INVESTIGATION OF DIELECTRIC PROPERTIES OF EVAPORITE MINERALS TO INTERPRET GPR DATA

Sohely Pervin and Douglas R. Schmitt

Department of Physics, University of Alberta

Summary

Proper interpretation of ground penetrating radar (GPR) images obtained in evaporite sequences requires knowledge of the dielectric properties of the constitutive minerals. Dielectric permittivity contrasts underneath the subsurface cause various reflections in GPR profiles and eventually provides important information about the objects. Measurement of dielectric properties of the evaporite minerals were performed over a frequency range of 10 MHz to 3 GHz using a commercially available RF impedance/material analyzer (Agilent 4991A) with an open-ended co-axial sensor (Agilent 85070A). Cold compression technique was used to prepare the samples from various mineral powders and core rock up to 250 MPa pressure. Furthermore, the mineral powder mixtures and the core rocks were grinded in a grinding machine and kept in an oven for couple of hours at about 80°C before pressurization. The porosity of the samples is reduced significantly due to the grinding, heating and high pressure. The permittivities measured from these synthetic samples were compared to the single crystals for accuracy. The changes of dielectric permittivity with the addition of additives according to their weight percentage were obtained as well.
Introduction

Potash ore is mined principally for its sylvite (KCl) that is an important industrial chemical with particularly use in agricultural fertilizer. Even the richest sylvite bearing potash contains substantial proportions of other evaporate minerals particularly halite (NaCl) and some carnallite. Formations adjacent to the best ores contain a wide variety of additional evaporates such as anhydrite, gypsum, calcite, and dolomite depending upon the depositional and burial history.

In the Saskatchewan potash mines, high quality ore zones are often bounded by thin ‘shale’ layers that are principally contaminated with anhydrite and calcite. Ground penetrating radars (GPR) attached to mining machines are often used to track these shale layers so that they can be avoided. As such, in order to best interpret the underground observations it is important to understand the reflections seen; but to do this fully requires appropriate knowledge of the physical properties of the constituent evaporate minerals.

This work is motivated by the need to better understand GPR images of underground potash mines in support of underground operations. Many of these new measurements reinforce the earlier work, but a novel aspect of this study is the measurement of ε on samples that are specially prepared by cold pressing of powders at pressures up to 300 MPa.

Theory

A transverse electromagnetic wave (TEM) travels in free space consists of alternating and in phase vector electric $\mathbf{E}$ (in V/m) and magnetic $\mathbf{H}$ (in A/m) fields that are both perpendicular to each other and to the propagation direction. The wave moves without loss at the speed of light $c = 299792458 \text{ m/s}$. The ratio of the magnitudes $|\mathbf{E}|/|\mathbf{H}|$ has real value equal to the intrinsic wave impedance $Z_0$ (in Ω).

$$ Z_0 = \frac{|\mathbf{E}|}{|\mathbf{H}|} = c \mu_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} $$

(1)

where $\varepsilon_0 (8.854 \ 187 \ 817 \times 10^{-12} \ \text{Farad m}^{-1})$ and $\mu_0 \ (4\pi \times 10^{-7} \ \text{Henry m}^{-1})$ are, respectively, the electrical permittivity and the magnetic permeability of free space. Note that $c = 1/\sqrt{\varepsilon_0 \mu_0}$ (Mohr, Taylor et al. 2008).
A given medium is electromagnetically characterized by three physical properties. In the below it is implicitly assumed that all of these properties are frequency dependent.

First, the magnetic permeability $\mu = \mu'\mu_0$ relates the magnetic flux density $B$ (in Wb/m$^2$) to the magnetic field intensity $H$ (in A/m) via $B = \mu_0 H$. The relative magnetic permeability may also be complex

$$\mu^* = \mu' - i\mu''$$

with the real $\mu'$ and imaginary $\mu''$ components describing the total induced magnetization and the magnetic hysteresis loss. In Geophysical investigations, it is more common to use the dimensionless volume magnetic susceptibility $k = \mu^*-1$. For the minerals studied here $|k| < 10^{-5}$ (Hunt and Moskowitz 1995) and for most cases it would be safe to take $\mu \approx \mu_0$. However, it may be prudent to include the magnetic permeability for this for more magnetic rocks.

The second is the DC electrical conductivity $\sigma$ (in S/m) describes the ability of free electrons and ions to drift under an externally applied electrical field.

Third and most importantly for radar frequencies is the dielectric permittivity $\varepsilon = \varepsilon^*\varepsilon_0$ describing the polarization of induced or oriented electric dipoles within a dielectric material by the same electrical field. In general, the relative dielectric permittivity $\varepsilon^*$ is complex and given by

$$\varepsilon^* = \varepsilon' - i\varepsilon''$$

where $\varepsilon'$ and $\varepsilon''$ are the real and the imaginary components, respectively. The real part $\varepsilon'$, often referred to as the dielectric constant despite its frequency dispersion, describes the ability of the material to store energy by polarization as a result of applying electromagnetic radiation (Nortemann, Hilland et al. 1997). The imaginary part $\varepsilon''$ describes the energy loss resulting from dielectric hysteresis. In the microwave region loss can be caused by both the conductively related motion of mobile conduction electrons or ions and by the dielectric hysteresis lag of dipole rotation behind the rapidly fluctuating electric field; together the relative equivalent dielectric permittivity $\varepsilon^{\text{eff}}$ to be expressed as

$$\varepsilon^{\text{eff}} = \varepsilon^* - i \frac{\sigma}{\omega\varepsilon_0}$$
where \( \omega \) is the angular frequency. This hysteresis results in heating of the material and is the principle employed in microwave cooking and rf heating. Alternatively, one could also consider the equivalent conductivity

\[
\sigma'_{\text{eff}} = \sigma + \omega \varepsilon' \varepsilon_0.
\] (5)

Examination of these show that the dielectric loss increases with frequency while conductivity is important at low frequencies. In free space case the electromagnetic wave’s propagation speed is equal to the speed of light \( c \).

\[
c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}
\] (6)

The propagation velocity of electromagnetic wave in a media where the magnetic permeability can be ignored is given by

\[
\nu = \frac{c}{\sqrt{\frac{\varepsilon'}{2} \left[ 1 + (1 + \tan^2 \frac{\delta}{2})^{1/2} \right]}}
\] (7)

that is less than \( c \). This wave has attenuation \( \alpha \) (in Neper/m)

\[
\alpha = \frac{\omega \sqrt{\mu_0 \varepsilon_0}}{\sqrt{2}} \left[ \sqrt{1 + \left( \frac{\sigma}{\omega \varepsilon'} \right)^2} - 1 \right]^{1/2}
\] (8)

The reciprocal of \( \alpha \) is called the skin depth and is defined as the depth at which the input energy reduces by \( 1/e \) where \( e \) is the base of the natural logarithm. This loss can be quantified in a number of ways with the loss tangent \( \tan \delta \)

\[
\tan \delta = \frac{\sigma'_{\text{eff}}}{\omega \varepsilon' \varepsilon_0} = \frac{\sigma + \omega \varepsilon' \varepsilon_0}{\omega \varepsilon' \varepsilon_0}
\] (9)

The reciprocal of \( \tan \delta \) is equal to the quality factor \( Q \) that is the ratio between the average stored energy per cycle to the energy lost per cycle. A consequence of this loss is that the \( \mathbf{E} \) and \( \mathbf{H} \) fields associated with the propagating wave are out of phase by angle \( \delta/2 \). The power dissipation which is the amount of energy lost per cycle is then given by:

\[
P = \sigma'_{\text{eff}} E^2.
\] (10)
Finally, the intrinsic electromagnetic impedance of a material may be given generally by

\[
Z = \sqrt{\frac{\mu}{\varepsilon_0 c^2}} = \sqrt{\frac{\left(\mu' - i\mu''\right)}{\varepsilon_0 - i\left(\varepsilon'' + \sigma / \omega\varepsilon_0\right)}} = c\mu_0 \sqrt{\frac{\left(\mu' - i\mu''\right)}{\varepsilon_0 - i\left(\varepsilon'' + \sigma / \omega\varepsilon_0\right)}}
\]

The impedances control the reflection and transmission of waves across an interface between two materials (1) and (2) of differing complex impedances \(Z_1\) and \(Z_2\), respectively. The plane wave reflection co-efficient (voltage«) \(R\) for the wave normally incidence from medium (1) is from the Fresnel equations is simply

\[
R = \frac{Z_2 - Z_1}{Z_2 + Z_1}
\]

\(R\) will generally be complex meaning that the reflected wave`s phase is rotated upon reflection by the angle \(\tan^{-1}(\text{Im}(R)/\text{Re}(R))\).

The above mentioned theory of the power dissipation of the electromagnetic wave in a medium is straightforward for a single component and an isotropic system. However naturally occurring rocks and minerals are not pure, but are contaminated with a variety of other minerals depending on their formation. Most rocks are also porous and to some extent saturated with liquids. Further, rocks are composed of grains with different sizes and distribution. All of the mentioned properties of the rock or mineral are proven to have a measurable effect on the complex dielectric permittivity and therefore complicate the interpretation of the GPR data.

**Examples**

Measuring the permittivity of single crystal using the coaxial line sensor technique requires a relatively large crystal with smooth and flat faces at least 2 cm across and with thickness at least 1 cm. High quality, pure and naturally occurring single crystals are not easily obtained; so in some case we manage to acquired large single crystals of high purity that are used primarily for IR optics. In Table (1) we present our measurements that have been conducted on single crystals. Couple of measurements were done on sylvite and halite single crystals. Dielectric permittivity of halite crystal is approximately 5.9 and sylvite crystal is approximately 4.8 which
are consistent with previous works (Olhoeft 1981). We also worked with calcite crystal and found the permittivity 8.2 but it could be different for different faces according crystal orientation.

Moreover, synthetic samples were prepared using cold compression technique. The dielectric permittivity from the cold compressed NaCl sample is 6 which is consistent with the single crystal (Figure 1). It turns out that using cold compression technique we are able to make sample whose porosity is really low or almost non-porous. Furthermore, we have added different percentage of LiF with NaCl according to their weight (Figure 2). It is clear from the figures that dielectric permittivity is decreasing with the addition of additive. Note that, for mineral mixtures the dielectric values depends on the homogeneity and heterogeneity of the constituent minerals.

**Conclusions**

Dielectric permittivity measurements were done on both single crystals and synthetic samples. The dielectric values of halite and sylvite crystals using co-axial probe technique are consistent with previous works. Moreover, the dielectric value of cold compressed NaCl are comparable with the result from halite single crystal. The mineral mixture values can be used to make models with different mixtures theories in future.

**Acknowledgements**

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**References**


Figure 1: Dielectric permittivity of cold compressed NaCl
Figure 2: Dielectric permittivity of cold compressed mixtures of LiF with NaCl. Dielectric permittivity is decreasing with the addition of LiF.
Table 1: Dielectric permittivity of single crystals

<table>
<thead>
<tr>
<th>Material</th>
<th>Dielectric permittivity</th>
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<tbody>
<tr>
<td>Halite</td>
<td>5.9</td>
</tr>
<tr>
<td>Sylvite</td>
<td>4.8</td>
</tr>
<tr>
<td>Calcite</td>
<td>8.2</td>
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