Permeability Calculations in Unconsolidated Sands

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Summary

Permeability is a property of the porous medium that measures the capacity and ability of the formation to transmit fluids. The rock permeability, k, is a very important rock property because it controls the directional movement and the flow rate of the reservoir fluids in the formation. This rock characterization was first defined mathematically by Henry Darcy in 1856. By analogy with electrical conductors, permeability represents reciprocal of residence which porous medium offers to fluid flow.

The permeability of a rock can be predicted using empirical relationships, capillary models, statistical models, and hydraulic radius theories. It generally has been recognized that grain size is the fundamental independent variable that controls permeability in unconsolidated sediments.

Sub-pore scale modeling is used in this study to evaluate the relation between grain size distribution and permeability of the porous media.

Introduction

One of the first accepted and frequently quoted relation between permeability and the properties of the porous medium was introduced by Kozeny (1927) and later modified by Carman (1937, 1956). The resulting equation is largely known as the Kozeny-Carman (KC) equation. This equation has taken several forms, including the following one that is commonly used (for water flood):

$$k = C\frac{g}{\mu_w \rho_w}\frac{\varphi^3}{S^2 D_R^2 (1-\varphi)^2} \quad (1)$$

Where,

$k =$ Absolute permeability,

$C =$ A factor to take into account the shape and tortuosity of channels,

$g =$ Gravitational constant,

$S =$ Specific surface ($\frac{m^2}{kg}$ of Solids),

$\mu_w =$ Water viscosity,

$\rho_w =$ Water density,

$D_R =$ Specific weight of solid ($\frac{\rho_s}{\rho_w}$),

$\varphi =$ Porosity.

This equation was developed based on the following assumption:

- Porous media is considered as a bundle of capillary tubes of equal length,
- The flow is laminar and steady,
- It is not taken into account velocity components normal to the tube's axes,
- Fluid is Newtonian and incompressible.

For porous media contains sphere shape grains the equation could be simplified as:

$$k = \frac{D_g^2}{180.\mu_w} \frac{\varphi^3}{(1-\varphi)^2} \quad (2)$$

Where, $D_g =$ Grain size.

Theory and/or Method

In this study, direct pore scale modeling was used to simulate single phase flow in a three dimensional homogeneous and heterogeneous porous medium patterns. Final goal of this study is to use this method to examine the effect of grain size distribution on the permeability and test the validity of the Kozeny-Carman equation for permeability prediction of different patterns with different grain size distribution. In contrast to the Kozeny-Carman equation, in direct pore-level modeling, Navier-Stokes and Continuity equations, as the governing equations of flow, are directly applied and solved on the 3-D porous medium without any simplification in medium geometry. Simulation result would be a velocity and pressure profile. Absolute permeability of the studied medium could be calculated using Darcy's equation:

$$k = \frac{v\mu L}{\Delta p} \quad (3)$$

Where, $L =$ Medium length in direction of the flow.

Three dimensional studied porous media are included sphere shape grains. To study the effect of grain size distribution on the absolute permeability of porous media, a “pattern generator” was developed to design and generate different patterns with different grain size distribution with known values. 3-D media were generated based on the process/object based reconstruction method. In this method the grain size is determined randomly using the Weibull Distribution:

$$r = (r_{max} - r_{min}) \times \left(-\theta - log(random\, no.) \times \left(1 - exp\left(\frac{-1}{\theta}\right)\right) + exp\left(\frac{-1}{\theta}\right)\right)^{\frac{1}{\gamma}} + r_{min} \quad (4)$$

The model is based on a random close packing of spheres with different radii.

The process based reconstruction reproduces a realistic imagination of 3D porous media. The idea of object/Process based reconstruction of porous media is simulation of subsequent steps of deposition, compaction and diagenesis. As the first step of the construction several grains are generated (using eq. (4)) and deposited. Grains position would be determined randomly. Shaking process is simulated to compact grains as much as possible.

Generated medium is introduced to COMSOL software, and single phase flow is simulated on the pattern applying Navier-Stokes and continuity equations. To validate sub-pore scale modeling, 8 regular cubical packing of uniform spheres were used to examine permeability prediction of this method. The porosity of all generated patterns is 47.6%. The only difference is grain size. Table 1 represents grain sizes of different studied mediums. Figure 1 shows a sample of generated pattern.

Table 1: grain sizes of different studied mediums.

Pattern No.	Grain Size Diameter (micron)	Pattern No.	Grain Size Diameter (micron)	Pattern No.	Grain Size Diameter (micron)	Pattern No.	Grain Size Diameter (micron)
1	16	3	24	5	36	7	50
2	20	4	30	6	40	8	60

To have the same boundary effect, the ratio between grain size and pattern length remains constant. It means all the generated patterns have the same number of grains.

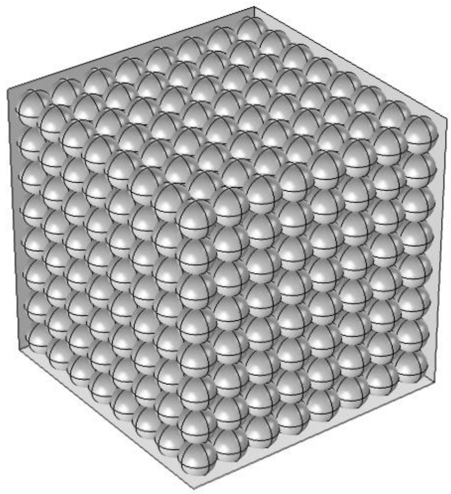

Figure 1: Generated regular cubical packing of uniform sphere.

Figure 2 is velocity profile as a result of pore-scale modeling.

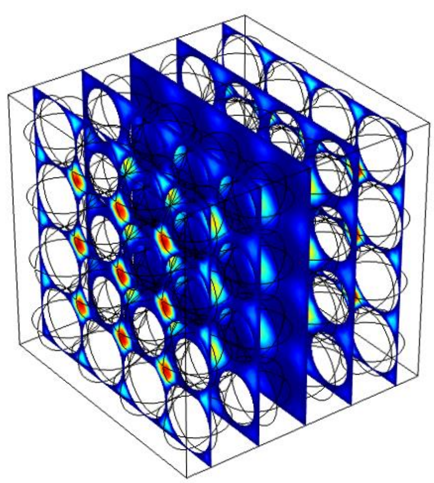

Figure 2: Velocity profile as a result of sub-pore scale modeling.

Kozeny-Carman equation is used to calculate the exact value of the medium properties. Figure 3 compare the permeability predicted values using Kozeny-Carman equation and sub-pore scale modeling approach.

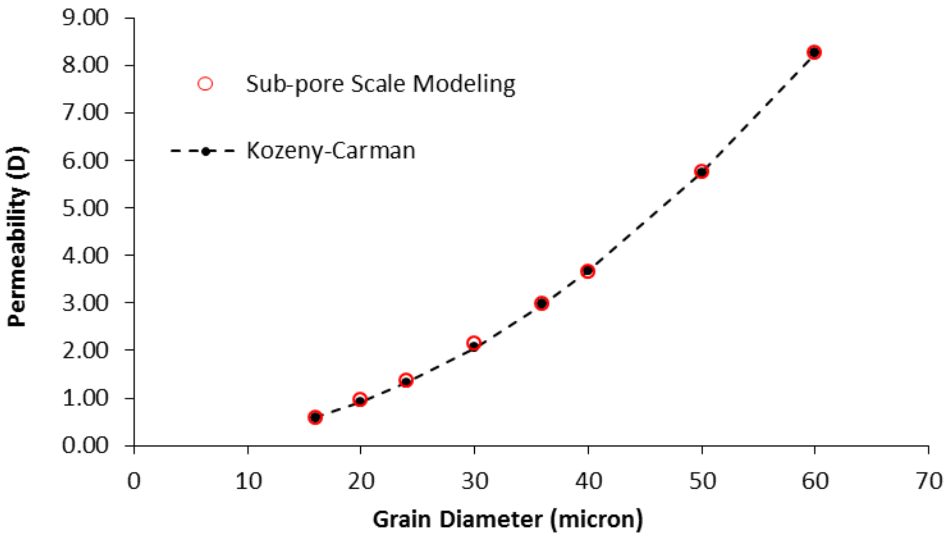

Figure 3: Permeability prediction using Kozeny-Carman equation and sub-pore scale modeling approach.

As the next step, after validation of the method, different non-regular patterns with non-uniform spheres were generated. Permeability predicted from sub-pore scale modeling approach is assumed as a true value and used to check the validity of Kozeny-Carman equation for permeability prediction. The results could be used to study grain size distribution effect on the medium permeability.