

# Verification of Predictions of Magnitude of Completeness Using Observations from an Earthquake Catalogue

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## Summary

We address the problem of designing a seismic network for monitoring induced seismic events that must meet specific performance criteria. We propose a method to assess a fundamental measure of performance without an earthquake catalogue: magnitude of completeness. The method is based on the site noise, instrument noise, and station distribution and can be used to model existing or hypothetical networks. We use the method to predict the performance of a seismic network installed in the vicinity of the New Madrid seismic zone. Using the catalogue of over 2000 events, we estimate the spatially varying magnitude of completeness for the earthquake catalogue using maximum curvature method. The observed magnitude of completeness is compared to the predicted value. A bootstrap sampling method is used to obtain a measure of uncertainty in the magnitude of completeness estimate. We find that predicted magnitude of completeness agrees reasonably well with the observed magnitude of completeness, though the observed magnitude of completeness tends to be slightly lower than the predicted result.

## Introduction

In the past decades, induced seismicity has grown from little more than a myth to a well-established scientific field. Despite the many advances made in the field, when it comes to designing networks for induced seismicity monitoring, station locations are chosen rather arbitrarily. Network performance is evaluated *a posteriori*, and if necessary a few more stations are added. This approach is impractical in a number of applications, but particularly for induced seismicity monitoring in which specific performance standards must be met even if no event is ever detected.

We propose a method to assess network performance of a hypothetical network. Station locations can be chosen according to the best compromise between minimizing site noise, improving azimuthal coverage, and increasing station density. For a hypothetical network we are able to assess a fundamental measure of network performance: magnitude of completeness. Our method allows for objective comparison of different network designs prior to station deployment and assessment of network performance to ensure that monitoring criteria are met.

We consider a case study with a catalogue of over 2000 events. Detailed records of seismic activity in the New Madrid Seismic Zone date back to 1974. For our purposes, we consider observed seismicity since 2002. This time period was chosen since there are relatively few station additions and equipment changes that are likely to strongly influence magnitude of completeness. We test the predictive seismic network performance modeling technique against the real earthquake catalogue. Spatially varying predicted magnitude of completeness is compared to values observed from an earthquake catalogue. In this study we will describe magnitude of completeness using just a single parameter estimating the location of the knee of the frequency magnitude distribution. (Some previous studies, e.g. Ogata and Katsura, 1993, have also included a parameter describing the confidence level.

This allows for definitions of magnitude of completeness based on probability of detection, but we did not think it was vital for verification of the predictive method.)

## Methodology

Our method for predicting network performance relies on three essential ingredients: the first is site noise. We map the site noise using data from existing stations and/or from a temporary deployment of a site noise survey network. A power spectral density probability density function (McNamara and Buland, 2004) is computed for the available stations through an SQLX analysis. We then interpolate between stations and extrapolate outside the polygon that bounds them. A second ingredient is the station distribution. Locations of existing and hypothetical stations to be included are specified by latitude and longitude. Instrument self-noise is constructed from models of published seismometer and digitizer self-noise specifications. Instrument noise is then summed with site noise to obtain the station noise for each sensor in the network. The third key ingredient is a one-dimensional velocity model, including estimates of the errors of layer velocity and layer boundary depth, as well as a local attenuation factor,  $Q$ . The velocity model plays an important role in determining the expected observed spectra of events.

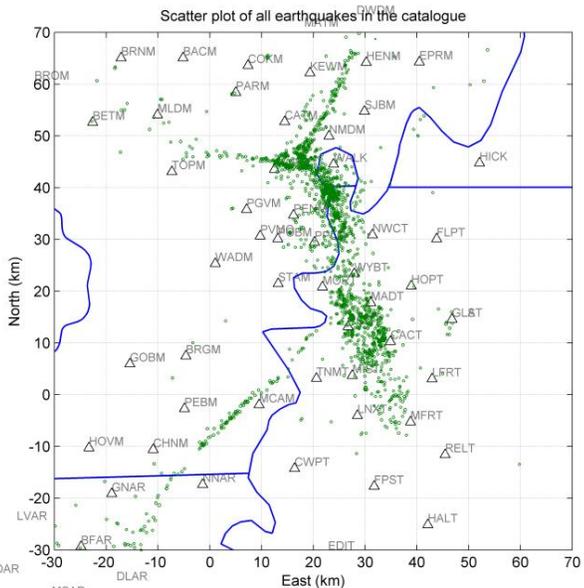
Magnitude of completeness then is estimated by computing the minimum detectable magnitude at each station for an event occurring at each point on a grid. The minimum detectable magnitude is determined by successively computing the signal-to-noise ratio (SNR) for different event magnitudes and requiring a minimum SNR threshold to be met. This minimum SNR threshold should be chosen to realistically estimate the minimum SNR required for the chosen event detection method, (e.g. STA/LTA triggering) but for an initial analysis we set it to 10 dB. Event spectra are estimated according to Brune (1970) with an additional factor to account for attenuation (Ackerley, 2012; Stabile et al, 2013).

The second component of this study is the verification of the predicted network performance using the observed catalogue. We divide our region of interest into a grid. All earthquakes within a given grid square are taken to make up the earthquake catalogue for that square. We require a minimum of 50 events in each grid square for the result to be considered reliable (Woessner and Wiemer, 2005 suggest 200, but in the interest of including more points of comparison we chose to lower this threshold). In selecting the ideal grid size we balance achieving the 50 event minimum in as many grid squares as possible while still allowing for spatial variation on small scales.

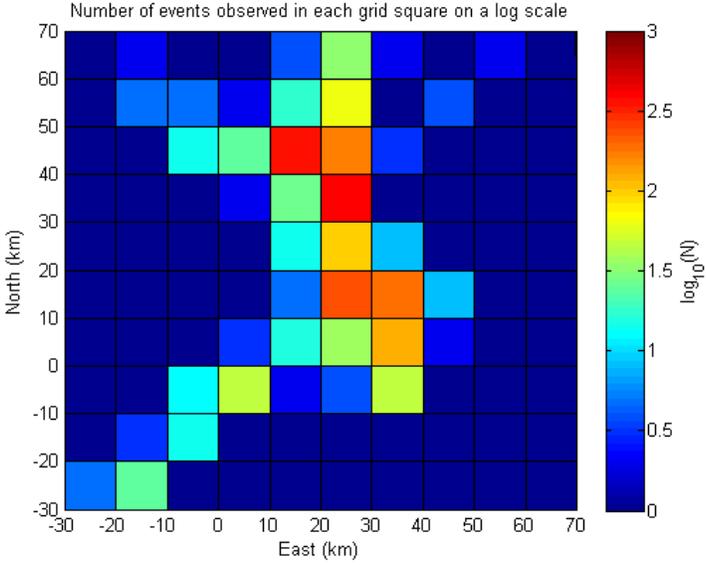
We now compute magnitude of completeness for the grid squares meeting the minimum number of events requirement. Woessner and Wiemer (2005) compare a number of different techniques for estimating the magnitude of completeness from an earthquake catalogue. We use the maximum curvature method (Wiemer and Wyss, 2000) which calculates the maximum of the first derivative of the frequency magnitude distribution. We apply a bootstrap sampling method and average the magnitude of completeness over 200 bootstrap samples (Woessner and Wiemer, 2005). This yields a stable estimate of magnitude of completeness and a measure of the uncertainty of the result.

## Examples

The New Madrid Seismic Zones consists of a mix of L28 sensors and Trillium 120 postholes. Figure 1 plots the catalogue of events and the distribution of stations in the network. The catalogue consists mostly of events clustered around the several known faults of the region. Figure 2 plots the base 10 logarithm of the number of events observed in each grid square. This can be thought of as a relative measure of the seismicity rate. Figure 3 plots the frequency magnitude distribution for an example grid square. Though just one example is shown, most of the data follow a similarly typical distribution. The number of stations available for processing has remained relatively constant for the duration, with the exception of a couple of minor changes to location or equipment at certain stations. These changes did not significantly influence magnitude of completeness for the network so temporal variations in magnitude of completeness due to increased data availability or quality should not be a concern.

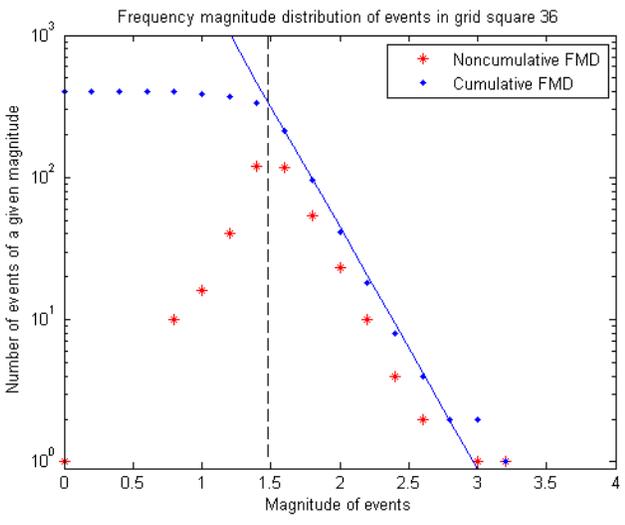


**Figure 1:** Scatter plot of the earthquake catalogue and the distribution of seismic monitoring stations relative to state lines.

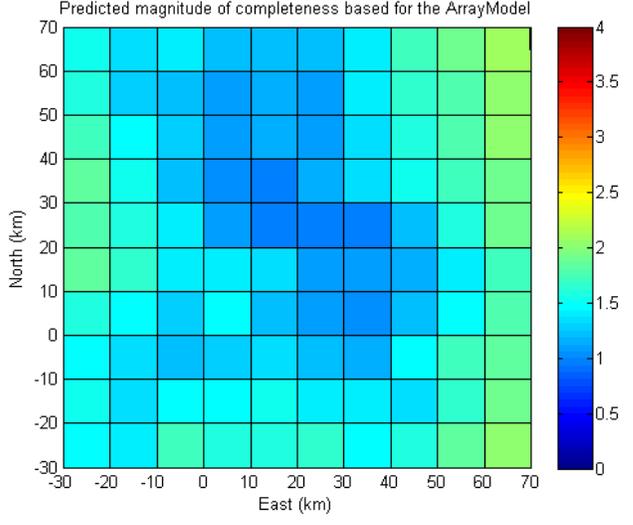


**Figure 2 :** Plot of the base 10 logarithm of the number of events in each grid square. We use only grid squares with at least 50 events.

We apply the methods discussed above to compute the predicted and observed magnitude of completeness for the network. The predicted magnitude of completeness for the region is plotted in figure 4. We observe that magnitude of completeness is fairly consistent throughout the regions of highest station density, with slight variations due to site noise and station spacing. Magnitude of completeness within the centre of array is typically between 1.0 and 2.0, rising toward the edges of the network.



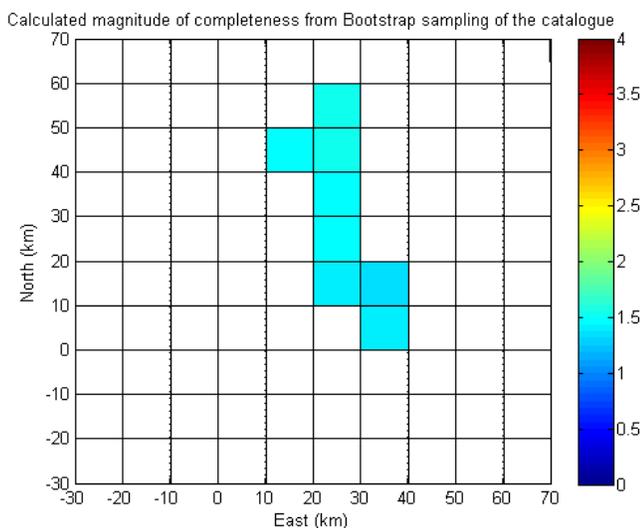
**Figure 3:** Frequency magnitude distribution for the grid square with its northwest corner at 40 km north, 20 km east. Bootstrap sampling yielded a magnitude of completeness of 1.48 for this grid square.



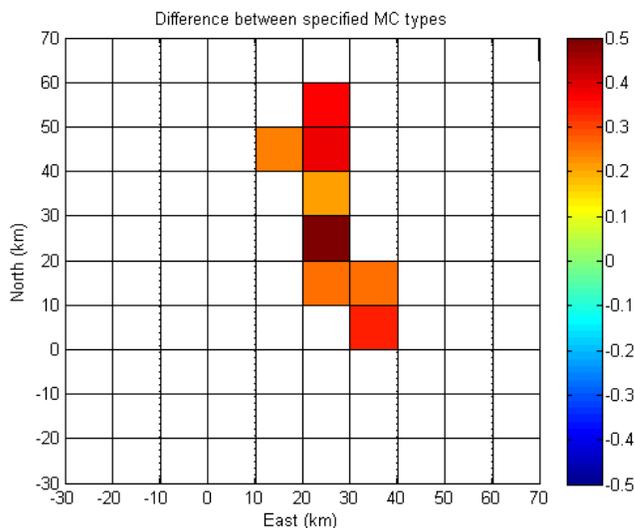
**Figure 4:** Predicted magnitude of completeness for the network in the New Madrid Seismic Zone. Typical values are between 1.0 and 2.0, rising at the edges.

The observed magnitude of completeness from the maximum curvature method yields results similar to the predicted performance, but consistently higher (their difference is plotted in figure 6) by about 0.3 magnitude units on average and with slightly less spatial variation. The maximum curvature method

reports magnitude of completeness near 1.5 (figure 5). In contrast, the predicted magnitude of completeness is between 1.0 and 1.5; unsurprisingly it is lowest near the centre of the network. We do not assess the magnitude of completeness in regions with insufficient events in the catalogue as the results are less reliable. The average difference is between the two methods is approximately 0.3 magnitude units.



**Figure 5:** Observed magnitude of completeness computed according to the maximum curvature with bootstrap sampling of the catalogue.



**Figure 6:** Difference between the observed magnitude of completeness and the predicted value. The result from the catalogue is consistently higher than the predicted value.

## Conclusions

The predicted and observed magnitudes of completeness agree well across most of the grid squares in which there were sufficient data to reliably estimate magnitude of completeness. Thus we conclude that the proposed method to estimate magnitude of completeness is sound though continued efforts are needed to eliminate tuning of parameters such as the minimum signal-to-noise ratio. The method provides an objective measure of performance that can be used to compare different networks, existing or hypothetical and to ensure that monitoring criteria are satisfied.

## Acknowledgements

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