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Noise attenuation via sequential rank-one matrix approximation

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Summary

Rank reduction filters are widely used in seismic data processing . They rely on L2 norm based sigular value decomposition with the assumption that the signal resides in a low dimensional linear subspace and therefore, requires the signal to belong to a linear structure. In this paper, we propose a sequential rank-one approximation algorithm to handle the case where the data matrix does not have purely linear structures embedded. Coherent signal that contains maximum energy is horizontally lined up and then, by applying rank one matrix approximation, this lined up signal can be extracted. We sequentially repeat this procedure for other (less energetic) coherent signals and extract them to obtain the final noise free result. We applied this method to synthetic data and real data to show the effectiveness of this algorithm.

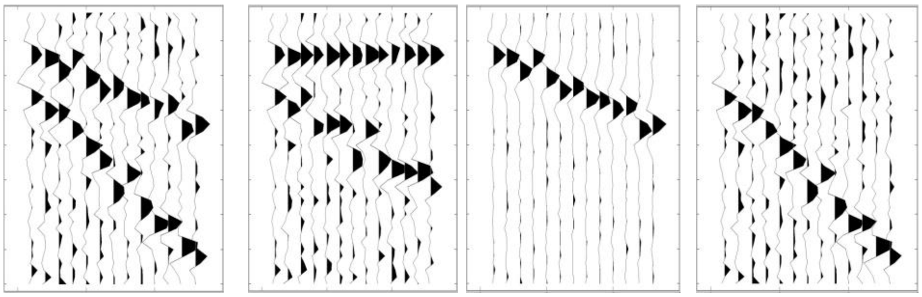
Introduction

Reduced-rank noise reduction, in which a signal matrix is approximated by another one with lower rank, may be viewed as an energy decomposition filtering of those parts of the corresponding spectrum with low energy (e.g. Scharf and Tufts, 1987). This technique is the underlying principle in the noise reduction algorithms proposed by, e.g. Trickett, 2008, where the key idea is to form a Hankel matrix from a frequency slice of the input signal, compute the singular value decomposition SVD of the matrix, discard small singular values to obtain a matrix with reduced rank and finally construct the output signal from this generally unstructured matrix by arithmetic averaging along its anti-diagonals. Success in applying this algorithm depends on whether or not the signal embedded in the data matrix can be mapped into a small subspace where only a few principal components can represent most of the energy possessed by the signal. Otherwise, to avoid signal leakage, a large number of components have to be used and as a result noise cannot be successfully removed. The requirement of linear events to fulfill this low rank reduction may not always be satisfied in the real world, especially for prestacked data that is recorded from a complicated geological area. Applying this method to a series of sliding windowed data may lessen the linearity problem. However, the smaller the window size is, the less noise can be removed because the ratio of rejected singular values (noise) to the retained ones (signal) could become smaller. Moreover, the signal events may experience static shifts and even if the shifts are very small, the ranks that are required to model the signal may need to increase a lot. Recently, investigators have pursued the idea of high dimension sorted data. The advantage of this high dimension sorted data is that it can provide multiple views of the data from different directions. Thus, where the events are linearly coherent in some directions then rank reduction along those directions may work efficiently.

As a summary, the key point in applying the rank reduction technique to a data matrix is that the matrix itself (or tensors for higher dimensions) is of low rank. Signal events in prestack seismic data are often curved and static-shifted and therefore, it may not be favourable to directly apply a rank reduction filter. Butler (2012) developed a time domain white noise suppression algorithm (T-WNS). To carry out this noise suppression, a dipping scan is first used to find a maximum energy direction and stack traces along this direction to obtain a model trace and then, this model trace is further refined by considering the static shifts along the maximum energy direction and finally, the model trace is distributed to different trace locations with proper time shifts that are calculated by cross correlating the model trace with data traces. This time domain algorithm works very well for most cases. In this paper, we refine this algorithm from threel aspects: firstly, a frequency domain radon transform operator is used to replace the dip scan. This can deal with fractional time shifts to make maximum energy searching more efficient; and secondly, instead of cross correlation for static shift correction, we use a multi-correlation coherence analysis that can be more robust for noisy data; and thirdly we use rank one matrix approximation to extract the event instead of distributing a stacked trace to each trace location to avoid improper trace scaling.

Method description

Our algorithm can be easily described with figure 1. In this figure, (a) the input contains two events and, based on maximum energy, one of the events is first horizontally lined up as shown in (b); with rank one extraction and time shift the event which had been aligned is extracted from the data as shown in (c). This is then subtracted from the data and finally the residual in (d) is ready for the next extraction.



(a) (b) (c) (d)

Figure 1. Method description: (a) the input contains two events and based on maximum energy, one of the events is first horizontally lined up (b); with rank one extraction and time shift the event that had been lined up is taken out from the data (c) and finally the residual in (d) is ready for the next extraction.

Based on the description above, our method can be defined by the following steps:

Step 1. Horizontally line up the event that contains the maximum energy.

We perform this stage with frequency domain radon transform. For a fixed frequency ω, the Radon Transform can be written as (Gu and Sacchi, 2009)

$$M(\omega, P) = \boldsymbol{R}\left(e^{i\omega\phi}\right)D(\omega, X)$$

where D(ω,**x**) and M(ω, **p)** represents data and radon transform frequency slice respectively; ***p*** is ray parameter vector, ***x*** is vector for offset; matrix ***R*** is the radon transform operator with

$$R_{i,j} = e^{i\omega\emptyset_{i,j}} = e^{i\omega p_i x_j}$$

With the radon transform, the ray parameter p_i corresponding to the maximum value in M is the direction for the maximum energy slant stack. Then alignment of the event in this direction can be obtained by multiplying elements in ith row in ***R*** with corresponding elements in D. Since events may not be completely linearly coherent from trace to trace due to curvature and static shift, the traces may need further up and down moves for a perfect line up. The time shift between any pair can be derived with cross correlation. However, because of noise, the time shift derived from each trace- pair may not be stable. Therefore, we apply multi-correlation coherence analysis that uses a least squares solver for a sytem of n(n-1)/2 overdetermined equations formed from

$$t_i - t_j = \Delta t_{i,j}\,; i = 1,2,\cdots n-1; j = i+1, i+2, \cdots, n$$

where t_i is for current event time at ith trace. The least squares solution to this equation system is (VanDecar and Crosson, 1990)

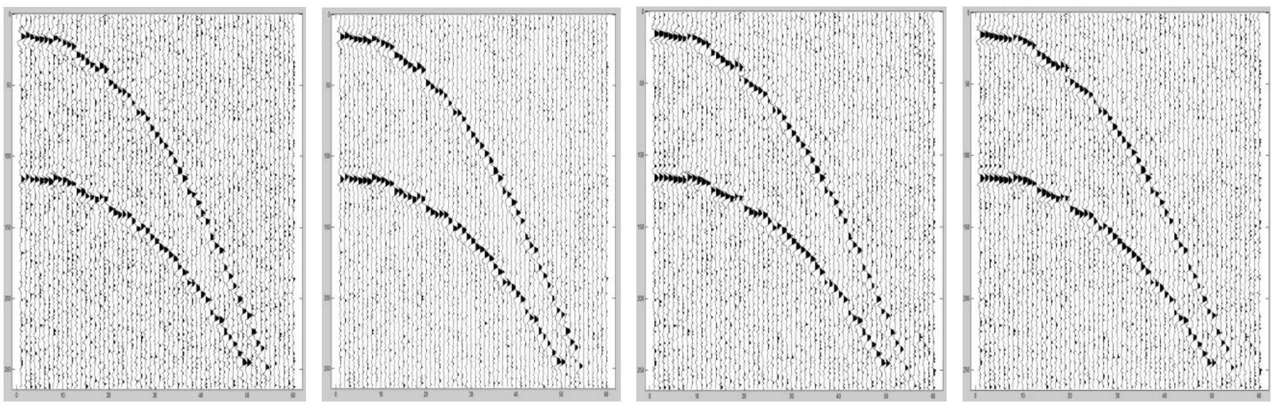
$$t_i^{est} = \frac{1}{n}\left(\sum_{j=i+1}^{n} \Delta t_{i,j} - \sum_{j=1}^{i-1} \Delta t_{j,i}\right)$$

Step 2. Fourier transform to T-X domain. Apply rank one approximation to the horizontally lined up event matrix to extract the event.

Step 3. Time shift the extracted events back to their original position and add it to the output; the residual of the data then goes to the next extraction. The final output is therefore the sum of all of the extracted events. In turn this “noise” should be subtracted from the original data to achieve the final result.

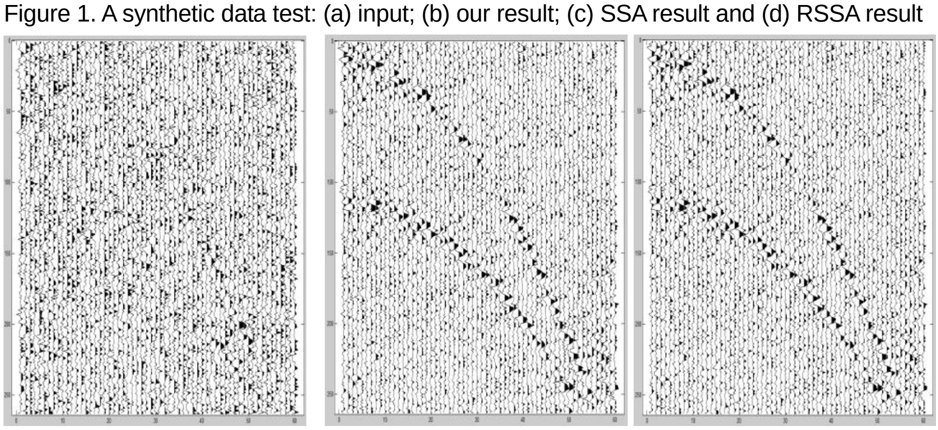
Examples

The first example is for synthetic data that contains two curved events with static shifts (figure 1a). As a comparison, we also run Cadzow filtering based singular spectrum analysis (SSA) and robust singular spectrum analysis (RSSA) (see Ke and Sacchi, 2012). An overlapping window (size 60 by 30) has been used for all three algorithms. The rank is fixed to three. The results from ours, SSA and RSSA are shown in figure 1 (b), (c) and (d). From the figure, the three algorithms attenuate the white noise well. However, the differences plot in Figure 2 shows that the result from our algorithm is best for preserving signal.



(a) (b) (c) (d)

Figure 1. A synthetic data test: (a) input; (b) our result; (c) SSA result and (d) RSSA result



(a) (b) (c)

Figure 2. Differences between input and (a) our result, (b)SSA’s result and (c) RSSA’s result.

The next example is for real land data that is used to test our algorithm and the T-WNS method (Butler, 2012), Figure 3. The input data contains both white noise and ambient noise (a); the filtered results from both algorithms cleaned out the noise ((b) from rank one method and (c) from T-WNS). As for ambient noise the result from rank one is better, which is due to the T-WNS distribution of the stacked model trace with improper scaling for matching the amplitude. Also, signal with small curvature can be preserved slightly better with rank one algorithm (see circled area). The difference plot is shown in (d) and (e).

Conclusions

We presented a sequential rank-one matrix approximation for white noise attenuation. From a physical point of view, this algorithm is a kind of principal component extraction. However, before going to principal component extraction, the data are processed to a condition that best favors low rank reduction approximation. Therefore, curved events and static shifts can be well handled.

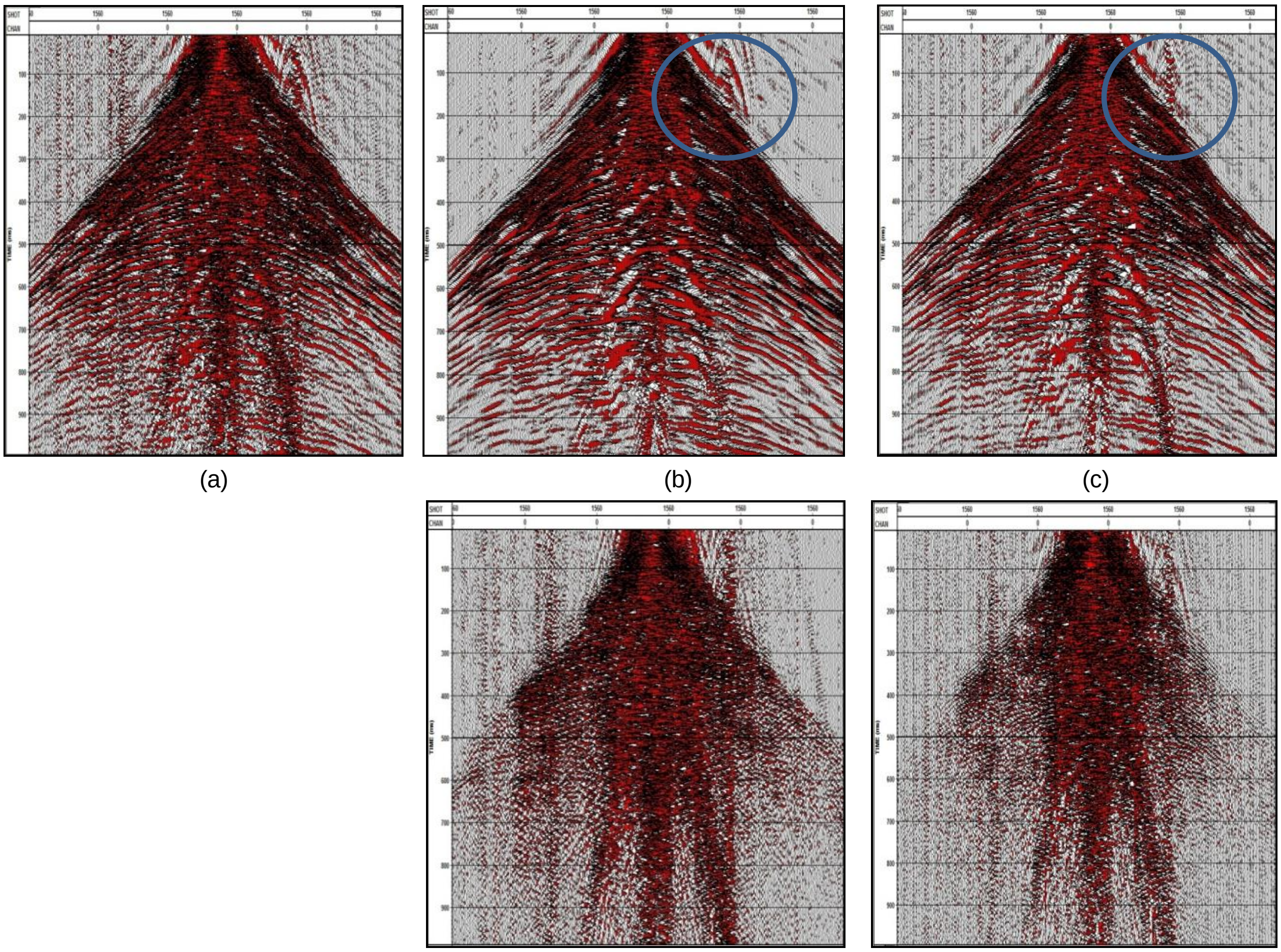


Figure 3. Real data test: (a) input data; results from (b) rank one and (c) T_WNS anddiffterence plot (d) for rank one and (e) for T_WNS.

Acknowledgements

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References

Butler, P., 2012, White noise suppression in the time domain, Abstract CSEG, 2012

Gu, Y. and M. Sacchi, 2009, Radon transform methods and their application in mapping mantle reflectivity structure, Surv. Geophys., vol.30, 327-354.

Scharf , L. L. and D. W. Tufts , Rank Reduction for Modeling Stationary Signals, IEEE Trans Acoust.Speec, Signal Processing, 35(3), 350-355, 1987

Trickett, S., 2008, F-xy Cadzow noise suppression: SEG Technical Program Expanded Abstracts, 27, 2586–2590

VanDecar, J.C. and R.S. Crosson, 1990, Determination of teleseismic relative phase arrival times using multi-channel cross-correlation and least squares, Bull. Seis. Soc. America, Vol. 80, No. 1, 150-169.