A framework for time-lapse AVO when contrasts are large

Shahin Jabbari*, University of Calgary, Calgary, Alberta, Canada
sjabbari@ucalgary.ca
and
Kris Innanen, University of Calgary, Calgary, Alberta, Canada

Summary

We apply perturbation theory in a treatment of the time-lapse amplitude-variation-with-offset (AVO) modeling problem. In it we express difference data, calculated from a baseline survey and a monitor survey, as an expansion in orders of both baseline interface properties and time-lapse changes. The linear forms are equivalent to those derived and used by Landro (2001), and higher order terms represent corrections appropriate for large contrasts. We conclude that (1) in many plausible time-lapse scenarios a meaningful increase in accuracy derives from inclusion of higher order corrections, and (2) coupling between baseline and time-lapse quantities is non-negligible when contrasts are large.

Introduction

Scattering theory has been used widely in many applications in seismology including time-lapse inversion (Zhang, 2006). In this paper we derive a self-consistent elastic perturbative treatment of the time-lapse AVO modeling problem, which is closely connected to scattering.

The behaviour of a reservoir can change over time due to production or employing enhanced oil recovery techniques to restore formation pressure and improve the fluid flow. Monitoring these changes with time-lapse seismic methods, facilitates management of a reservoir and extends the useful life of an oilfield. In time-lapse monitoring a baseline survey and one or more monitor surveys, are acquired over an interval of time during which the geological/geophysical characteristics of a reservoir change (Lumley, 2001). Changes in the pressure or fluid saturation in a reservoir can be an indicator to determine the difference data between the baseline and monitor survey. Time-lapse amplitude variations with offset (AVO) methods have been applied to analyze these changes (Landro, 2001).

Theory and Methods

Time-lapse AVO connotes the analysis of changes to the offset or angle dependence of reflection coefficients from the baseline to the monitor survey. Consider an incident P wave striking the boundary between two elastic media which are incidence medium and reservoir with rock properties $V_{P0}$, $V_{S0}$, $\rho_0$ (above) and $V_{PBL}$, $V_{SBL}$, $\rho_{BL}$ (below). The reservoir properties change to $V_{PM}$, $V_{SM}$, $\rho_M$ in the monitor survey.

Reflection coefficients for the baseline and monitor survey are calculated by solving the Zoeppritz equations, once with the target properties [BASELINE] and once with the target properties [MONITOR]. Difference reflection coefficients are determined by subtracting the baseline reflection data from the monitor reflection data.

To model this difference in a physically interpretable way, we introduce two groups of perturbation parameters; perturbations “a” representing the change from the incidence medium to the target medium
in the baseline survey, and perturbations “b” representing target medium changes from the baseline to monitor survey:

\[
\begin{align*}
a_{VP} &= 1 - \frac{V_{P}^2}{V_{P}^{2BL}}, \quad a_{VS} = 1 - \frac{V_{S}^2}{V_{S}^{2BL}}, \quad a_{\rho} = 1 - \frac{\rho_{0}}{\rho_{BL}}, \\
b_{VP} &= 1 - \frac{V_{P}^2}{V_{P}^{2M}}, \quad b_{VS} = 1 - \frac{V_{S}^2}{V_{S}^{2M}}, \quad b_{\rho} = 1 - \frac{\rho_{BL}}{\rho_{M}},
\end{align*}
\]

Substituting these perturbations into the two requisite instances of the Zoeppritz equations (modeling the baseline and the monitoring reflection amplitudes), we derive a series expansion for the difference data reflection as follows.

\[
\begin{bmatrix}
R_{PP} \\
R_{PS} \\
T_{PP} \\
T_{PS}
\end{bmatrix}
_{BL} = b_{BL} \begin{bmatrix}
R_{PP} \\
R_{PS} \\
T_{PP} \\
T_{PS}
\end{bmatrix}
_{PP}^{BL}(\theta) = \frac{\det(P_P)}{\det(P)}
\begin{bmatrix}
R_{PP} \\
R_{PS} \\
T_{PP} \\
T_{PS}
\end{bmatrix}
_{M} = b_{M} \begin{bmatrix}
R_{PP} \\
R_{PS} \\
T_{PP} \\
T_{PS}
\end{bmatrix}
_{PP}^{M}(\theta) = \frac{\det(P_P)}{\det(P)}
\]

Where \(P_{BL} \) and \(P_{M} \) are Zoeppritz metrics for the baseline and monitor survey. \(P_{P} \) is the P matrix with the first column replaced by the vector b. R’s and T’s are reflection and transmission coefficients for PP and PS waves.

\[
\Delta R_{PP}^{(M)}(\theta) = R_{PP}^{(M)}(\theta) - R_{PP}^{(BL)}(\theta)
\]

\[
\Delta R_{PP}^{(1)}(\theta) = \Delta R_{PP}^{(1)}(\theta) + \Delta R_{PP}^{(2)}(\theta) + \Delta R_{PP}^{(3)}(\theta) + \ldots
\]

\[
\Delta R_{PP}^{(1)}(\theta) = \frac{1}{4} (1 + X^2) b_{VP}^2 - 2(BX)^2 b_{VS} + \left( \frac{1}{2} - 2(BX)^2 \right) b_{\rho}
\]

\[
\Delta R_{PP}^{(2)}(\theta) = \frac{1}{4} (1 + X^2) b_{VP}^2 + (B^2 X^2 - 2(BX)^2) b_{VS}^2 + \left( \frac{1}{4} - \frac{1}{4} B^2 X^2 - (BX)^2 + B^3 X^2 \right) b_{\rho}^2
\]

\[
+ (2B^3 X^2 - (BX)^2) b_{\rho} b_{VP} + (2B^3 X^2 - (BX)^2) a_{VP} b_{VS} + (\frac{1}{4} X^2) a_{VP} b_{VP}
\]

\[
+ (2B^3 X^2 - (BX)^2) a_{\rho} b_{VS} + (2B^3 X^2 - (BX)^2) b_{\rho} a_{VS} + (2B^3 X^2 - \frac{1}{2} BX^2) a_{\rho} b_{\rho}
\]

Where \(X = \sin(\theta) \) and \(B = \frac{V_{S0}}{V_{P0}} \)
Numerical behaviour of truncations of the difference AVO series

We will complete this initial discussion with an examination of the numerical influence of the low order nonlinear terms of the series. The linear term can be shown to be equivalent to the form used by Landro (2001). We conduct a numerical test using the same parameters used by Greaves and Fulp (1987). In their research, a 3-D seismic survey had been provided over a period of 15 months on the Holt sand reservoir. There was an increase in gas saturation which caused a measurable decrease in elastic parameters, approximately 5% in density, and 15%-35% in the velocity. In Figure 1 we plot the exact difference reflection coefficient associated with such a change (black curve). This curve is compared with the linear (blue curve) and higher order approximations (red and green curves) embodied in the equations above (Figure 1). The second and third order approximations are in a good agreement with the exact difference in the pre-critical regime. Since we truncate our approximations beyond first order in $\sin^2\theta$, this is expected and serves to define the domain of their applicability (higher order terms in the incidence angle can be used if so desired. We conclude that the nonlinearity of the relationship between the difference reflection coefficient and perturbations in both the baseline medium and the time-lapse changes may be significant and non-negligible in geophysically plausible scenarios.

Figure 1: $\Delta R_{pp}$ for the exact, linear, second order, and third order approximation. Elastic incidence parameters: $V_{P0} = 3000$ m/s, $V_{S0} = 1500$ m/s and $\rho_0 = 2.000$ gm/cc; Baseline parameters: $V_{PBL} = 4000$ m/s, $V_{SBL} = 2000$ m/s and $\rho_{BL} = 2.500$ gm/cc; Monitor parameters: $V_{PM} = 3400$ m/s, $V_{SM} = 1700$ m/s and $\rho_{M} = 2.375$ gm/cc.

Conclusions

Perturbation theory can be used to pose time-lapse seismic monitoring problems in a quantitative and interpretable and easily analyzable way. Forms for elastic difference reflection coefficients that closely resemble standard linearized AVO equations are derivable, with nonlinear corrections that include coupling terms between baseline and time-lapse changes.

Numerical studies indicate that in geophysically plausible (though reasonably large-contrast) scenarios these nonlinear terms can have significant impact in pre-critical regimes.

Ongoing research includes validation of these methods with synthetic and laboratory data, and recasting in terms of relative changes $\Delta V_p/V_p$, $\Delta V_s/V_s$, and $\Delta \rho/\rho$, which is more commonly used in AVO analysis. Future research includes incorporating these results in our growing body of general scattering/perturbation theoretic treatment of time-lapse seismic modelling and inversion.

Acknowledgements

We wish to thank sponsors, faculty, and staff of the Consortium for Research in Elastic Wave Exploration Seismology (CREWES) for their support of this work.

References

