Integration geoConvention 2013 | geoscience engineering partnership

Maximum Entropy Acquisition Design and Optimal Interpolation

Mauricio D. Sacchi, Md. Mafijul Islam Bhuiyan, and Nasser Kazemi, University of Alberta, Edmonton, Canada
msacchi@ualberta.ca

Abstract

Acquisition design plays a very significant role in seismic data processing and imaging. An optimized seismic acquisition design requires less resources and therefore, it can reduce the total cost of seismic exploration. Non-linear optimization methods can be used to deduce the position of sources and receivers that can lead to an optimal grid. By optimal grid, we understand a *feasible* distribution of sources and receivers that is amenable of reconstruction via known wavefield restoration methods. For this purpose, we propose to maximize the entropy of the sampling function of the grid subject to acquisition constraints. Maximum entropy grids produce sampling functions that can be easily reconstructed by existing interpolators. The design of source-receiver patterns that lead to maximum entropy mid-point offset grids is a non-linear optimization problem that we would like to explore.

Introduction

Different key input parameters such as field geometry, fold, and bin size have significant influence on seismic acquisition design (Cordsen et al., 2000). In conventional land data acquisition design, the field geometry is generally assumed dense and orthogonal not only to avoid spatio-temporal aliasing artifacts but also to obtain high-fidelity and high-resolution seismic data. Classical acquisition techniques often require resources that drastically increase the total cost of the survey. Hence, we would like to design the sparsest acquisition where reconstruction algorithms can optimally operate (Liu and Sacchi, 2004; Xu et al., 2005; Abma and Kabir, 2006; Trad, 2009; Trickett et al., 2010). In recent year important advances in seismic signal reconstruction have been made. However, little understanding exits in ways of linking acquisition design to the ability of existing interpolators to reconstruct seismic data. We discuss a methodology to perturb field operations in a way sampling enables high quality reconstructions.

Entropy: A measure of grid dissorder

We first introduce the entropy of a vector via the following expression (Sacchi et al., 1994)

$$H = -\frac{1}{N\log(N)}\sum_{k=1}^{N} q_k \log(q_k) \tag{1}$$

with

$$q_k = \frac{x_k^2}{\sum_n x_n^2/N} \tag{2}$$

minimum entropy is achievable for sparse vectors and maximum entropy is achieved by random vectors x. The definition was normalized in such a way $-1 \leq H \leq 0$. We use this definition of entropy also for multidimensional structures by summing over all samples of the multidimensional signal x.

Sampling function

One can characterize a regular grid by its sampling function. For instance, we can consider a 2D lattice of regular grid points occupy by observations $d^{obs}(n,m)$ and represent it in terms of the ideal fully sampled data $d(n,m)$ as follows

$$d^{obs}(n,m) = s(n,m) \times d(n,m) \tag{3}$$

where $s(n,m) = 1$ when there is an observation and $s(n,m) = 0$ when the grid point is empty. It is possible to show that the Discrete Fourier Transform (DFT) of the observed data is related to the DFT of the data in the ideal grid via

$$D^{obs}(k_x,k_y) = S(k_x,k_y) \otimes D(k_x,k_y)\,, \tag{4}$$

where $\otimes$ is the symbol for discrete convolution and $S(k_x,k_y)$ is the sampling function. Naghizadeh and Sacchi (2010) has demonstrated that the recovery of $d(n,m)$ from $d^{obs}(n,n)$ depends on $S(k_x,k_y)$. Clearly this can be generalized to data that depends on four dimensions and time or frequency.

In Figure 1 we portrayed 2 sampling operators and their associated sampling functions. In seismic data interpolation one would like to avoid regular decimation because it leads to spectral replicas that conduce to alias. In general, spectral replicas like those in Figure 1c an d are not problematic if the signal that one would like to record is band-limited. The sampling operator and sampling function in Figures 2a and b shows a case with decimation in x and y that can lead to sampling scenarios that cannot be easily interpolated or reconstructed via known Fourier reconstruction methods (Liu and Sacchi, 2004; Xu et al., 2005; Abma and Kabir, 2006; Trad, 2009) . On the other hand, Figures 2c and d show that the same number of observations distributed randomly in the original grid lead to a sampling function that resembles the sampling function of the fully sampled data but corrupted by background noise. The sampling function in Figure 2d conduces to situations that can be easily reconstructed by Fourier methods (Naghizadeh and Sacchi, 2010) and by local directional transforms (Hennenfent and Herrmann, 2008).

Let assume we have a 2D grid that has been decimated in a regular manner in the x and y coordinates. This corresponds to the sampling operator and sampling function indicated with $j = 0$ in the Figure 3. We propose to flip occupied grid points by unoccupied grid points in a random fashion and repeat the flipping 1000 times. Figure 3 shows the evolution of the sampling operator and of the sampling function for $10, 200, 500$ and 1000 flips. The same number of grid points are occupied for all the simulations (16×16 points). The idea of flipping is to allow us to start with a regular (aliased) configuration with a fix number of observations and slowly evolve into a grid with the same number of observation but with a sampling function amenable of interpolation. Clearly, after $j = 200$ flips one see that the strong replicas associated to regular sampling in x and y have disappeared. Figure 4 shows the entropy H of the sampling function versus number of flips. In other words, we computed the amplitude of the DFT of the sampling operator $s(n,m)$ and evaluated its entropy, $H(|S(k_x,k_y)|)$. We observe that by randomly perturbing the original grid we increase the entropy of the sampling function. This makes us believe that entropy can be one of the many metrics that one could optimize for finding optimal grids.

This analysis serves to understand the importance of grid randomization to minimize alias. However, data are often recorded by field patterns that respond to logistic constraints (sail lines, cables, obstacles, etc). We would like to investigate ways of transforming data recorded in the field into grids that can guarantee an optimal reconstruction via existing interpolation methods.

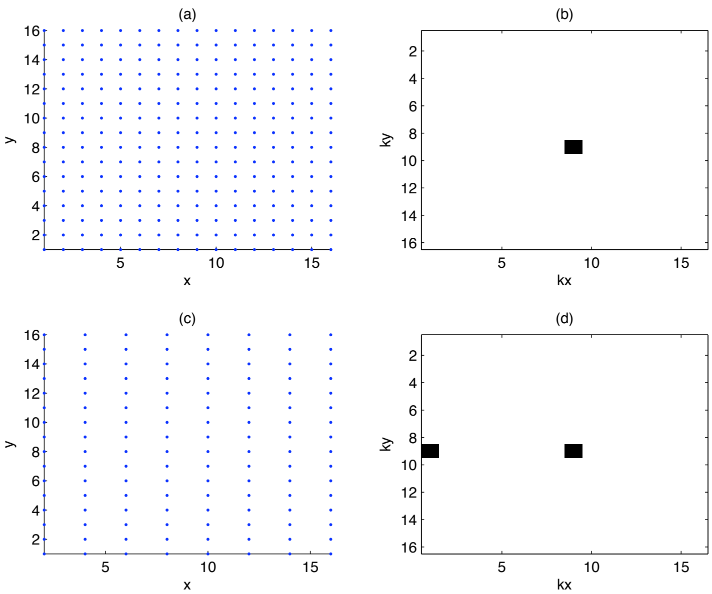

Figure 1: a) A 2D lattice where all points are occupied by observations. b) Sampling function of a). c) A grid lattice where every second x-line was removed. d) The sampling function of c).

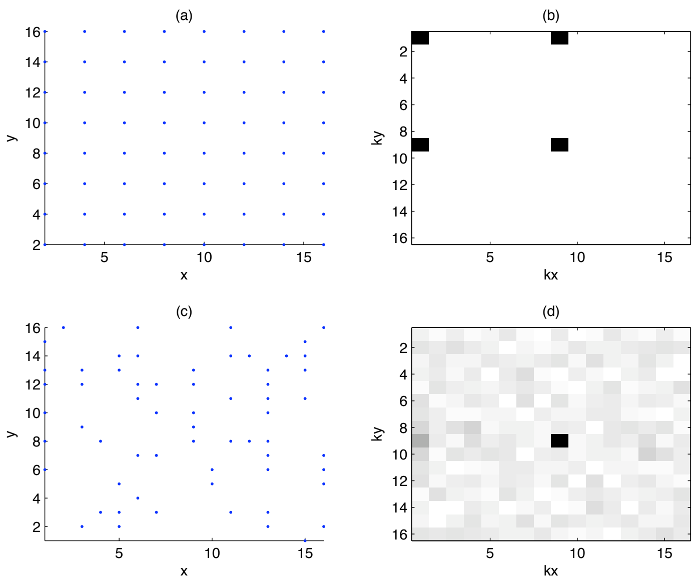

Figure 2: a) A 2D lattice where every second x and y lines were removed. The grid has 16×16 points but only $64 = 8 \times 8$ are occupied with information. b) Sampling function of a) where alias is evident. c) In this case about 64 points are occupying random positions in the original grid. d) The sampling function of c).

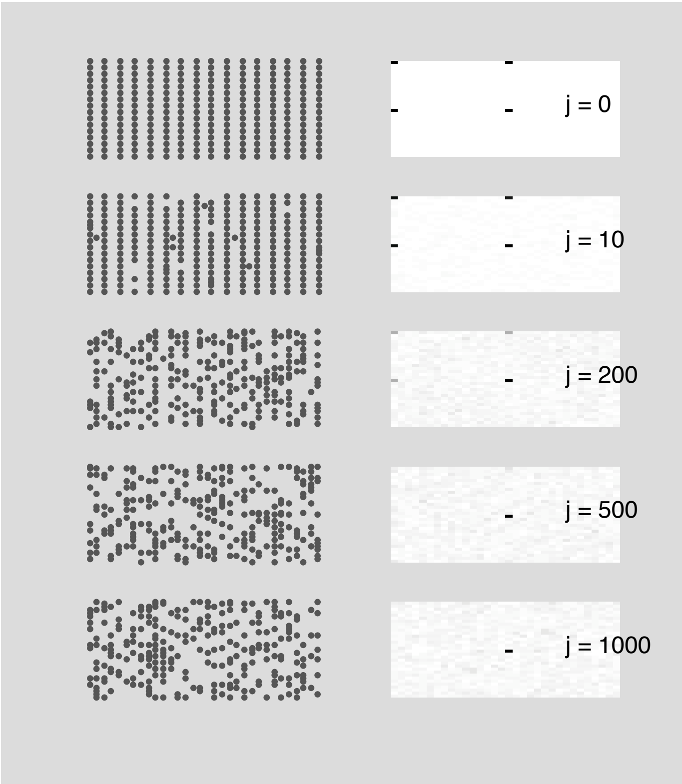

Figure 3: Grids (left) and Sampling Functions (right). We started with a grid of size 32×32 occupied by 16×16 observations. $j = 0$ indicates that three was no flipping of occupied by non-occupied points. The rest of the images indicate grids and associated sampling functions after $j = 10, 200, 500$ and 1000 random flips. Each flip entails making an occupied grid point becoming unoccupied and vice-versa. This guarantees that the number of observations is kept constant and equal to 16×16 for all cases.

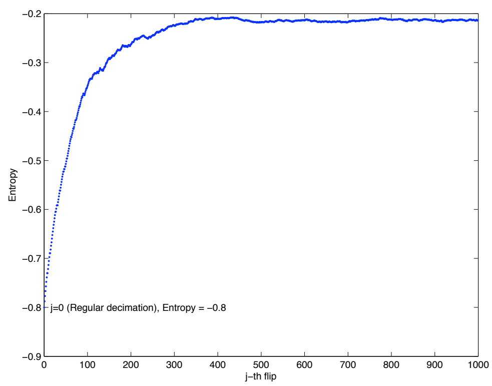

Figure 4: Evolution of the entropy of the sampling function versus number of flips. Maximum entropy grids lead to wavefield sampling scenarios that can be reconstructed by existing methods (e.g. MWNI (Liu and Sacchi, 2004), ALFT (Xu et al., 2005), POCS (Abma and Kabir, 2006) and Cadzow reconstruction (Trickett et al., 2010)).

Binning

Data are recorded in source-receiver coordinates. The transformation from source-receiver to midpoint-offset is invertible and we will call this transformation $\mathscr{G}$. When the data are transformed from source-receiver coordinates to midpoint-offset bins using for instance, bin centering, the transformation is non-invertible because more than one trace could end up in the same bin. We denote the operation of binning $\mathscr{B}$.

The geometry in the source-receiver manifold can be represented by Ω_x. The lattice Ω_x can be regular or irregular but for simplicity we assume a configuration that is doable by current acquisition practices. We also assume that Ω_x has a given number of sources and receivers occupying geographical positions. Moreover, one can also assume that not all sources have produced data that were acquired by all receivers. The survey in the midpoint-offset grid can be represented by Ω_y. Clearly, Ω_x indicates a collection of natural coordinates for sources and receivers and Ω_y indicates the position of the data in a regular midpoint-offset grid where not all grid points have been occupied by data. The transformation can be represented as follows

$$\Omega_y = \mathscr{B}\mathscr{G}\Omega_x \tag{5}$$

We can now compute the sampling function of the grid Ω_y which we denote $|S_y(\mathbf{k})|$ where $\mathbf{k}$ indicates the vector of wave-numbers associated to the DFT of the grid Ω_y. For instance, $\mathbf{k}$ can indicate midpoint inline, midpoint cross-line, offset and azimuth wave-numbers.

A cost function for acquisition design

Consider now that one has N_x traces in the original acquisition manifold Ω_x. The number of traces that are populated in the grid Ω_y is called N_y. In addition we call M_y the total size of the grid Ω_y. For instance, if the grid Ω_y is composed of 20×20 midpoints and 10 offset and 5 azimuths $M_y = 20 \times 20 \times 10 \times 5$. We now define the grid density

$$\rho_y = \frac{N_y}{M_y}$$

and the grid efficiency

$$\eta_y = \frac{N_y}{N_x}\,.$$

In an ideal scenario η_y should be close to one. However, $\eta_y = 1$ can lead to a low grid density and therefore, to a questionable reconstruction of Ω_y via our interpolators. To increase ρ_y, we need to use a coarse midpoint-offset grid that will lead to a decrease in η_y. The latter occurs because multiple traces will be placed on a given bin. In addition, one would like to force values of ρ_y to be close to a target density that permits our interpolator to reconstruct the data. For instance, MWNI (Trad, 2009) can reconstruct volumes that are populated by $\approx 10\%$ of traces. We call the desired values for these variables $\bar{\rho}_y$ and $\bar{\eta}_y$. One can pose acquisition design as the problem of finding the geometry Ω_x that maps to a grid Ω_y. Putting it all together, we now have a problem where the goal is to maximize the cost function H subject to constraints. Given a grid Ω_y, our unknowns are the distribution of data in Ω_x that properly populates the grid Ω_y. Mathematically, this can be expressed as follows

$$\begin{aligned} \text{Find } \Omega_x \text{ by maximizing} \quad & H(|S_y|) \\ \text{subject to} \quad & \eta_y - \bar{\eta}_y \approx 0 \\ \text{and} \quad & \rho_y - \bar{\rho}_y \approx 0\,. \end{aligned} \tag{6}$$

Where we remind the reader that $|S_y|$ indicates the multidimensional Fourier transform of the sampling operator. The latter is a function of Ω_x because $\Omega_y = \mathscr{B}\mathscr{G}\Omega_x$. This problem lacks of practicality because it does not consider field acquisition practices. We propose to start with an initial geometry Ω_x that is realizable in the field and find realizable source and receiver perturbations, $\Delta\Omega_x$, such that maximize H. The non-linear optimization problem can be summarized as follows

$$\begin{aligned} \text{Find } \Delta\Omega_x \text{ by maximizing} \quad & H(|S_y|) \\ \text{subject to} \quad & \eta_y - \bar{\eta}_y \approx 0 \\ & \rho_y - \bar{\rho}_y \approx 0 \\ & \Omega_x + \Delta\Omega_x \in \mathscr{F}_x \\ \text{with} \quad & \Omega_y = \mathscr{B}\mathscr{G}(\Omega_x + \Delta\Omega_x) \end{aligned} \tag{7}$$

where $\mathscr{F}_x$ is a feasible set of solutions. In other words, $\mathscr{F}_x$ is needed to avoid solutions that are not realizable as one needs to consider obstacles and source-receiver deployment and logistic constraints. We envisage using Genetic Algorithms or Simulated Annealing to solve the optimization problem.

Conclusions

The final destination of this proposal is to define an optimum acquisition design framework that will identify the smallest possible number of shots and receivers locations that guarantee proper recovery of a denser acquisition via existing interpolators. At present time we are also investigating ways of incorporating subsurface information in the acquisition design problem.

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