Model parametrization strategies for Newton-based acoustic full waveform inversion

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Summary

This paper studies the effects of model parameterizations on velocity model building using full waveform inversion with Newton-based optimization method. Three types of model parameterizations were considered. These include velocity, slowness and slowness squared. Numerical results demonstrate that the quality of the reconstructed velocity model and convergence rate of the Newton method based acoustic full-waveform inversion is greatly affected by the type of model parameterization used. A high convergence rate of FWI with Newton-based optimization method can be achieved by using either slowness or slowness squared as model parameterization in comparison with velocity parameterization.

Introduction

Full waveform inversion (FWI) aims to estimate subsurface model parameters through local optimization methods (Lailly, 1983; Tarantola, 1984; Stekl and Pratt, 1998; Pratt et al., 1998; Virieux and Operto, 2009). The inversion is often performed using iterative gradient methods. For large scale problems, due to computational cost of simulating multiple sources on every iteration, the gradient-based methods are the most practical and popular optimization engine for FWI. However, gradient-based optimization methods are generally very slow and have difficulties of properly scaling the reconstructed model parameters in comparison to Newton's methods. The most common method to improve the rate of convergence in the gradient optimization algorithm is to scale the gradient using diagonal components of a pseudo-Hessian matrix (Shin et al., 2001).

On the other hand, the Newton-based methods that require the Hessian matrix have fast convergence rate compared to gradient-based methods. However, fast convergence rate in Newton-based method comes at the expense of solving the inverse of a large dense Hessian matrix or a set of large system of dense Hessian matrix have to be solved. Even though the Newton-based methods have faster convergence rate, the choice of model parameterization influences the behavior of the Hessian matrix and thereby affects the convergence rate and the quality of the reconstructed model parameters. In the acoustic full waveform inversion, the Earth's model estimation is often carried out in velocity model parameterization. However, other types of model parameterizations can also be employed such as slowness and slowness squared. These three model parameterizations do not change the shape of the misfit function but they affect how the gradient is scaled, the convergence rate and the accuracy of the results by changing the characteristic nature of the Hessian matrix.

In this paper, Singular Value Decomposition (SVD) analysis on the Hessian matrix from a small portion of the Marmousi velocity model is used to examine the influences of the three model parameterizations. The Gauss-Newton (GN) and full Newton (FN) methods are used for full waveform inversion.

Theory

Waveform inversion often uses the least-squares misfit defined as the l_2 norm of residual between the observed data $\mathbf{d}^{obs}$ and the model data $\mathbf{d}^{cal}$,

$$J(\mathbf{m}) = \frac{1}{2}\sum_{\omega_i}^{N_\omega}\sum_{s,r}^{N_s,N_r}(\mathbf{d}^{obs}_{s,r}(\omega_i) - \mathbf{d}^{cal}_{s,r}(\omega_i))^\dagger(\mathbf{d}^{obs}_{s,r}(\omega_i) - \mathbf{d}^{cal}_{s,r}(\omega_i)) + \mu R(\mathbf{m}), \tag{1}$$

where $\dagger$ is the complex conjugate transpose, ω is the angular frequency, $R(\mathbf{m})$ is the regularization term and μ is the regularization parameter. The parameters N_s and N_ω represent the number of sources and frequencies, respectively. In Newton's formulation, the model perturbation update ($\triangle\mathbf{m}$) is estimated by solving the following linear system of equations

$$\Re e[\mathbf{H}]\triangle\mathbf{m} = -\Re e[\mathbf{g}], \quad (2)$$

where $\Re e$ represents for real part, and $\mathbf{H}$ and $\mathbf{g}$ are the Hessian matrix and the gradient of the misfit function, respectively.

Even though our main goal is to estimate velocity, in the acoustic wave approximation, we can parameterize the model parameter $\mathbf{m}$ in terms of velocity (v), slowness (v^{-1}) or slowness squared (v^{-2}). The shape of the misfit function in equation (1) does not change by the type of the model parameterization used in the numerical optimization scheme. However, the choice of model parameterization influences the eigenvalue spectrum of the Hessian matrix and the direction of gradient. Consequently this also influences the convergence behaviour of the algorithm. The change in the direction of gradient comes from the scaling term of the partial derivatives of the Helmholtz operator.

Examples

Figure 1 shows the portion of the Marmousi velocity model that is used to analyze the impact of model parameterization choice on the Hessian matrix. The analysis focus on the eigenvalues and eigenvectors obtained from the Singular Value Decomposition (SVD) of the Hessian matrix.

From SVD analysis of the Hessian matrix and making use of equation (2), the gradient vector of the objective function is the sum of the projected model perturbation along the eigenvectors. The direction of eigenvectors corresponding to largest and smallest eigenvalues point in the direction of greatest and smallest curvature of the objective function, and provide the well and poorly retrieved parts of the model parameter, respectively. Figure 2 shows the first three eigenvectors in depth corresponding to the largest three eigenvalues. Plots from bottom to up are the eigenvectors in depth associated with the 1^{st}, 2^{nd} and 3^{rd} largest three eigenvalues, respectively. The red, green and blue colors represent eigenvectors from velocity, slowness and slowness squared model parameterizations, respectively. The eigenvector corresponding to the largest eigenvalue provides the most importance part of the model parameters that can be retrieved by FWI. As we see from the plots, unlike the eigenvectors that arises from slowness or slowness squared model parameterization, the magnitude of eigenvectors results from the velocity model parameterization decreases with depth. As a result, with the velocity model parameterization in the acoustic approximation FWI, the velocity model of very shallow parts would be properly retrieved well than the deep part of the model, where the velocities are the highest. In the case of slowness squared parameterization, the eigenvectors are almost depth independent and are strong with depth (see Figure 2) compared to eigenvectors from the slowness or velocity parameterization. Hence, the deep part of the model would be properly reconstructed.

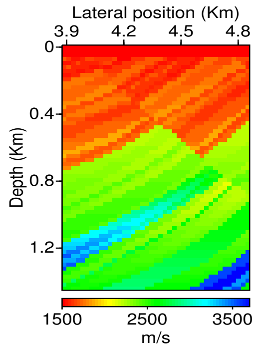

Figure 1 *Portion of Marmousi velocity Model.*

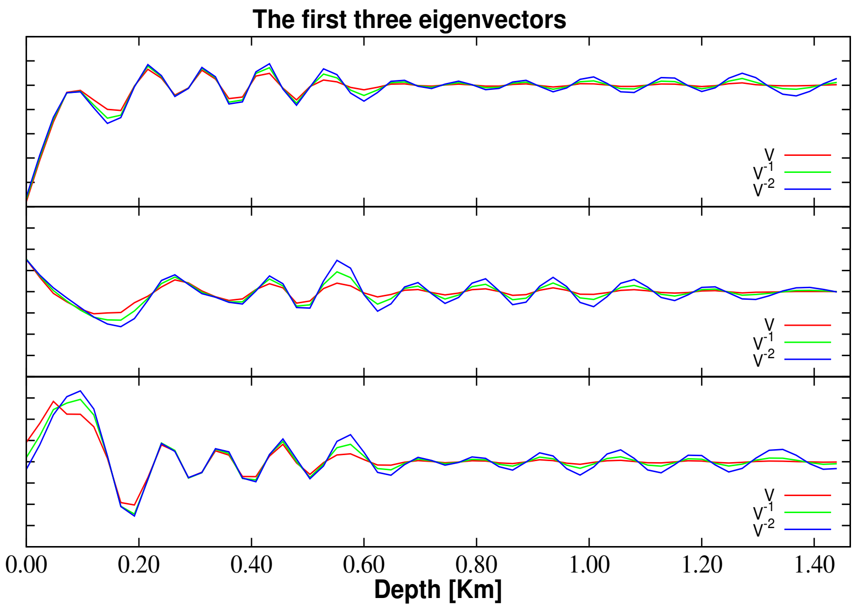

Figure 2 *The first three model eigenvectors of the approximate Hessian matrix against depth corresponding to the largest eigenvalues of the Hessian matrix.*

Next we consider Marmousi velocity model shown in Figure 3a. For inversion, 95 sources and 192 receivers at the surface are considered, and a set of 8 discrete frequencies were selected between 3.1 Hz and 21 Hz. Figures 4 depict the reconstructed velocity models using GN and FN methods from noisy data set with SNR=10. The deep part of the model, where velocities are the highest, are well determined in the case slowness or slowness squared parameterization. Thus, the solutions from slowness and slowness squared parameterizations have more balanced amplitude in shallow and deep parts of the model.

The convergence rates of the GN and FN methods for the three types of model parameterizations are shown in Figures 5. As can be seen from the figure, in the slowness squared parameterization, the reduction in the misfit function by GN or FN method is faster than the one obtained from slowness or velocity parameterization. This is because the use of slowness or slowness squared parameterization provides proper scaling of the descent direction both in the deeper and shallow parts of the model.

Conclusions

We examined effects model parameterizations of 2D acoustic full waveform inversion designed to estimate a reliable velocity model using Newton-based optimization methods. Velocity, slowness and slowness squared model parameterizations were considered. The use of Hessian matrix in FWI numerical algorithms plays a

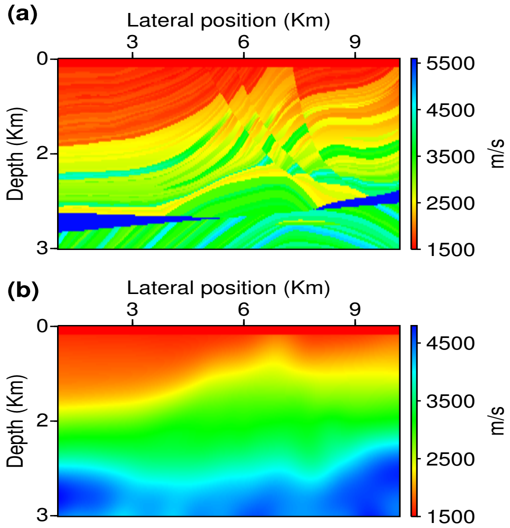

Figure 3 *True Marmousi velocity model (a) and smooth velocity model used as starting model for inversion (b).*

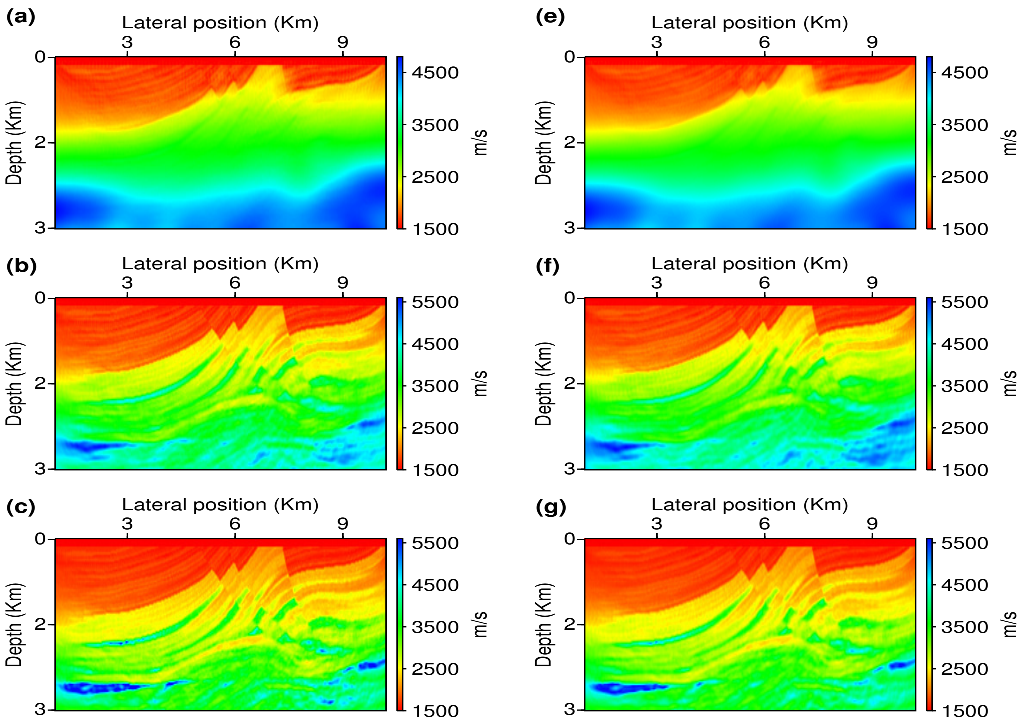

Figure 4 *Reconstructed velocity models after the end of all frequency data components using Gauss-Newton and Full Newton. From top to bottom are the reconstructed velocity models using velocity v, slowness v^{-1} and slowness squared v^{-2} parameterization, respectively. On the right from top to bottom (a), (b) and (c) are results obtained using Gauss-Newton and on the left (d), (e) and (f) are results obtained using Full Newton method obtained from noisy data with SNR=10.*

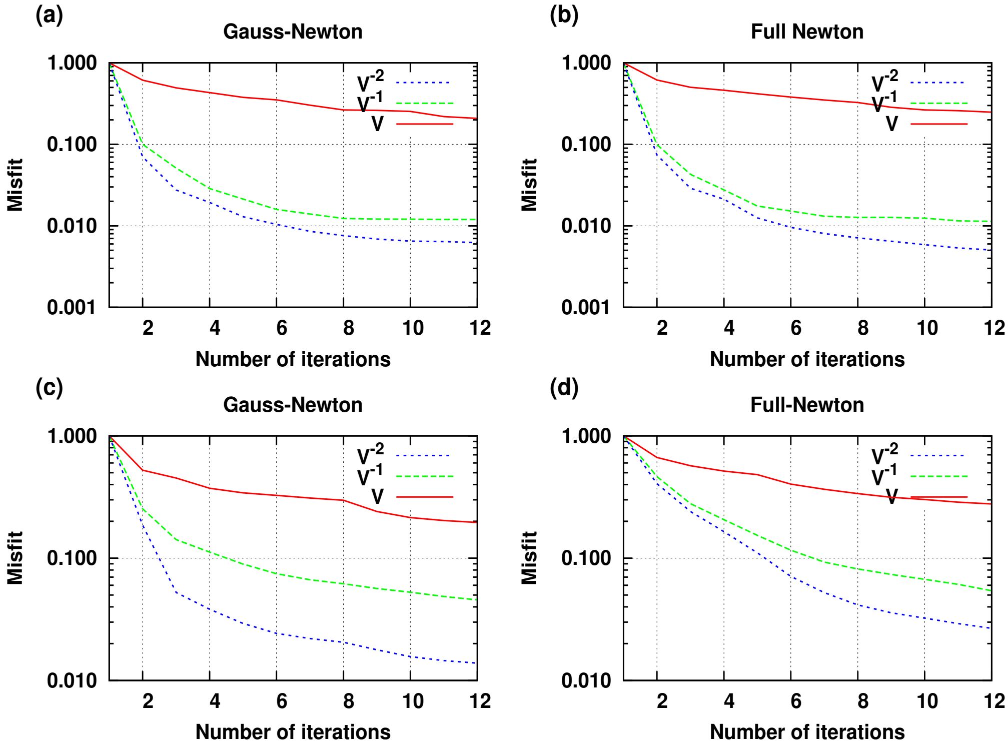

__Figure 5__ Data misfit reduction of the Marmousi velocity model for three model parameterizations: velocity v, slowness v^{-1} and square of slowness v^{-2}. On the left, from top to bottom are misfit functions for 4.70 Hz and 20.02 Hz data sets using Gauss-Newton method, and on the right from top to bottom are misfit functions using Full Newton optimization method.

crucial role in improving resolutions and amplitudes of deeper parts of the model by re-calibrating energies of gradients in shallow and deeper parts of the model. Eigenvectors analysis of the Hessian matrix show that slowness or slowness squared model parameterization makes the eigenvectors almost depth independent as compared to the case of velocity parameterization. As a result the deeper parts of the model would be properly reconstructed and superior convergence rates of Gauss-Newton and full Newton methods can be achieved by the use of slowness squared parameterization.

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