Integration geoConvention 2013 geoscience engineering partnership

Spectral-element Simulations of Elastic Wave Propagation in Exploration and Geotechnical Applications

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Summary

The spectral-element methods (SEMs) have long been applied to model wave propagation for seismological research by providing more geometrical flexibility and accurate wave simulations for complex earth models over traditional numerical techniques such as the finite-different method. In this report, we investigate the applicability of SEM for exploration and geotechnical purposes utilizing an existing software package SPECFEM2D (http://www.geodynamics.org/cig/so ftware/specfem2d). Two examples are given including a marine survey for a subsalt structure, and wave simulations for an open-pit mine. Wavefield snapshots and synthetic seismograms are produced to illustrate the simulation results.

Introduction

One of the fundamental tasks in seismology is to solve the elastic seismic wave equations accurately. With the advancing of computational power, numerical methods for the calculation of synthetic seismograms have been contributing significantly to seismological research since the late 70's (Chaljub et al., 2007). Among numerical approaches used to model the propagation of seismic waves, the finite-difference (FD) methods are the most widely used (Komatitsch, Tsuboi, & Tromp, 2005) due to its simple formulation. They employee the differential form of the equation of motion and essentially replace derivatives of velocity vectors and stress tensors by their finite-difference equivalents (Levander, 1988; Virieux, 1986).

However, classical methods such as FD and pseudo-spectral methods have inherent limitations with velocity models that exhibits complex geometries, including undulated surface topography and major internal discontinuities, which although could be overcome, is numerically costly (Komatitsch et al., 2005; Robertsson, 1996).

In comparison, finite element methods (FEMs), another category of numerical techniques in the space-time domain, honor the geometrical model complexity by sophisticated mesh design and have been widely used to predict natural phenomena (Bathe, 2008). The basic idea of FEMs is to mesh the whole model into a set of finite elements, derive a set of relationships for the momentum equation over the subspace defined by the element set, and assemble these relations into a linear system to solve for the approximate wavefield values on the whole model (Reddy, 2005). Spectral-element methods (SEMs) are essentially higher degree finite-element methods. By providing more accurate wavefield presentations, they are especially appropriate for problems of seismic wave propagation.

Theory and/or Method

In FEMs and SEMs, the integral formulation is applied by first taking the dot product of the momentum equation with an arbitrary test vector $\mathbf{w}$, and then integrating by parts over the volume $\boldsymbol{\Omega}$ to impose the boundary conditions (Komatitsch et al., 2005).

$$\int_{\Omega} \rho \mathbf{w} \cdot \partial_t^2 \mathbf{s}\ d^3\mathbf{x} = -\int_{\Omega} \nabla \mathbf{w} : \mathbf{T}\ d^3\mathbf{x} + \int_{\Sigma_s} \mathbf{m}(\mathbf{x}_s, t) : \nabla \mathbf{w}(\mathbf{x}_s) d^2\mathbf{x}_s + \int_{\Gamma} \mathbf{n} \cdot \mathbf{T} \cdot \mathbf{w}\ d^2\mathbf{x}$$

The three terms on the right hand side of this equation involve contributions from stress, source, and tractions on boundaries respectively ($\mathbf{n}$ is the outward normal unit vector and on the free surface $\partial\Omega$, $\mathbf{n} \cdot \mathbf{T} = \mathbf{0}$) (Komatitsch et al., 2005). It is clearly from the integral formulation above that in FEMs/SEMs, free surface condition is satisfied naturally through the boundary integration term (Komatitsch et al., 2005; Tromp, Komatitsch, & Liu, 2008). Another advantage of the FEMs/SEMs is that it can overcome the limitations of geometrical complexity of 3-D regional velocity models, including the variations of surface topography and major internal discontinuities (Komatitsch et al., 2005). For global wave propagation, no absorbing boundary condition needs to be applied (Komatitsch et al., 2005); however in local or regional simulations, which is the usual case for exploration and mining cases, absorbing boundary conditions are applied on all edges except the top free surface.

In this report, a 2-D SEM based package, SPECFEM2D is applied to velocity models with fluid layers, complex geometry, and/or undulations of major discontinuities and illustrates the ability of SEM in handling various scenarios in industrial applications. Moreover, even though only elastic media are used for the given examples, SPECFEM code has the capability to further simulate wave propagation in viscoelastic and poroelastic media.

Generating mesh is the first as well as the critical step of the modeling processes for FEM/SEM (Casarotti et al., 2007). Generally, a stable 2-D SEM simulation requires an unstructured all-quadrilateral (all-hexahedral in 3-D case) conforming mesh with grid sizes constrained by required simulation accuracy. Incremental time stepping duration is in turn determined by the model structure and mesh grid sizes. (Casarotti et al., 2007). Meshing for models with simple and regular geometries may be performed within the SPECFEM package; however, meshing software such as CUBIT (Sandia National Laboratory, http://cubit.sandia.gov) is necessary to generate high quality quadrilateral mesh for models with geometrical complexity.

Examples

First, SEM simulations of models with increasing model complexity are explored for exploration applications. In Example 1 (Fig 1), a marine survey scenario with a subsalt structure underneath the ocean floor is presented. The 2-D meshes designed through CUBIT illustrate the advantages of SEM in accommodating fluid layer, abrupt changes in internal boundaries, and asymmetric geometry. The first simple model includes no water layer, the second model is simulated with a flat sea floor, and the third model includes both water layer and an uneven sea floor. Taking into account the velocity variations, we meshed the structures with different element sizes (See Fig 1 for velocity model information). A Ricker pressure source was located 2 meters below the free surfaces with a source frequency of 50 Hz in all simulations. Snapshots at time t=0.74 second for all simulations are shown on the right panels with wavefronts identified and labeled. The effects of water layer and the salt-dome geometry are clearly demonstrated by differences in the advance of various wavefronts.

Example 2 (Fig 2) simulates an open-pit mine scenario, where the complex surface topography is adapted by the SEM through the unstructured mesh design. In this case, the seismic source is a Ricker pressure source with 100 Hz frequency, and located at 626 m depth. For comparisons, we simulated three 2-D models: a) a flat surface model, b) a bowl shaped open-pit model, c) a staircase open-pit model. The open pits of models b) and c) are approximately 1.6 km wide and 426 m deep. In the staircase model, the general bench width is 15 m, while the working bench width is 40 m; the bench height is 17.8 m, the bank width is 12 m, and thus the face angle is 56 degrees. The same meshing element size is used for the three models. Synthetic seismograms (X-components of particle velocities) are calculated, along with peak particle velocity (PPV) for P-waves at surface stations (red dots in Fig 2), which may be used in further analysis of seismic hazards. The models used in these cases are fully elastic with no attenuation. However, due to geometrical spreading effect, wave amplitudes decrease with increasing distance from the source, as proven by the PPV plots where the further a station is from the source, the smaller PPV is observed. PPV lateral variations due to surface topography are clearly shown as well.

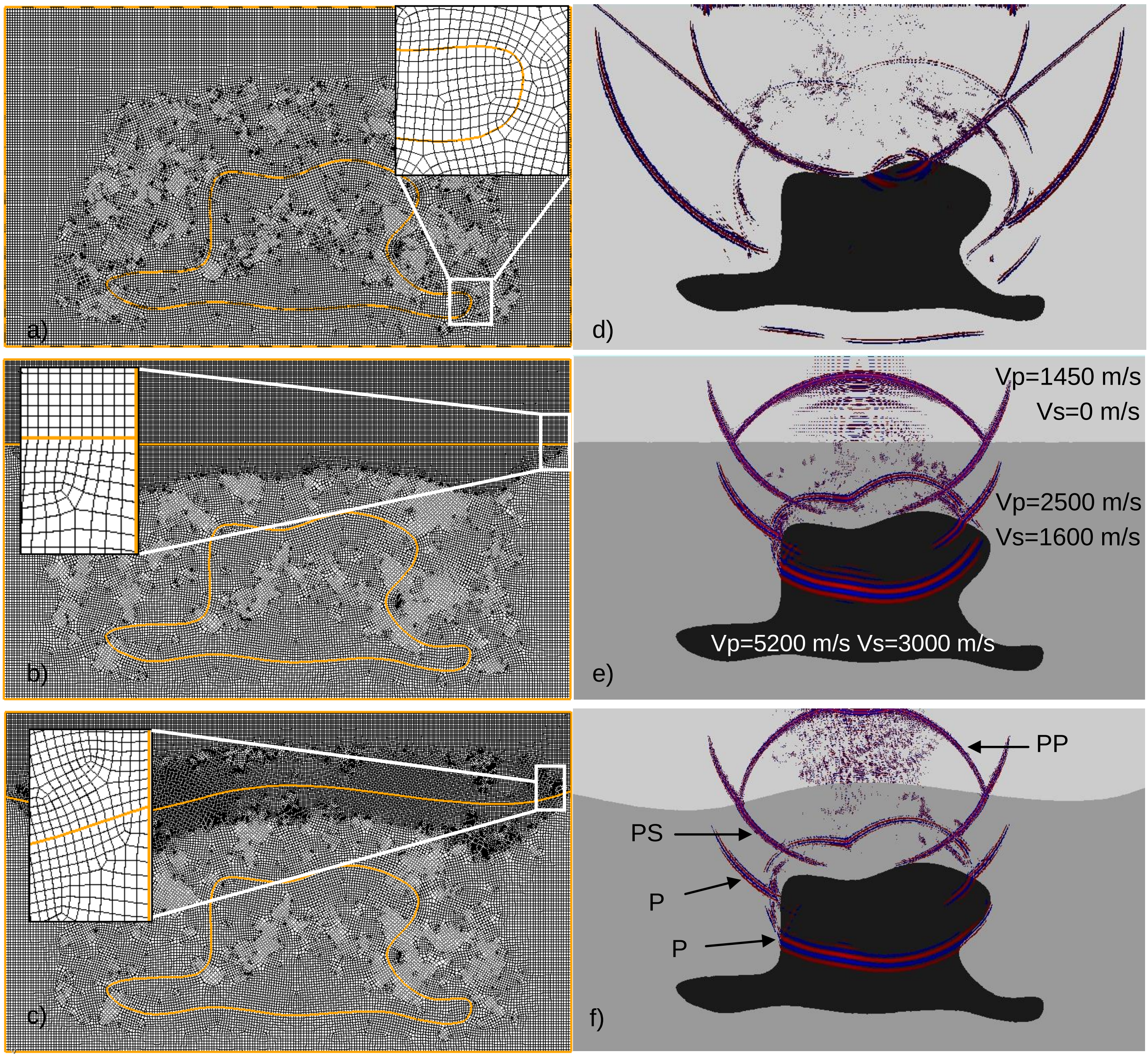

Figure 1: Subsalt simulations; the left column shows three meshed models with increasing complexity, and the right column shows their corresponding SEM simulation snapshots at t=0.74 s; the sizes of all models are 4000 m X 2400 m.

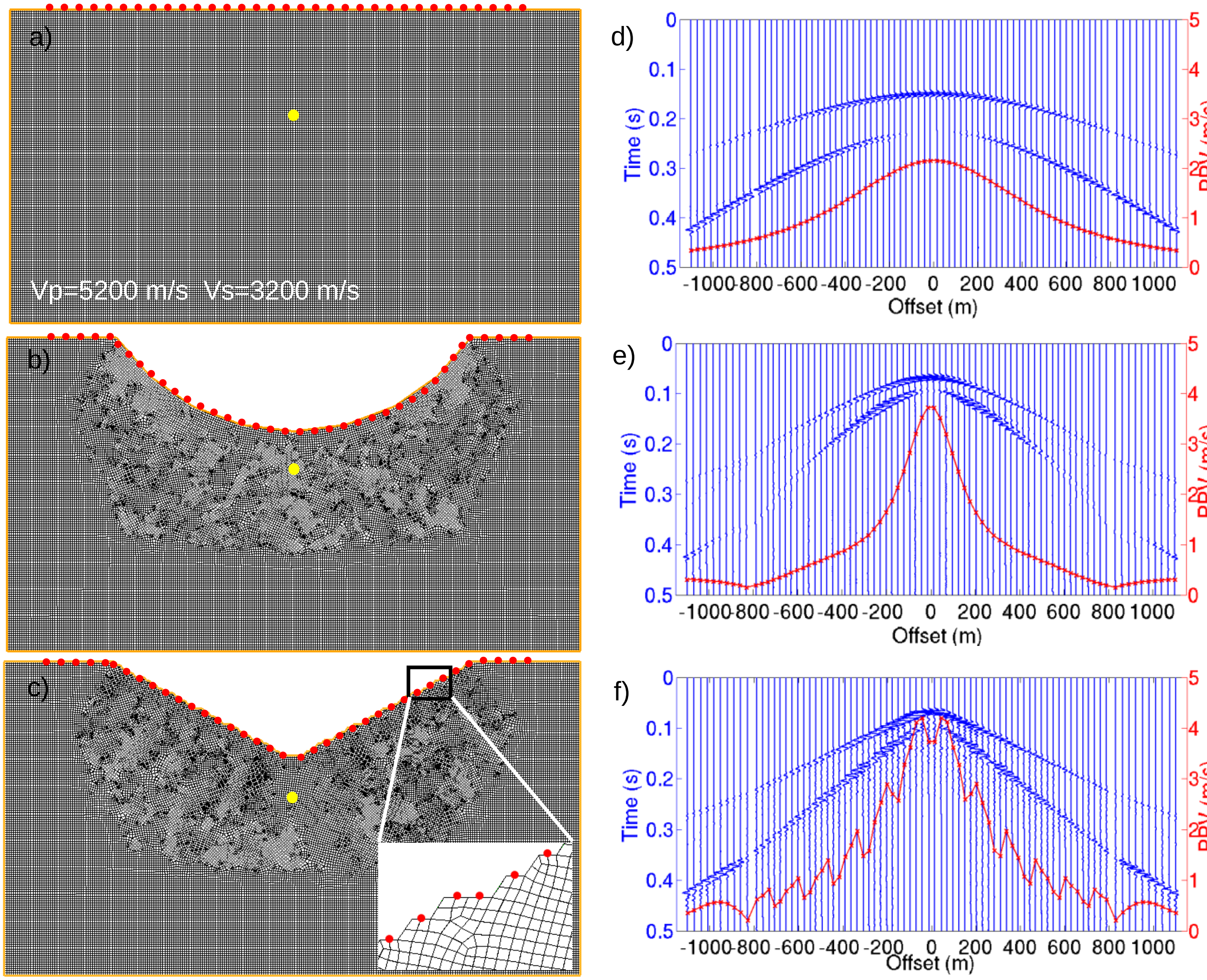

Figure 2: Open-pit mine simulations; the left column illustrates three meshed models with increasing complexity of surface topography, and the right column shows their corresponding calculated peak particle velocity (PPV) and synthetic seismograms; red dots indicate the receiver locations, and yellow spots highlight the source locations; sizes of all models are approximately 2616 m X 1427 m.

Conclusions

This study shows that spectral-element methods can be naturally applied to wave simulations for exploration and geotechnical applications, due to their flexibilities in dealing with model complexities, such as the presence of fluid layer, highly irregular surfaces, and abrupt boundary variations. They have demonstrated great potential in solving industrial problems more accurately and effectively than classical FD methods. SEMs may also prove to be an essential tool for our future simulations of realistic 3-D models.

Acknowledgements

This research is supported by NSERC and the SUMIT ORF program. SEM simulations in this report are performed based upon SPECFEM package (http://www.geodynamics.org/cig/software/specfem2d). We would like to thank the developers of SPECFEM2D for the continuing support we receive.

References

Bathe, K. J. (2008). The Finite Element Method. In B. W. Wah (Ed.), *Wiley Encyclopedia of Computer Science and Engineering* (pp. 1253-1264): John Wiley & Sons, Inc.

Casarotti, E., Stupazzini, M., Lee, S. J., Komatitsch, D., Piersanti, A., & Tromp, J. (2007). *CUBIT and Seismic Wave Propagation Based Upon the Spectral-element Method: An Advanced Unstructured Mesher for Complex 3D Geological Media.* Paper presented at the 16th International Meshing Roundtable, Seattle, Washington, U.S.A. .

Chaljub, E., Komatitsch, D., Vilotte, J.-P., Capdeville, Y., Valette, B., & Festa, G. (2007). Spectral element analysis in seismology. *IASPEI Monograph on Advances in Wave Propagation in Heterogeneous Media, 48*, 32.

Komatitsch, D., Tsuboi, S., & Tromp, J. (2005). The spectral-element method in seismology. In G. a. L. Nolet, A. (Ed.), *The Seismic Earth* (pp. 205). Washington DC: AGU.

Levander, A. R. (1988). Fourth-order finite-difference P-SV seismograms. *Geophysics, 53*(11), 12.

Robertsson, J. O. A. (1996). A numerical free-surface condition for elastic/viscoelastic finite-difference modeling in the presence of topography. *Geophysics, 61*(6), 14.

Tromp, J., Komatitsch, D., & Liu, Q. (2008). Spectral-Element and Adjoint Methods in Seismology. *COMMUNICATIONS IN COMPUTATIONAL PHYSICS, 3*(1), 32.

Virieux, J. (1986). P-SV wave propagation in heterogeneous media: Velocity-stress finite-difference method. *Geophysics, 51*(4), 13.