GeoConvention 2015
GEOSCIENCE
new horizons
Telus Convention Centre • 4-8 May 2015 • Calgary, AB Canada

Noise attenuation via robust low rank matrix factorization to singular Spectrum Analysis

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Summary

The Singular Spectrum Analysis (SSA) method is an efficient tool for seismic data white noise attenuation. When data contains outlier noise this subspace based technique faces problems due to its L2 norm optimization that is very sensitive to outliers. In this paper, we solve the SSA problem with a robust low rank matrix factorization algorithm that also uses the L2 norm but with constraints on both the rank of the matrix and the number of outliers. The basic idea of this method is similar to the recently developed nuclear norm minimization that constrains outliers, but our formulation and implementation are simpler. In addition, any *a priori* information regarding outliers can be easily incorporated to make the performance of this method more effective. We applied this method to synthetic data and real data to show the effect of this algorithm.

Introduction

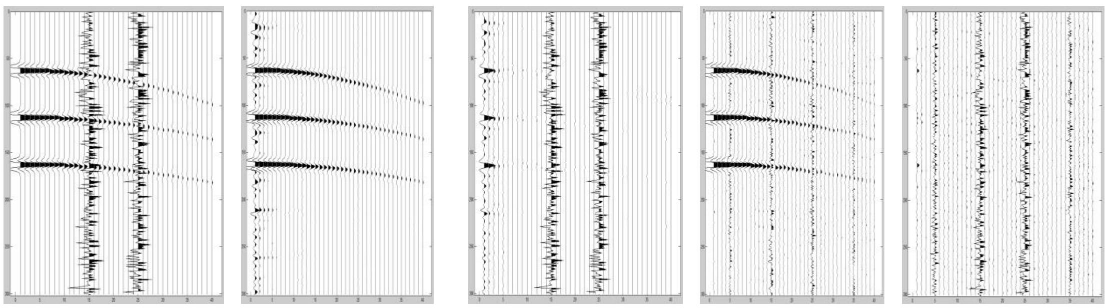
Seismic data is not only contaminated with white noise and linear shot noise, but also noise bursts that can be caused by faulty channels shown as outliers because of the very high amplitude. These outliers can severely degrade the performance of methods based on L2 norm optimization. In recently developed subspace techniques for noise attenuation and trace interpolation, e.g. eigen-image analysis (Trickett, 2003), Cadzow filtering based singular spectrum analysis (SSA) (Sacchi, 2009; Trickett, 2008, Kreimer and Sacchi, 2012; Gao et al., 2013), all of them rely on an L2 norm optimization technique such as SVD (singular value decomposition). Therefore, application of those algorithms to attenuate noise may be limited only to white noise. Consequently there has been a growing interest in L1 norm based subspace optimization approaches to deal with the problem of outliers, e.g. (P. Markopoulos, 2014). However, while there exists a fast L1 norm computation technique for a real valued data matrix, e.g. Kundu (2014), it is hard to apply this to complex-valued problems such as SSA. The L1 norm is generally a NP hard problem and as a relaxation, the nuclear norm can be used to constrain the behavior of outliers, e.g. Xu and Caramanis(2012).

As an improved SSA, the robust singular spectrum analysis (RSSA) developed by Chen and Sacchi (2013) remedies the outlier problem in SSA. In this algorithm, the misfit of residual is used as a weighting function that gives potential outliers a smaller weight when applied in a least squares solver to update low rank matrices. However, as pointed out by Kriegel et al (2008) we do not know in practice which points are outliers and thus need to be assigned a lower weight. An improper weight may also lead to signal leakage.

Based on observations that outlier traces appearing in seismic data are very sparse, we solve the SSA problem with a robust low rank matrix factorization algorithm that also uses the L2 norm for minimizing the error function but with constraints on both the rank of the matrix and the number of outliers. The procedure of seeking low rank approximation matrices is similar to that in RSSA (Chen and Sacchi, 2014) but instead of a weighting function to suppress outliers, we simply constrain the number of outliers. Even though the problem formulation is similar to the nuclear norm constraint of outliers (Cheng Sacchi, 2014), our constraint of outliers can be considered as a simplified L0 norm constraint (Lu and Zhang, 2010). Therefore, our algorithm is simpler to understand and implement. Moreover, any *a priori* information regarding knowledge about the outliers can be easily incorporated to make the performance of this method more effective.

Robust low rank matrix factorization

In order to clearly describe our method, we first investigate an example. Figure 1a shows synthetic data where the signal amplitude decreases dramatically from near offset to far offset. Two noise traces with high amplitude are added to the data. The results produced from RSSA and SSA are shown in figure 1b and 1c.

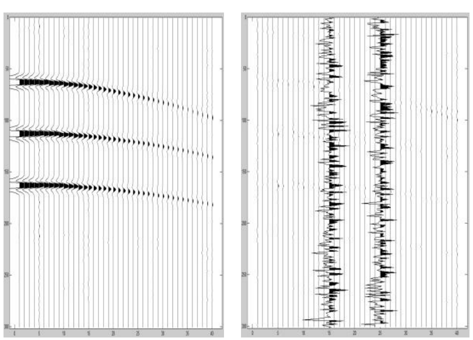


(a) (b) (c) (d) (e)

Figure 1. (a) input data; (b) result via RSSA and its misfit (c); (d) result via SSA and its misfit (e).

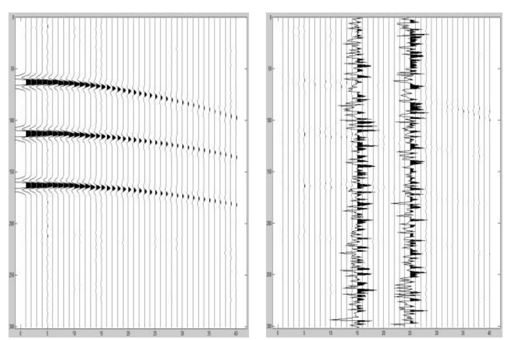
The RSSA attenuates the outlier traces but the traces at near offsets are also affected; conversely, SSA cannot remove the outliers but the amplitudes at near offset are preserved.

Figure 2 shows the SSA result with the data where the outliers are zeroed out. The result shows that the two traces that occupy the same locations as the outliers are also well reconstructed. What if two more traces are zeroed out? Figure 3 shows this result is almost the same as that where only two outliers are zeroed. This experiment supports what we mentioned above: L2 norm minimization of the data matrix where all the outliers have been removed is the optimal solution. This experiment tells us that if only a small number of outlier points, not necessarily equal to the exact number of outliers, are ignored from the data matrix, we can still optimally reconstruct the low rank structure embedded in the data matrix. The proof of this can be read from, e.g. Lu and Zhang (2010).



(a) (b)

Figure 2. (a) result via SSA with outliers removed and (b) misfit



(a) (b)

Figure 3. (a) result via SSA with two more traces removed and (b) misfit

Our robust matrix factorization can now be summarized as:

1. Input an m by n complex matrix H (Hankel matrix) and apply SVD to obtain initial m by k matrix U and n by k matrix V.
2. Calculate residual R=H-UV'.
3. Calculate matrix S that contains the difference between R and its median value.
4. Update matrix H: H – S (S is the matrix of outlier points), subject to the condition that non-zero elements in S are less than some predefined σ and are larger than a predefined threshold β.
5. Alternatively update U and V via *min* || H- UV'||$_2$
6. If there is no convergence, go to 2.

The formulation of this robust matrix factorization can be written as L0 norm constraint (Lu and Zhang, 2010)

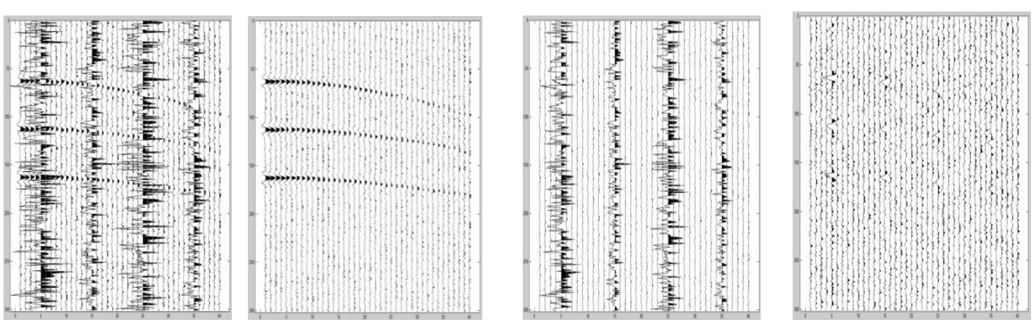
$$min\ (rank\ (UV') + \gamma||S||_0);\quad s.t. ||H - S - UV'||_2 < \epsilon$$

Comparing our formula with nuclear norm optimization, the only difference is $||S||_*$ is replaced by$||S||_0$. However, this small difference can make implementation easier and further, *a prior* a priori information about outliers can be easily incorporated.

It should be stressed that initializing U and V with SVD may be necessary because improper initialization may lead to a minimization to some local minimum (Markopulos et al., 2013); the advantage of low rank matrix factorization over rank reduction via SVD is not only computational efficiency but also, based on our simulations experience, the convergence is faster for the matrix completion problem.

Examples

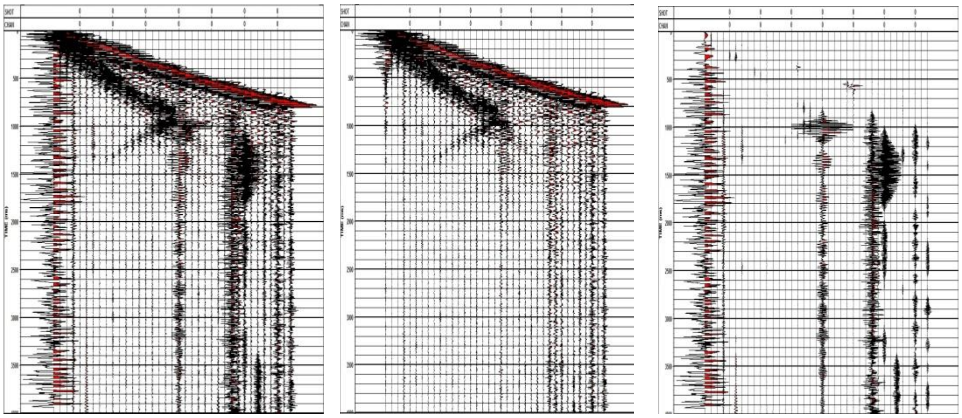
Our first example is to test on synthetic data with white noise and outliers as shown in figure 4. The input data contains three events that are contaminated with both white noise and outliers (a). Because of the curvature of events we fixed the rank to 6 to avoid signal leakage. The noise filtered result (b) shows that outliers are well removed and white noise is also reduced (c). In order to show the misfit clearly, (d) presents the difference between the data with the outliers zeroed and the result, which shows that the signal is very well preserved.



(a) (b) (c) (d)

Figure 4. (a) input data; (b) result from our algorithm; (c) misfit (d)misfit excluding outliers

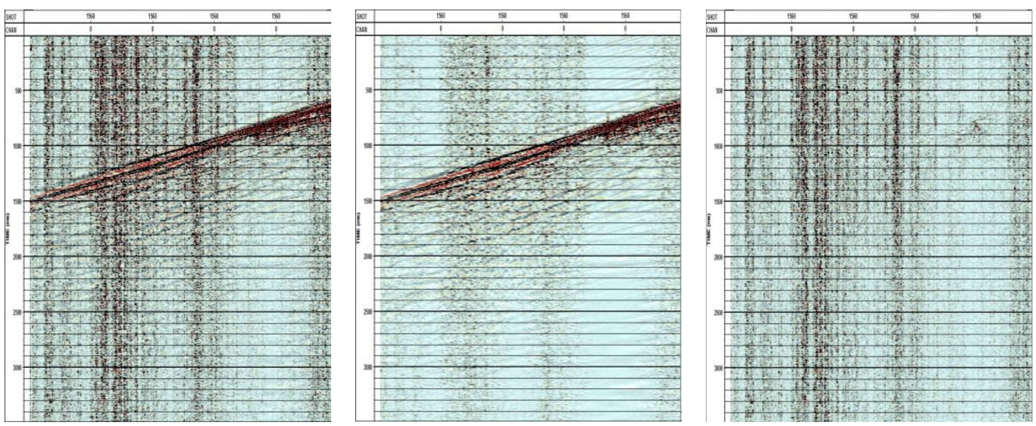
The second example is for real VSP data Figure 5. The input data contains strong amplitude noise traces (a). Our algorithm almost perfectly removes the strong noise (b) and the signal is well preserved (c).



(a) (b) (c)

Figure 5. VSP data: (a) input data; (b) filtered result and (c) difference.

The third example is for land data, as shown in Figure 6. The input data contains strong ambient noise. The filtered result and its difference with the input are shown in (b) and (c) respectively.



(a) (b) (c)

Figure 6. land data (a) input data; (b) filtered result and (c) difference.

Conclusions

We presented a robust low rank matrix factorization algorithm for attenuation of white noise plus outlier noise. This algorithm can be called L0 norm constraint optimization of low rank matrix approximation because the number of sparse elements that are excluded from the data matrix is a constraint for optimizing the sought-for low rank matrix approximation. Even if the formulation of this algorithm is similar to nuclear norm optimization, it is much simpler regarding implementation and physical understanding. It is intersting to see that the optimization can be carried out by excluding some outlier points from the data matrix, which is actually the basic technique used in matrix completion. Because of the power of low rank matrix completion for data reconstruction, the excluded points from the data matrix can be well reconstructed provided the matrix has a low rank structure. Examples for synthetic data and real data shows our algorithm performs successfully to achieve those goals.

Acknowledgements

We thank K. Chen and M. Sacchi for providing their RSSA Matlab code.

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