(\frac{\beta_{\mathrm{E0}}}{\alpha_{\mathrm{E0}}}\right)^2 \sin^2\sigma $$

$$ {}^{\mathrm{P}}_{\mathrm{P}}\mathbb{V}^{Q_{\mathrm{P}}}_{\mathrm{AE}} = Q^{-1}_{\mathrm{P0}} \quad (5) $$

From above equations, it is evident that the viscoelastic scattering potential is complex. The real part is the elastic scattering potential and the imaginary part is the term induced by the anelasticity of the medium. In above relations A stands for fractional perturbation, for instance $A_\rho = 2\frac{\rho_2-\rho_1}{\rho_2+\rho_1}$.

In Figure .1 we plot the elastic and anelastic parts of the density and S-wave velocity components of the potential for scattering of an inhomogeneous P-wave to an inhomogeneous P-wave. We observe that the anelastic density component is comparatively small and the major contribution comes from the elastic part. In the limit of normal incidence, the absolute value of the density part of the elastic scattering potential goes to its maximum value, and the anelastic part approaches to zero. For S-wave velocity component of the scattering potential, similar to the density component, the major contribution to the reflectivity is from the elastic part. In this case both elastic and anelastic components approach zero for normal incidence as expected.

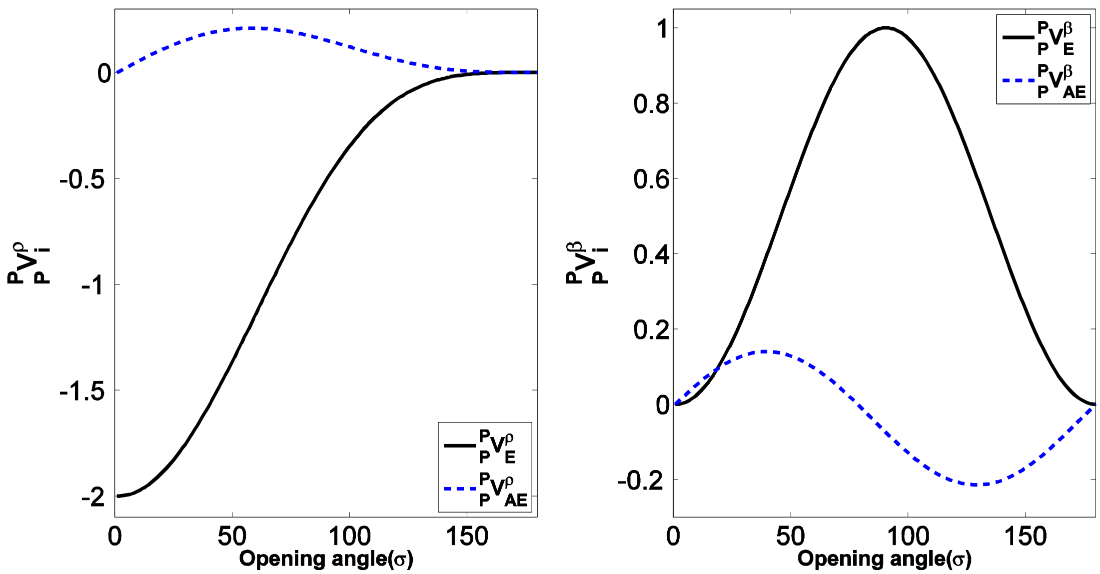

Fig.1. Elastic and anelastic density (left) and S-velocity(right) components of the viscoelastic potential for scattering of incident inhomogeneous P-wave to inhomogeneous reflected P-wave versus of opening angle, for background attentaution angle 60° . Quality factor of P-wave for reference medium is to be 10 and for S-wave is 7. Also the S-to P-velocity ratio for reference medium is chosen to be 1/2. Solid line is for elastic part and dash line for anelastic part.

Conclusions

The seismic response of the real earth deviates from the elastic-isotropic model often used to frame the seismic wave propagation problem. Here we investigate viscoelasticity in its capacity to reproduce the effect of dissipation on the propagation of a wave. Full formal theory for viscoelastic seismic waves exists, but the most powerful versions of it have largely been restricted to layered media. Exact, closed-form solutions for viscoelastic waves in arbitrary multidimensional media are not in general available, but, to first order, scattering formulations can provide interpretable approximate forms. These forms are important for obtaining physical insight into interactions of seismic waves with dissipative media, but also for posing and solving inverse scattering and full waveform inversion problems.

The scattering potential in displacement space is obtained by sandwiching the scattering operator between the incident and reflected polarization vectors. Since for the viscoelastic waves, polarizations are complex, the viscoelastic scattering potential we obtained is a complex function whose real part is elastic scattering potential and whose imaginary part is the related to the anelasticity of the medium. In contrast to the elastic scattering potential that only alters the amplitude of the outgoing field, the viscoelastic scattering potential alters both amplitude and phase of the outgoing field. Anelasticity appears to have more significant effect on converted waves than on conserved modes.

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