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Introduction to Moment Tensor Inversion of Microseismic Events

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Summary

Understanding the source mechanisms of microseismic events is important for understanding the fracturing behavior and evolving stress field within a reservoir, knowledge of which can help to improve production and minimise seismic risk. The most common method for calculating the source mechanisms is moment tensor inversion, which can provide the magnitudes, modes and orientations of fractures. An overview of the methodology is presented including problems and limitations.

Introduction

Microseismic monitoring is extremely valuable for tracking the performance of hydraulic fracturing treatments within reservoirs. An improved understanding of the physical processes governing induced seismicity is important, both for maximising production and reducing seismic hazard. In particular, seismic source mechanisms can provide insights into the fracturing behaviour of the reservoir and surrounding rocks, and an understanding of the evolution of the stress field. These insights can contribute to an advanced knowledge of fracture type, propagation and connectivity.

The principle method for calculating seismic source mechanisms is moment tensor inversion. Moment tensor inversion utilises the seismic radiation pattern in order to calculate the seismic moment tensor, a matrix of 9 force couples that are used to describe the source mechanism:

$$\mathbf{m} = \begin{bmatrix} M_{xx} & M_{xy} & M_{xz} \\ M_{yx} & M_{yy} & M_{yz} \\ M_{zx} & M_{zy} & M_{zz} \end{bmatrix}. \qquad (1)$$

Due to the conservation of angular and linear momentum, the seismic moment tensor is considered symmetric, meaning that only six of the nine components are independent. The off-diagonal elements form balanced double-couples, avoiding net torque or rotation in the tensor. The moment tensor characterises the event magnitude, fracture type (e.g. double couple, tensile), and fracture orientation. Several techniques for performing moment tensor inversion can be used. This paper introduces several of the inversion methods and their advantages and limitations.

Theory and Methods

Before discussing the methodology, it should be noted that an important factor when applying moment tensor inversion, and especially when the mechanism is not assumed to be double-couple, is the distribution of the seismic sensors. The results of the inversion will be more reliable if the sensor locations allow for a good sampling of the focal sphere, and therefore ideally as many sensors as possible should be deployed, surrounding the region in which the events are occurring. Of course in practice this is not usually possible, but it is desirable to deploy the sensors in a configuration as similar to this as possible. If the focal sphere is not adequately sampled there are very few constraints on the mechanism and therefore the results can be meaningless. Assumptions for the source mechanism are necessary in these cases (e.g. assume a double-couple mechanism), but these assumptions may be invalid and due to the poor sampling the orientation may be poorly defined.

The majority of earthquakes in global seismology are caused by shear faulting and are therefore described by a double-couple (DC) source mechanism. Many source inversion methods therefore assume a double-couple source. However in recent years it has become apparent that many microseismic events may also include a volumetric component in their source mechanisms, especially if recorded in volcanic environments or during hydraulic fracturing treatments, where fluids influence the fracturing behaviour. These volumetric components can be large and it is therefore important to take them into account, as a failure to do so can greatly affect interpretations of fracturing behaviour and related stress fields.

A purely volumetric source is known as an isotropic (ISO) source. Seismic sources can also be described using a compensated linear vector dipole (CLVD) (Knopoff and Randall, 1970), where no volumetric change or shearing takes place. This describes when one dipole is compensated by the two other dipoles, which are half the magnitude. However it must be noted that moment tensor solutions can be non-unique. For example, a mechanism described as CLVD by the moment tensor can also be described by other possible mechanisms. A CLVD mechanism can be created by two different double couple geometries with different moments of M_0 and $2M_0$, or by a ring-fault mechanism (Ekström, 1994).

The simplest method with which to calculate moment tensors is using the first arrival polarity method. In this method, the mechanism is often assumed to be double-couple. The orientation of the mechanism can therefore be determined by the polarity of the P-wave arrivals (i.e. first arrivals) at each sensor. Again, sensor distribution is vital, for example if all of the sensors are located to the southeast of the source location it is very plausible for the P-wave polarity to be identical for all sensors and therefore impossible to determine the mechanism orientation with any accuracy (figure 1).

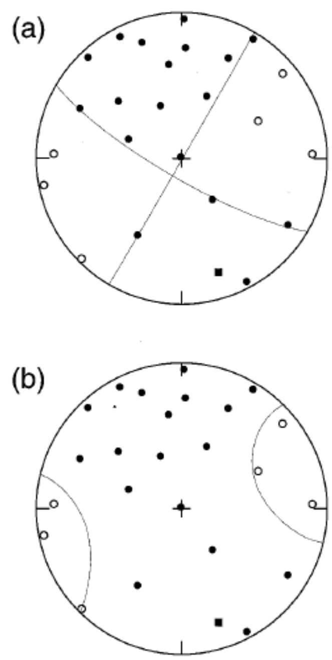

Figure 1. Limitation of the first arrival method. Examples of two different focal mechanisms that fit a high quality dataset. Open and solid circles indicate inward and outward motions, respectively. (a) shows a pure double-couple mechanism, and (b) a mechanism with a large isotropic component. Modified from Julian et al. (1998).

An extension of this method is to include the P-wave amplitudes in the inversion to better constrain the orientation of the P-wave radiation pattern, and similarly the P/S amplitude ratio can be used (e.g. Hardebeck and Shearer, 2003). These methods should provide more accurate solutions but it is important to take into account amplitude variations between sensors due to propagation effects when using only P-waves (geometrical spreading, attentuation, station site effects etc).

A more accurate but more computationally expensive method for calculating source mechanisms is 3D full waveform moment tensor inversion (e.g. Chouet et al., 2003; De Barros et al., 2011; Eyre et al., 2013, Zecevic et al., 2013). As the name suggests, in this method the full waveform data recorded on all components at each of the stations are inverted to calculate the seismic moment tensor. The data consist of both the source contribution and a path (propagation) contribution. The propagation effects can be removed by modelling the propagation of seismic waves between source and receiver locations as accurately as possible, producing Green's functions. Green's functions are the displacement responses recorded at the receivers when an impulse force (or moment) function is applied at the source position in a viscoelastic Earth (i.e. the medium response). The *n*th component of the displacement *u*, recorded at position **x** and time *t*, can be written as:

$$u_n(\mathbf{x},t) = M_{pq}(t) * G_{np,q}(\mathbf{x},t), \qquad n, p, q = x, y, z \qquad (2)$$

where M_{pq} is the force couple in the direction *pq* and $G_{np,q}$ are the spatial derivatives of the *n*th components of the Green's functions generated by the moment M_{pq}. The asterisk indicates convolution and the summation convention applies. The Green's functions can be calculated using several methods such as ray-tracing, and for this the seismic velocity structure should be modelled as accurately as possible.

In the frequency domain, equation (2.26) is linear and can be written as:

$$u_n(\mathbf{x},\omega) = M_{pq}(\omega) \cdot G_{np,q}(\mathbf{x},\omega), \qquad n, p, q = x, y, z. \qquad (3)$$

Therefore the inversion is often performed in the frequency domain, and is solved separately for each frequency. This can be represented in matrix form as:

$$\mathbf{u} = \mathbf{Gm} \qquad (4)$$

where **u** is the data matrix, **G** contains the Green's functions, and **m** contains the moment tensor and single forces components. This can be solved through the least squares method (Menke, 1984):

$$\mathbf{m}^{est} = \left(\mathbf{G}^T \mathbf{W} \mathbf{G}\right)^{-1} \mathbf{G}^T \mathbf{W} \mathbf{u} \qquad (5)$$

where $\mathbf{m}^{est}$ is the estimated moment tensor matrix solution, **W** is a diagonal matrix of weights for the quality of the data and *T* denotes the transpose matrix. The weighting matrix can play an important role in the inversion procedure if noise varies significantly between stations. The quality of the inversion results can be tested through the evaluation of the misfit between the calculated and observed data.

For a better understanding of the source mechanisms, the moment tensors can be decomposed into their principal components using the methods of Vasco (1989). This is based on the singular value decomposition of the six time-dependent moment tensor components. This leads to an estimation of a common source-time function and its contribution to each component of the moment tensor, thus giving a source-time history of the source process and its mechanism. The eigenvalues of the scalar moment tensor give the source mechanism and the eigenvectors give the orientation of the principal axes.

The disadvantages of the full waveform method are that a good knowledge of the velocity structure is necessary, computing the Green's functions can be computationally expensive and the method performs better for low frequency data as it can become unstable at higher frequencies.

For each method, the solution can be decomposed into the percentage of isotropic, CLVD and double-couple source mechanisms. One method for implementing this is that of Vavryčuk (2001), where the percentages of the individual components ($c_{component}$) constituting the moment tensor can be calculated using:

$$c_{ISO} = \frac{1}{3}\frac{tr(\mathbf{M})}{\left|M_{|\max|}\right|} \cdot 100, \qquad (10)$$

$$c_{CLVD} = 2\varepsilon\left(100 - \left|c_{ISO}\right|\right), \qquad (11)$$

$$c_{DC} = 100 - |c_{ISO}| - |c_{CLVD}| \tag{12}$$

where $M_{|\max|}$ is the maximum absolute value of the eigenvalues of the moment tensor. The double-couple percentage is always positive, while the isotropic and CLVD percentages are positive for tensile sources and negative for compressive sources. The angle between the slip vector and the fault plane *α* can also be calculated using:

$$\alpha = \sin^{-1}\left(3\frac{M^*_{\max} + M^*_{\min}}{|M^*_{\max}| + |M^*_{\min}|}\right) \tag{13}$$

which is 0° for pure double-couple and 90° for pure tensile events.

Conclusions

Moment tensor inversion can be used to constrain the source mechanisms of microseismic events. Methods using P- and S- arrivals can be used to give an approximate solution, but the more computationally expensive full-waveform inversion is likely to give more accurate solutions, especially for fluid-induced seismicity where seismic sources may contain volumetric components. However if the velocity model is poorly constrained the P/S method is preferred. For the accurate determination of seismic source mechanisms, monitoring networks should be designed to maximise the sampling of the seismic radiation pattern.

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