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Spectral decomposition with real seismic wavelet

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Summary

Spectral decomposition has been widely used in seismic interpretation. There are several approaches used, from the popular Short Time Fourier Transform (STFT) and Continuous Wavelet Transform (CWT) to less frequently used methods such as Matching Pursuit, S-transform, Chirprit Transform, Wavelet Packet Transform, etc. Each approach has its advantages and disadvantages, but they all are common in that they can be simplified as some kind of operation between the seismic data and serial kernel functions with closed form expressions. In STFT, sine/cosine and some window functions are used; in CWT, a mathematic wavelet; in S-Transform, the Gaussian function. The advantages of these methods are their speed and the capability to invert back to the original time domain from the spectral decompositions. The disadvantage of these approaches is that it may not always suit the seismic data. A method is proposed that is similar to CWT, however, instead of using a wavelet derived from a mathematic expression, an actual wavelet is extracted from the seismic data. Since the real wavelet does not have a mathematic expression, we may not be able to transform back from the spectral decomposition. However, this is regarded as a limited deficiency, since inversion to the original data may not be required in many cases. The direct benefit, compared to CWT, is that there is much less ringing or wavelet effect. This approach maintains high resolution and largely reduces the side effects of CWT. The proposed method uses an algorithm similar to CWT, wherein the seismic data is convolved with groups of dilated, squeezed, and stretched seismic wavelets. If the closed form expression of a wavelet is known, squeezing and stretching can be easily done but doing this with the discrete seismic wavelet is challenging. Special care has to be taken. The approach is demonstrated using a real seismic data set with promising results.

Theory and Method

In seismic, spectral decomposition is done by convolving a seismic trace $s(t)$ with a kernel function $h(t,\tau)$. It can be expressed as

$$S(\omega,\tau) = < s(t), h(t,\omega,\tau) >, \tag{1}$$

where *t* represents time, ω represents frequency and τ represents time shift. In STFT[1],

$$h(t,\omega,\tau) = W(t-\tau)e^{-i\omega t}, \tag{2}$$

where *W* is the window function, can be boxcar, Hanning, Gaussian etc. In S-transform[2],

$$h(t,\omega,\tau) = \frac{|\omega|}{\sqrt{2\pi}} e^{-\frac{(t-\tau)^2\omega^2}{2}} e^{-i\omega t}. \tag{3}$$

In CWT[3],

$$h(t,\sigma,\tau) = \frac{1}{\sqrt{\sigma}}\psi\left(\frac{t-\tau}{\sigma}\right), \tag{4}$$

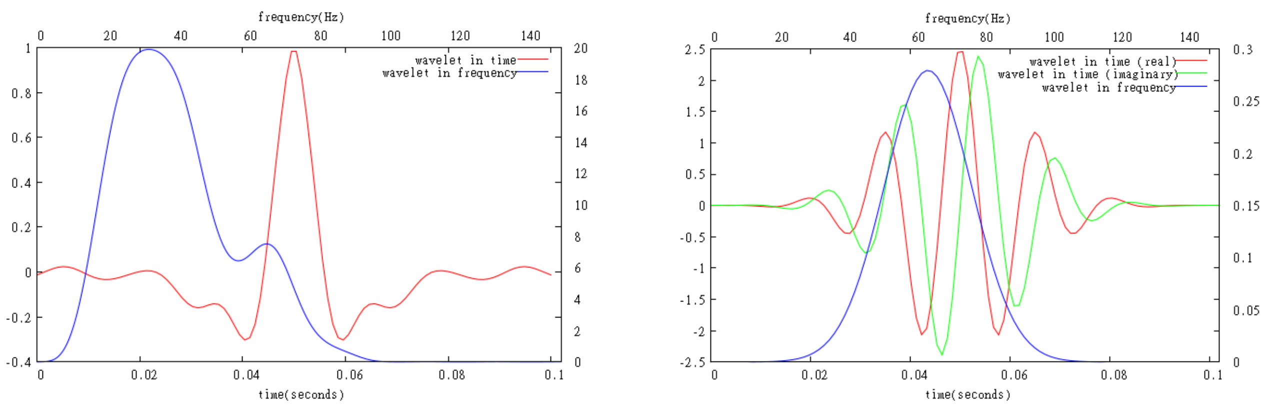
where ψ is the mathematic wavelet that can have different forms, among which Morlet wavelet ($\frac{1}{\sqrt{2\pi}} e^{i\omega_0 t} e^{-t^2/2}$) is the most popular one and Mexican hat wavelet is another frequently used one. Note that in equation (4) σ, which is called scale or scale factor, is used instead of ω. They are linked by the central frequency in interpretation. In CWT, ψ is called mother wavelet. It can be squeezed and stretched simply by changing the scale σ. This can be done in either the time or the frequency domains because of the relationship

$$x\left(\frac{t}{\sigma}\right) \leftrightarrow |\sigma| X(\sigma\omega). \tag{5}$$

By the way, ψ in equation (4) has to meet the so called admissibility condition[3],

$$\int_{-\infty}^{\infty} \frac{|\Psi(\omega)|^2}{|\omega|} d\omega < \infty, \tag{6}$$

to guarantee the convergence and invertibility. In seismic, spectral decomposition with CWT has some advantages comparing to STFT thus gradually becomes the prior option when doing the interpretation. However, its ringing and wavelet effects have also been well noticed and recognized. These artifacts occur mainly because the used wavelets do not suit seismic data in many cases. In this abstract, the author proposes a new spectral decomposition approach that uses the real seismic wavelet rather than the closed form mathematic wavelets. The real seismic wavelet is actually extracted from the seismic data and it varies with different data set. In this approach the extracted seismic wavelet are squeezed and stretched and then convolved with the seismic trace. The amount of squeeze and stretch is determined by equation (5). For example, giving a seismic wavelet with the dominant frequency 32Hz, to compute spectral decomposition at 64Hz, σ needs to be equal to 2. If σ is not an integer number, the squeeze and stretch cannot be simply done. There are different approaches to work around this. I choose to use an interpolate-resampling approach. The wavelet is first densely interpolated and then resampled according to the amount of squeeze and stretch desired.



(a) Extracted wavelet (b) Morlet wavelet

Figure 1: The extracted wavelet from Blackfoot P-wave seismic data (a) and Morlet wavelet (b).

Figure 1 (a) shows an extracted wavelet from Blackfoot (Blackfoot field near Strathmore, Alberta, Canada) P-wave seismic data in both the time (red) and frequency domains (blue). Figure 1 (b) shows a Morlet wavelet in both the time and frequency domains. Morlet wavelet is a complex wavelet, so both the real parts (red) and the imaginary parts (green) are shown. Figure 2 shows the extracted wavelet (blue) and its many squeezed and stretched versions in the time domain. Excessive squeeze will undoubtedly cause distortion. However, the stretch is relatively much safer. Once the extracted wavelet has been squeezed and stretched based on the desired frequencies used in spectral decomposition, they are subsequently convolved with the seismic trace to produce the spectra. It can be done either in the time or frequency domains.

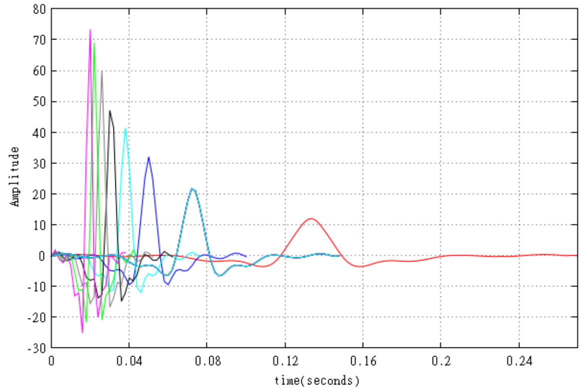
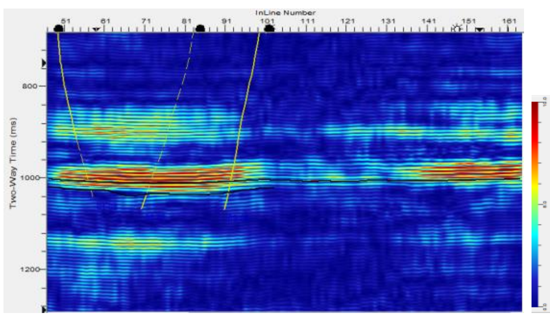


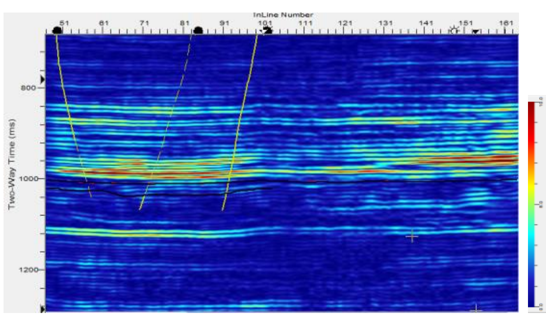
Figure 2: The extracted wavelet (blue) and its squeezed and stretched versions.

Example

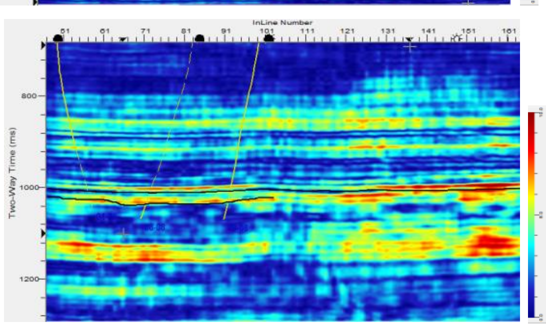
To illustrate the new approach, the spectral decomposition at 72Hz using three methods on a cross line in P-wave Blackfoot data are displayed in Figure 3. Comparing to STFT, both the new approach and CWT clearly exhibit higher time resolution at this high frequency, but the result from the new approach has much less ringing effect than the result from CWT. Pay attention to the high energy (corresponding to warm colors) events. The comparisons at other frequencies are similar.



(a) CWT result at 72Hz



(b) Result via the new approach at 72Hz



(c) STFT result at 72Hz

Figure 3: Comparison of three spectral decomposition methods on a cross line in Blackfoot.

Conclusions and discussions

A new spectral decomposition approach which uses the extracted seismic wavelet rather than the mathematic wavelets is proposed. The extracted wavelet is numerically squeezed and stretched and then convolved with seismic trace. The result shows much less ringing or side lobe effect than the result from CWT but has the same high resolution characteristics as CWT.

Since the squeeze and stretch of the wavelet in the new approach have to be done numerically, the computation cost will increase. However, the additional calculations are only needed to do once for the entire data set because the extracted wavelet will be used throughout the entire data set, therefore the increased computation cost can be ignored. In case multiple seismic wavelets are used, the increased computation cost is still very small.

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