Kirchhoff Imaging with Adaptive Green’s Functions for Compensation for Dispersion, Attenuation, and Velocity Imprecision
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Summary
Wide band seismic surveys present the challenge of imaging properly at all frequencies. Dispersive velocity fields, imprecision in the velocity field, and waveform changes due to attenuation all degrade the imaging at higher frequencies. Here I present a method for deriving and applying adaptively a short, white operator to compensate for these challenges, and to effect a better image at the higher frequencies.

Introduction
The seismic surveys acquired for KOSP (Kai Kos Dehseh Oil Sands Partnership) in northern Alberta use a wide frequency band for the Vibroseis sweep, from 8 Hz up to either 180 or 210 Hz. It is reasonable to assume that across such a wide frequency band the seismic velocities may differ.

Commonly, attenuation compensation is used to adjust the wavelet to one which all frequencies appear to propagate at the velocity of the asymptotic high frequency. If we know the attenuation constant ‘Q’, and if the model for attenuation is strictly true (e.g. linear energy loss with particle motion), then we could precisely align the velocities at all frequency. Even if we could do this, the uncertainty of the velocity in the presence of noise at the dominant frequency (typically about 60 Hz on this survey) might be significant relative to the period at the higher frequencies. An error of ⅓ period at the dominant frequency would be a full period at the upper frequency limit. Such an error would not allow the higher frequencies to image properly.

Commonly, the greatest attenuation occurs in the shallowest layer. Surface-consistent deconvolution attempts to compensate for the different wavelet after having passed through the near surface. However, there are difficulties with surface-consistent deconvolution arising from the difficulty in determining the spectrum of the signal in noise-contaminated data. Some attempts have been made to recover improved spectral estimates of the signal (Barrett, 2008), but the problem remains difficult.

The approach taken in this presentation is to modify the Green’s function within a Kirchhoff prestack migration by deriving an optimal matching operator between a previously derived pilot image trace and every input trace at every image trace position. This matching operator is then modified to maintain the spectral characteristics of the input data, that is, a phase-only transformation is derived. These separate operators are applied to the input trace before the Kirchhoff summation.

Theory and Method
A specific attenuation model or dissipation process could provide a relation describing the velocities as a function of frequency. This may be sufficient to define the velocity relationship to be used for migration, or it might not. Inherent simplifications in the physical model, unknown inhomogeneities and
anisotropies in the actual earth, and errors in parameter estimation may yield velocity relationships which do not correspond to the observed data. Rather than using a deterministic approach, here I will use a data adaptive approach to imaging across the spectrum. Consider the Kirchhoff migration integral

$$U(x) = \iiint \sigma R(x_s, x, t) G(x, x_s, x_r, t, v) \, dt \, dx_s \, dx_r$$

where $x_s$ and $x_r$ are the source and the receiver locations; $x$ denotes an image point within the acquisition surface $\Sigma$; $v$ is the velocity field; $R$ is the recorded data; and $G$ is the Green’s function, relating the surface to the image point in terms of traveltimes as well as compensating for amplitude changes along the raypath.

Commonly the Green’s function is taken to be a weighted delta function $V(x, x_s, x_r, v) \delta(t - \tau(x, x_s, x_r, v))$ which selects the value of the recorded trace that would correspond to the computed traveltimes from source and receiver points to the image points. The Green’s function may also contain an inverse wavelet, and compensation for oblique raypaths. It may contain an operator which deterministically compensates for a dispersive velocity field, although this is not common. It may also contain the phase shift due to passage of the ray through caustics using a calculation of the KMAH (Keller, Maslov, Arnold and Hörmander) index.

Here I instead define the Green’s function to be a short convolutional operator derived using a set of input pilot image traces. The pilot uses a previous conventional migrated volume, decimated to spikes at local extrema in order to include a subset of the high frequency component of the data.

Let $H_x(t, z)$ be the pilot image trace at the surface position including the image point $x$. This can be either with respect to time or depth, depending on the output space of the migration. I shall use depth $z$ in the discussion below. Then we demigrate this pilot image trace using the inverse of the Green’s function delta function. This operation just remapping the travel times in the opposite direction as the migration of the input to the image trace. Let $Y_{sr}(t)$ be the demigrated version of $H_x(z)$:

$$Y_{sr}(t) = \int H_x(z) V^{-1}(t) \delta(t - \tau(z, x_s, x_r, v)) \, dz$$

Then for every sample on this trace which is not a peak or trough, I replace the sample with a zero. This produces a trace $\hat{Y}_{sr}(t)$ containing an approximate subset of the reflectivity, and including high frequency within the trace.

Next I derive a Wiener matching operator $W_{sr}(t)$ which optimally transforms the recorded trace $R(x_s, x_r, t)$ into the modified demigrated pilot trace $\hat{Y}_{sr}(t)$, with a specified small number of samples in the operator. $\| R_{sr} \otimes W_{sr} - \hat{Y}_{sr} \|^2$ is minimized.

Next I derived a modified matching operator $\hat{W}_{sr}(t)$ which only applied a phase transformation to the data, without modifying the amplitude spectrum. This is accomplished by taking the Fourier transform of $W_{sr}$, scaling each complex Fourier component by the inverse of its complex amplitude, and inverse Fourier transforming.
The operator $\hat{W}_{srx}(t)$ is applied to the recorded trace $R(x_s, x_r, t)$ before the migration and Kirchhoff summation. So the modified Kirchhoff migrated image trace $U_x(z)$ is given by

$$U_x(z) = \iiint R(x_s, x_r, t) \otimes \hat{W}_{srx}(t) V(t) \delta(t - \tau(x(z), x_s, x_r, v)) \, dt \, dx_s \, dx_r.$$  

The matching operator may be considered to be combined as part of the Green’s function, but for implementation purposes, the operator is applied by convolution, and the delta function as a trace resampling.

Note that there is a separate convolutional operator for every input trace and image trace pair. In the Kirchhoff migration each input trace is migrated onto every image position in the output canvas, and each pair will have distinct operators. It is very reasonable to derive the operators in a surface-consistent manner, with separate source and receiver operators for each surface station in the acquisition, but that approach has not been applied here.

**Examples**

A prestack Kirchhoff time migration module was written incorporating the usual software design. Field data traces are input to the module in any order. The spatial and temporal extents of the output image canvas are specified by user parameters. As the traces are read, the partially migrated images are gradually accumulated. Partial image datasets are periodically output for disaster recovery. Parallelization may be accomplished either by partitioning the input dataset, by partitioning the output canvas, or some combination of the two.

Provision was made in the PSTM for orthorhombic isotropy in the velocity field, although accurate estimation of the azimuthal component is difficult and the estimate made from offset-vector tile migration did not significantly alter the imaging in our case. Only vertical transverse isotropy (VTI) was used, with an effective $\eta$ incorporating both the effects of the anisotropy and the curved raypaths of the depth-dependent velocity.

Below is a sample line from a standard PSTM on a dataset acquired over KOSP oil sands property in northern Alberta:

![Figure 1: Full migrated image (left), and migrated with adaptive Green’s function (right).](image-url)
The adaptive Green’s function appears to align some of the acquisition linear noise effects, but more significantly appears to slightly improve the vertical resolution. Occasionally one sees a slightly stronger intermediate event. The difference, however, is not dramatic.

Next consider the amplitude spectra displayed for each trace along a line across the project area:

![Figure 2: Tracewise amplitude spectra for standard PSTM (left) and with adaptive Green’s functions (right).](image)

The upper limit of the frequency of the adaptive Green’s function data depends on the sweep frequency in this merged dataset. Older data were acquired to 180Hz, and newer data to 210Hz. I did not apply a post-migration spectral whitener or deconvolution, yet the output frequency band is appropriate for the full spectrum of the acquisition.

**Conclusions**

On these example data we achieved more coherent energy passing through the Kirchhoff migration, without post-migration deconvolution. We achieved coherent migration up to the frequency limits of our Vibroseis acquisition. The visual change in resolution is small. However, the question remains open as to how high a frequency could possibly be coherently resolved by adaptively modifying the Green’s function. This method is a result from a research program at Statoil, and does not represent current best practice in Statoil.

**Acknowledgements**

Thank you to the KOSP partners, Statoil and PTTEP, for allowing me to present this work, to Ion Geophysical for the pre-migration processing of the data, and to my lovely wife, Claudia, for encouraging me along the way.

**References**

