

## A theoretical analysis of linear and nonlinear shear wave time-lapse difference AVO

Shahin Jabbari and Kristopher A. Innanen

University of Calgary, CREWES Project

### Summary

Multicomponent time-lapse amplitude variation with offset (AVO) may improve predictive capability of time-lapse difference data analysis. In this paper, the difference data during the change in a reservoir from the baseline survey relative to the monitor survey are described for shear waves. We have defined a framework for the difference reflection data,  $\Delta R_{SS}(\theta)$ , in orders of baseline interface contrasts and time-lapse changes. The framework is tested using numerical examples from real time-lapse cases used by Veire et al. (2006) and Landrø (2001). Results indicate that second order corrections to linear time-lapse AVO expressions, involving products of elastic time-lapse perturbation and baseline medium perturbation, match numerical examples used in this paper with reduced error in comparison to linearized forms. The second order term represents corrections appropriate for large P-wave and S-wave velocity and density contrasts in the reservoir from the time of the baseline survey to the time of the monitor survey. We conclude that in many plausible time-lapse scenarios the increase in accuracy associated with higher order corrections is non-negligible for shear waves as well as P waves.

### Introduction

Multicomponent surveying has been developed rapidly in seismology including time-lapse monitoring (Stewart et al., 2003). Time-lapse studies monitor the fluid flow and pressure changes over time in the reservoir due to production or employment of enhanced oil recovery techniques. In the time-lapse monitoring process, a baseline survey is acquired prior to production of a reservoir. This is followed by seismic surveys (monitor surveys) over a particular interval of time when geological/geophysical characteristics of a reservoir may change. Comparison of repeated seismic surveys over months, years, or decades add the dimension of calendar time to the seismic data (Greaves and Fulp, 1987; Lumley, 2001; Arts et al., 2004). Time-lapse seismic images display the results which are not predicted by reservoir modeling and emphasize the effect of the production rather than lithological variation (Waal and Calvert, 2003; Tura, 2003). The 4D time lapse signal is sensitive to the changes in impedance and interval parameters, which lead to measurable changes in seismic amplitudes. Linear and nonlinear elastic time-lapse difference for P-P sections and converted wave has been discussed by (Jabbari and Innanen, 2013 and 2014). The study described here focuses on applying the perturbation theory in a time-lapse amplitude variation with offset (time-lapse AVO) environment to derive a framework for modeling and analysis of the time-lapse difference data for the shear wave.

Linearized P-P AVO modelling of time-lapse difference data was discussed by Landrø (2001).

### Theory

We consider two seismic experiments involved in a time-lapse survey, a baseline survey followed by a monitoring survey. The P-wave and S-wave velocities and the density change from the time of the baseline survey relative to the monitoring survey. This pair of models is consistent with an unchanging cap rock overlying a porous target which is being produced. Let  $V_{P0}$ ,  $V_{S0}$ ,  $\rho_0$  and  $V_{Px}$ ,  $V_{Sx}$ ,  $\rho_x$  be the rock properties of the cap rock and reservoir. Amplitudes of reflected and transmitted P and S waves impinging on the

boundary of a planar interface between these two elastic media are calculated through setting the boundary conditions in the Zoeppritz equations which can be rearranged in matrix form e.g.:

$$S \begin{bmatrix} R_{SS} \\ R_{SP} \\ T_{SS} \\ T_{SP} \end{bmatrix} = c_S \quad (1)$$

Where

$$S \equiv \begin{bmatrix} \sin \phi & -\sqrt{1 - (B^{-1})^2 \sin^2 \phi} & \frac{F_x \sin \phi}{\sqrt{1 - F_x^2 \sin^2 \phi}} & -\sqrt{1 - E_x^2 \sin^2 \phi} \\ -\sqrt{1 - \sin^2 \phi} & -B^{-1} \sin \phi & \sqrt{1 - F_x^2 \sin^2 \phi} & E_x \sin \phi \\ -2 \sin \phi \sqrt{1 - \sin^2 \phi} & B^{-1}(1 - 2 \sin^2 \phi) & 2A_x F_x^2 \sin \phi \sqrt{1 - F_x^2 \sin^2 \phi} & -A_x E_x (1 - 2F_x^2 \sin^2 \phi) \\ 1 - 2 \sin^2 \phi & 2 \sin \phi \sqrt{1 - (B^{-1})^2 \sin^2 \phi} & A_x F_x (1 - 2F_x^2 \sin^2 \phi) & 2A_x F_x^2 \sin \phi \sqrt{1 - E_x^2 \sin^2 \phi} \end{bmatrix} \quad (2)$$

$\phi$  is the S-wave incident angle, and

$$c_S \equiv \begin{bmatrix} \sin \phi \\ \sqrt{1 - \sin^2 \phi} \\ 2 \sin \phi \sqrt{1 - \sin^2 \phi} \\ 1 - 2 \sin^2 \phi \end{bmatrix}. \quad (3)$$

The ratio of elastic parameters used here are defined as:

$$A_x = \frac{\rho_x}{\rho_0}, \quad B^{-1} = \frac{V_{P_0}}{V_{S_0}}, \quad E_x = \frac{V_{P_x}}{V_{S_0}}, \quad F_x = \frac{V_{S_x}}{V_{S_0}}. \quad (4)$$

We now form an auxiliary matrix  $S_s$  by replacing the first column in  $S$  with  $c_S$ . The reflection coefficient is determined by:

$$R_{SS}(\phi) = \frac{\det(S_s)}{\det(S)}. \quad (5)$$

$R_{SS}$  for the baseline and monitor surveys are calculated using the method explained above, where rock properties for cap rock are the same, but reservoir properties change from  $V_{P_b}$ ,  $V_{S_b}$ ,  $\rho_b$  at the time of the baseline survey to  $V_{P_m}$ ,  $V_{S_m}$ ,  $\rho_m$  at the time of the monitor survey. If we replace  $x=b$  for the baseline survey and  $x=m$  for the monitor survey in equations (2) and (4), the reflection coefficients can be calculated for both.

The difference data reflection coefficient between the baseline and monitor survey is then calculated as:

$$\Delta R_{SS}(\theta) = R_{SS}^m(\theta) - R_{SS}^b(\theta). \quad (6)$$

In our time lapse study we have considered two groups of perturbation parameters (Innanen et al., 2013; Stolt and Weglein, 2012). The first group expresses the perturbation caused by propagating the wavefield from the first medium to the second medium in the baseline survey:

$$b_{VP} = 1 - \frac{V_{P_0}^2}{V_{P_b}^2}, \quad b_{VS} = 1 - \frac{V_{S_0}^2}{V_{S_b}^2}, \quad b_\rho = 1 - \frac{\rho_0}{\rho_b}. \quad (7)$$

The second group is to account for the change from the baseline survey relative to the monitor survey, the time lapse perturbation, we define:

$$a_{VP} = 1 - \frac{V_{Pb}^2}{V_{Pm}^2}, \quad a_{VS} = 1 - \frac{V_{Sb}^2}{V_{Sm}^2}, \quad a_\rho = 1 - \frac{\rho_b}{\rho_m}. \quad (8)$$

Applying equations (7) and (8), elastic parameters may re-defined in terms of perturbations in P-wave and S-wave velocities and the densities as:

$$\begin{aligned} A_m &= (1 - b_\rho)^{-1} \times (1 - a_\rho)^{-1}, \\ E_m &= \frac{V_{P0}}{V_{S0}} \times (1 - b_{VP})^{-\frac{1}{2}} \times (1 - a_{VP})^{-\frac{1}{2}}, \\ F_m &= (1 - b_{VS})^{-\frac{1}{2}} \times (1 - a_{VS})^{-\frac{1}{2}}. \end{aligned} \quad (9)$$

These forms are substituted into equation (2), and the determinants and determinations in equation (5) are expanded in orders of all six perturbations, and  $\sin^2(\phi)$ :

$$\Delta R_{SS}(\theta) = \Delta R_{SS}^{(1)}(\theta) + \Delta R_{SS}^{(2)}(\theta) + \dots \quad (10)$$

## Results and numerical analysis

The linear and second order terms in the difference data reflection coefficient for shear wave in terms of the baseline and time-lapse perturbations, and  $\sin^2(\phi)$  are as follows:

$$\begin{aligned} \Delta R_{SS}^{(1)}(\phi) &= \left[ \frac{1}{4} (7 \sin^2 \phi - 1) \right] a_{VS} + \left[ \frac{1}{2} (4 \sin^2 \phi - 1) \right] a_\rho \\ \Delta R_{SS}^{(2)}(\phi) &= \left[ \left( \frac{7}{4} - \frac{V_{S0}}{V_{P0}} \right) \sin^2 \phi - \frac{1}{8} \right] a_{VS}^2 \\ &+ \left[ \left( 1 + \left( \frac{1}{4} \right) \frac{V_{P0}}{V_{S0}} - \frac{V_{S0}}{V_{P0}} \right) \sin^2 \phi - \frac{1}{4} \right] a_\rho^2 \\ &+ \left[ \left( 1 - 2 \frac{V_{S0}}{V_{P0}} \right) \sin^2 \phi \right] (a_{VS} a_\rho + a_{VS} b_\rho + b_{VS} a_\rho) \\ &+ \left[ \left( \frac{7}{4} - 2 \frac{V_{S0}}{V_{P0}} \right) \sin^2 \phi \right] (a_{VS} b_{VS}) \\ &+ \left[ \left( \left( \frac{1}{2} \right) \frac{V_{P0}}{V_{S0}} - 2 \frac{V_{S0}}{V_{P0}} \right) \sin^2 \phi \right] (a_\rho b_\rho) \end{aligned} \quad (11)$$

We examine the derived linear and non linear difference time lapse AVO terms qualitatively with numerical examples. In the first example, we apply the data used by Veire et al. (2006). They used two synthetic models for the reservoir: a baseline scenario with a water saturation of 10% and an effective pressure of 2 MPa . In the monitor survey, the water saturation and effective pressure are 50% and 8 MPa respectively. These changes altered the seismic parameters and caused 15%, 11%, and 1% increase respectively in P-wave and S-wave velocities and density. In the second example, the data used by Landrø (2001) are employed. Typical values for P-wave and S-wave velocities and density for the cap rock and reservoir (preproduction and post production), which are the same as Gullfaks 4D project, are used. In the Gullfaks field, there are +13%, -2%, and +4% changes in the reservoir in P-wave and S-wave velocities and density respectively due to the production. The exact difference data are compared with our derived linear and second order approximations in Figure 1. The second order time-lapse AVO approximations are in better agreement with the exact difference data, especially for for higher contrasts in seismic parameters and when angles are below the critical angle which correspond to the range of the study in this paper.

In the current research, we extended this work by formulating a framework for the difference reflection data for shear wave. This framework is expressed as an expansion in orders of both baseline interface properties and time-lapse changes from the time of the baseline survey to the time of the monitor survey.

For the validation of this framework for linear and non linear  $\Delta R_{SS}$ , the results has been tested using numerical examples. A comparison of the theoretical results for the linear and second order approximations with the the numerical analysis indicates that including higher order terms in approximating the difference data improves the accuracy of calculating time lapse difference reflection data, particularly for large contrast cases. We conclude that in many plausible time-lapse scenarios, the increase in accuracy associated with higher order corrections demonstrated in this paper enhances time-lapse modelling.

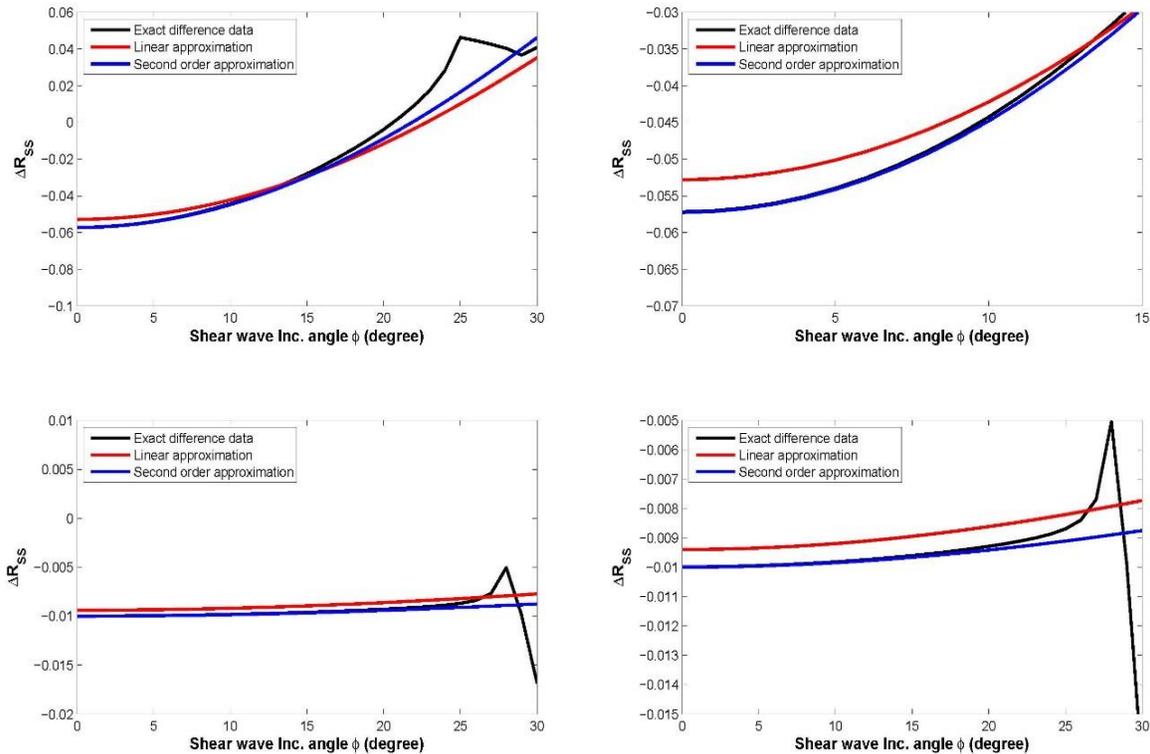


Figure 1:  $R_{SS}$  for the exact, linear, and second order approximations. Top left: +15%, +11%, and +1% changes in P- and S-wave velocities and density respectively in the reservoir after production, Veire et al. (2006); bottom left: +13%, -2%, and +4% changes in P- and S-wave velocities and density respectively in the reservoir after production, Landrø (2001); top and bottom right: zoomed in for better differentiation.

## Conclusions

Time-lapse measurements provide a tool to monitor the dynamic changes in subsurface properties throughout the development phase during the exploitation of a reservoir. Changes in geological and geophysical properties of a reservoir such as fluid saturation and pore pressure produce measurable changes in elastic parameters such as P-wave and S-wave velocities and the density. Landrø (2001) provided a linear approximation for the time lapse difference reflection data only for P-P sections. Also, the accuracy of Landrø's equation diminishes when the contrast between the reservoir at the time of the baseline and monitor is relatively large (Landrø, 2001). A framework was formulated for the difference reflection data including linear and non linear terms in  $\Delta R_{PP}$ ,  $\Delta R_{PS}$  and  $\Delta R_{SP}$  by Jabbari and Innanen (2013 and 2014).

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