Interpolation artifacts and bidimensional ensemble empirical mode decomposition

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Summary

Scatter point interpolation plays a significant role in bidimensional empirical mode decomposition (BEMD) implementation. The type of interpolant has a large influence on the final decomposition results and should ideally be adapted to the target image. Fortunately, interpolation artifacts can be reduced by bidimensional ensemble empirical mode decomposition (BEEMD).

Introduction

The empirical mode decomposition (EMD) method developed by Huang et al. (1998) is a powerful signal analysis technique for non-stationary and nonlinear systems. EMD decomposes a signal into a sum of intrinsic oscillatory components, called Intrinsic Mode Functions (IMFs). Each IMF has different frequency components, potentially highlighting different geologic and stratigraphic information (Magrin-Chagnolleau & Baraniuk, 1999; Han & Van der Baan, 2011). Furthermore, high-resolution time-frequency analysis is possible by combining EMD with the instantaneous frequency. The resulting time-frequency resolution promises to be significantly higher than that obtained using traditional time-frequency analysis tools, such as short time Fourier and wavelet transforms (Han and Van der Baan, 2013). Furthermore, Bekara & Van der Baan (2009) utilize EMD in frequency-distance (f-x) domain to suppress the random and coherent noise.

As a 2D extension of EMD, Linderhed (2002,2005) and Nunes (2003) proposed the Bidimensional Empirical Mode Decomposition (BEMD) algorithm, which decomposes images into Bidimensional Intrinsic Mode Functions (BIMFs). Initial BIMFs contain the higher spatial and frequency information; the later BIMFs and the residual are mainly composed of slow oscillations which illustrate the major trend of the original image. Like IMFs, BIMFs are potentially helpful for signal analysis. It has for instance been used for rainfall analysis, image enhancement and geologic feature extraction (Sinclair and Pegram, 2005; Qin et al., 2008; Huang et al., 2010).

Obligatory choices in any BEMD implementation surround decisions on how to detect local extrema, how to interpolate scatter data points, and what stopping criteria to use. These decisions will impact the kind of BIMFs that are ultimately extracted. Nunes and Delechelle (2009) discuss extrema point detection based on neighboring window or various morphological operations. For the scatter data interpolation, thin-plate spline radial basis function (TPS-RBF), cubic spline, B-spline and Delaunay triangulation methods are usually used in the BEMD applications (Damerval et al., 2005; Nunes and Delechelle, 2009). Instead of scatter point interpolation, finite-element method and order statistics filter are employed to estimate the upper and lower envelopes for computation purpose (Xu et al., 2006; Bhuiyan et al., 2008). Stopping criteria controls the number of iterations thus balancing performance versus computation time.

Like EMD, mode mixing may restrict applications of BEMD. Overshoot and undershoot may occur as well depending on the type of chosen interpolant, leading to blurred and unrepresentive BIMFs.

In this paper, we first compare Delaunay triangulation, cubic and TPS-RBF interpolation methods to illustrate how they may impact analysis results. Next, we apply BEMD using these three interpolants onto seismic data to demonstrate potential pitfalls. Finally, we propose bidimensional ensemble empirical mode decomposition (BEEMD) and illustrate how it can improve image analysis by reducing interpolation artifacts.

Bidimensional empirical mode decomposition

BEMD decomposes an image into its Bidimensional Intrinsic Mode Functions (BIMFs) based on the local spatial and spectral scales. As an extension of EMD, the definition of BEMD is based on the paper of Huang et al. (1998) and the sifting process can be described as below (Linderhed, 2002; Nunes and Delechelle, 2009):

1. Find all the local maxima and all the local minima points of the image.

2. Create upper and lower envelopes by proper interpolation of the local maxima and local minima points of the image.

3. For each point, take the mean envelope of the upper and the lower envelopes.

4. Subtract the mean envelope from the input image.

5. Check the residual between the original image and the mean image; determine whether it meets the stopping criteria. If not, repeat the process from step 1 with the residual as input images. If yes, define the residual as a BIMF and subtract it from image.

6. Find next BIMF by starting over from step 1 with the residue between the image and former BIMF as input signal.

Through the sifting procedure above, the mean envelope of each BIMF is guaranteed to be zero or nearly zero, and the BIMFs are locally orthogonal, two properties which are shared with 1D IMFs. The only difference is the number of local extrema and the number of zero crossings; for EMD, the number of local extrema and the number of zero crossings must be equal or differ by at most one, however, due to the properties of an image, it is impossible to satisfy this property for BEMD (Bhuiyan et al., 2008).

Scatter point interpolation

Point interpolation plays a significant role in any BEMD implementation, and the ideal produced envelopes should go through each data point and wrap the whole image. Different interpolation methods are suited for different images. For the smooth images, the aim of interpolation is to find a stable, continuous and smooth envelope. On the other hand, for the images which have many discontinuities, interpolation should be sharp. The sharp feature guarantees to avoid overshoot and undershoot problems, whereas smooth interpolation can not.

To obtained the upper and lower envelopes, we test three interpolation methods: Delaunay triangulation interpolation (Sapidis and Perucchio, 1991), cubic interpolation and thin-plate spline radial basis function (TPS-RBF).

Figure 1(a) is a synthetic image with smooth features. The blue dots are local maxima and red dots are local minima. Figures 1(b)-1(d) are envelopes obtained using all three interpolats. The envelopes obtained by Delaunay triangulation interpolation manifests sharp and discontinuous features; the ones obtained by cubic interpolation tend to be smoother; from comparison, the envelopes of TPS-RBF show the smoothest results, as the second derivative is guaranteed to be continuous. In this case, TPS-RBF preserves the features contained in the smooth test image best.

Another synthetic image with discontinuous features is shown in Figure 2(a). In this case, cubic interpolation (Figure 2(c)) and TPS-RBF (Figure 2(d)) exhibit overshoot and undershoot artifacts; however, Delaunay triangulation (Figure 2(b)) produces the most satisfactory image.

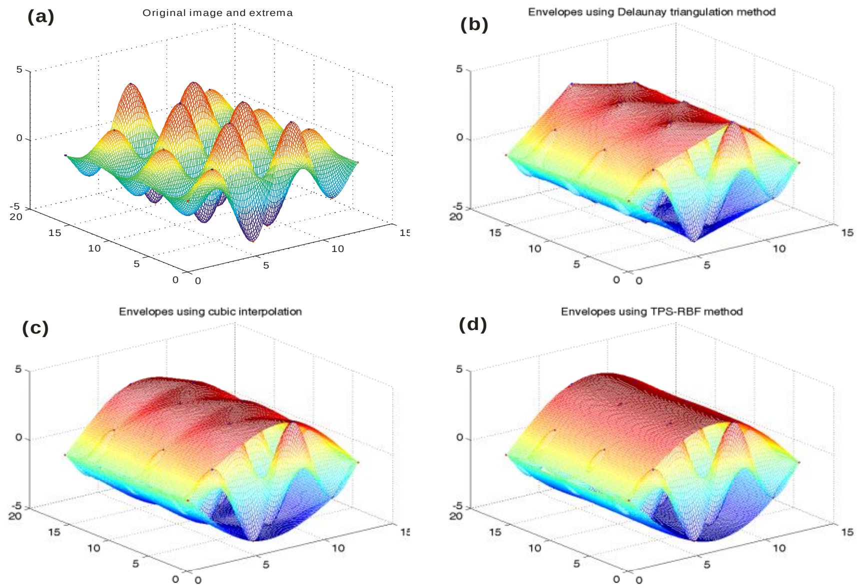

Figure 1: Smooth test image. (a). Test image. Blue dots are local maxima and red dots are local minima. (b). Delaunay triangulation creates discontinuous slopes between each triangle part; (c). Cubic spline produces the smoother envelopes; (d). TPS-RBF yields the smoothest envelopes.

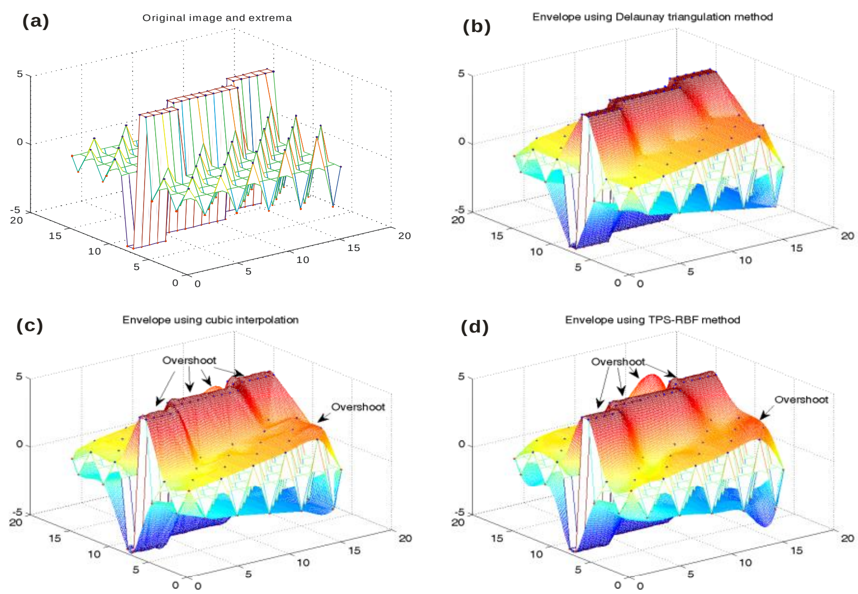

Figure 2: Test image with discontinuities. (a) Test image. Blue dots are local maxima and red dots are local minima. (b). Delaunay triangulation produces envelopes without overshoot; (c). cubic spline creates smoother envelopes with only some overshoot; (d). TPS-RBF produces the smoothest envelopes with severe overshoot.

Application of BEMD on seismic data

Figure 3 shows an image of seismic data representing two geologic subsurface features, namely a buried channel and a fault. Both features are identified by arrows. The image contains both smooth and sharply delineated features, making this a relevant test for identifying the effect of the interpolant on the resulting BIMFs.

Figure 4(a) to Figure 4(c) display the first BIMF component of BEMD using 3 different interpolation methods. The outputs from Delaunay triangulation (Figure 4(a)) and cubic interpolation (Figure 4(b)) are similar. They both highlight the channel and fault features clearly. The one from TPS-RBF (Figure 4(c)) fares less well. Overshoot and undershoot make the boundaries of the channel fuzzy, and there is no clear identification of the fault. The fault is not visible on the later BIMFs either.

In the next section we demonstrate how noise-injection using bidimensional ensemble EMD can alleviate interpolation artifacts, thereby facilitating any interpretation without the need to adapt the interpolant each time to the image.

Bidimensional ensemble empirical mode decomposition

Based on the dyadic filter bank of EMD (Flandrin et al., 2004), Wu and Huang (2009) propose the ensemble empirical mode decomposition (EEMD), which enhances the application of EMD series method. Following their idea, we propose the bidimensional ensemble empirical mode decomposition (BEEMD).

BEEMD is a noise-assisted analysis method. It injects noise into the decomposition algorithm to stabilize its performance.

The implementation procedure for BEEMD is simple:

(1). Add a fixed percentage of Gaussian white noise onto the image,

(2). Decompose the resulting signal into BIMFs,

(3). Repeat steps (1) and (2) several times, using different noise realizations;

(4). Obtain the ensemble averages of the corresponding individual BIMFs as the final result.

The added Gaussian white noise series are zero mean with a constant flat spectral and spatial spectrum. Their contribution thus cancels out and does not introduce any image components not already present in the original image, which is helpful to avoid mode mixing.

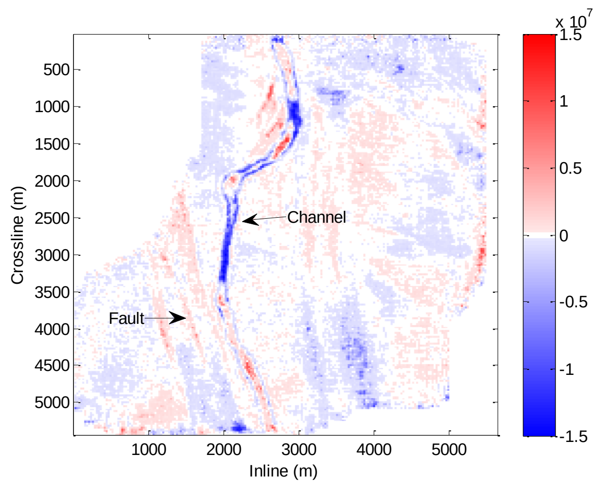

Figure 3: Seismic test image with smooth and discontinuous features. The channel and subtle fault are identified by arrows.

Application of BEEMD on seismic data

We apply BEEMD algorithm onto Figure 3 with all three interpolants using 50 noise realizations with 10% added noise. This time, all BIMF1 (Figure 4(d) to Figure 4(f)) show similar results, always identifying both the fault and channel features. Both Delaunay triangulation and cubic interpolation produce similar results to a single BEMD; yet the TPS-RBF outcome has been greatly improved by eliminating most interpolation artefacts due to overshoot and undershoot (compare with Figure 4).

Conclusions

BEMD can aid in image analysis; yet the type of chosen interpolant can greatly affect the extracted bidimensional intrinsic mode functions. Non-smooth interpolants such as Delaunay triangulation are best for images with many sharply delineated features and discontinuities. Very smooth interpolants such as TPS-RBF are superior if inherent features exhibit smooth gradients as well or if instantaneous frequencies are also desired. Cubic splines seem to cover a convenient middle road, rendering them suitable as all-purpose interpolants.

Ideally however the interpolant is adapted to each image, making automated interpretations more challenging. On the other hand noise-injection using bidimensional ensemble empirical mode decomposition (BEEMD) can alleviate many interpolation artifacts, thereby faciliting any interpretation without the need to adapt the interpolant each time to the image.

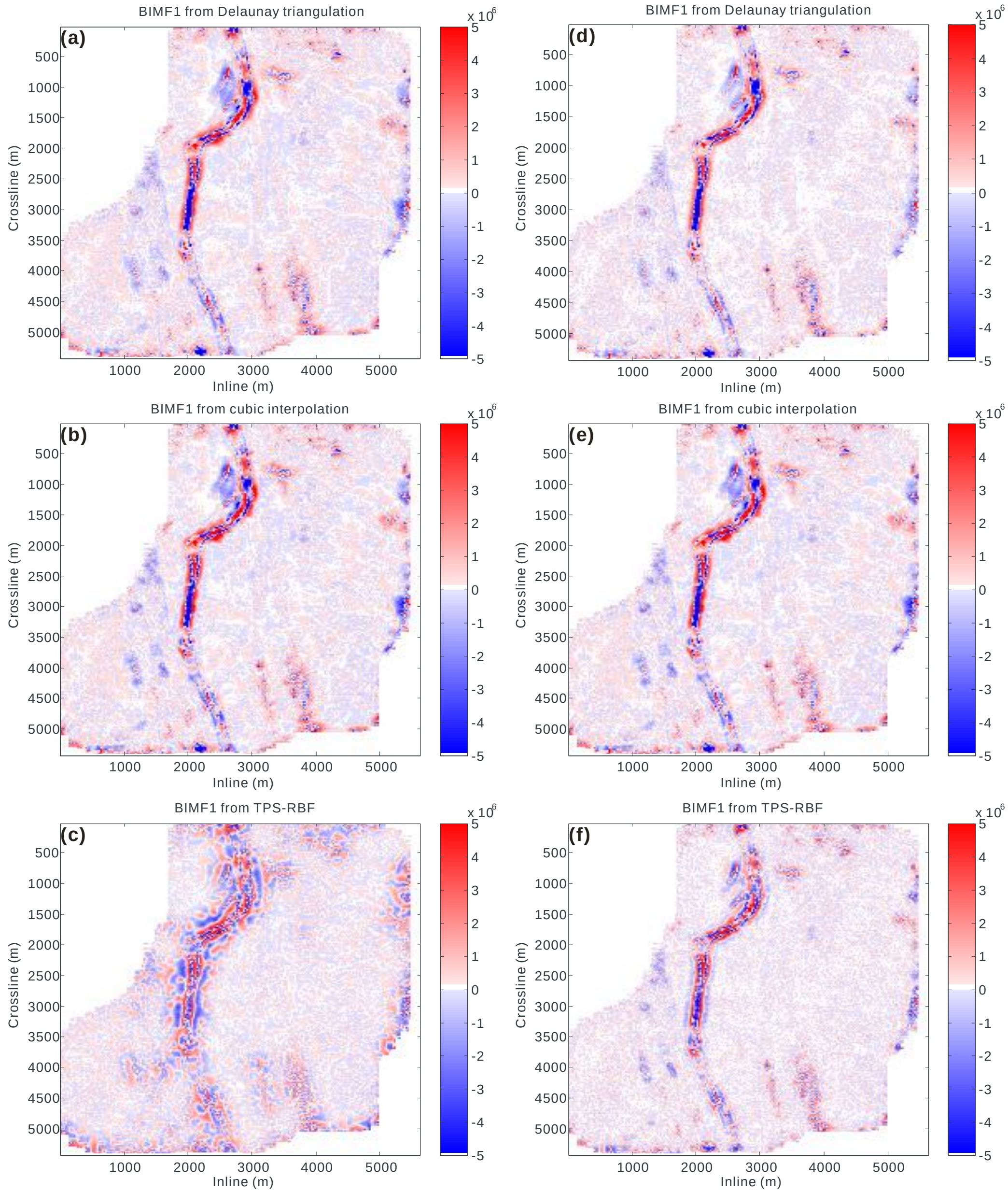

Figure 4: (a) to (c) are BIMF1 after BEMD results using Delaunay triangulation, cubic interpolation and TPS-RBF, respectively. The outputs from Delaunay triangulation and cubic interpolation highlight the channel and fault features. Overshoot and undershoot artifacts spread out the channel boundaries in TPS-RBF method. (d) to (f) are BIMF1 after BEEMD with 50 realizations using Delaunay triangulation, cubic interpolation, and TPS-RBF respectively. All three interpolants now produce similar results.

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