Crack-tip stress field, Coulomb failure, and the spectral characteristics of tensile rupture

David W. Eaton* and Chad M. Hogan, Department of Geoscience, University of Calgary
eatond@ucalgary.ca

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Summary

By combining recent empirical observations with classical theory from fracture mechanics and earthquake seismology, we develop a simple geodynamical framework for simulation of frac-induced microseismicity. Using analytical expressions for crack-tip stress, we apply the Coulomb failure criterion to obtain a scalar field with units of stress, known as the Coulomb stress field. This approach is motivated by many previous studies in earthquake seismology that have demonstrated a possible link between co-seismic Coulomb stress change and the distribution of aftershocks. Subject to excedance of a threshold value of time-dependent Coulomb stress, microseismicity is postulated to occur at random locations and with event magnitudes that satisfy the Gutenberg-Richter magnitude-recurrence relation. We also consider representative source spectra for both tensile and shear events, which are parameterized by P- and S-wave corner frequencies and DC displacement coefficients. This analysis shows that pure tensile events may be distinguished from shear slip events based on low S/P amplitude ratios. For both types of events, velocity spectra, i.e. those computed directly using the output of geophones and seismometers, exhibit a spectral peak near the corner frequency that scales with seismic efficiency, η (the ratio of radiated seismic energy to deformation energy). Finally, a crack that opens and closes very quickly (delay time τ ~ 50 ms) is characterized by periodic resonance and spectral notches that may be diagnostic of this phenomenon.

Introduction

A number of researchers have proposed that, following an earthquake, areas of increased Coulomb stress correlate with increased aftershock activity, whereas areas of decreased Coulomb stress correlate with regions of relative quiescence (e.g. King et al. 1994; Stein, 1999). In addition, microseismic sources can be characterized in various ways, including moment-tensor analysis and the determination of spectral properties such as corner frequency and related parameters. Moment-tensor inversion is based on the inversion of amplitude radiation patterns, ideally with both P and S waves, and requires a good coverage of the solid angle around the source zone to achieve a robust solution (Eaton and Forouhideh, 2011). Given the relatively narrow receiver aperture for many microseismic surveys used for hydraulic fracture monitoring (HFM), there is a potential advantage in the use of spectral analysis since it can, in principle, be applied based on recordings from a single sensor or cluster of sensors. Ultimately, the objective of microseismic source characterization is to discriminate between various modes of failure to provide insight into the effectiveness of hydraulic fracture stimulation.

Theory

Coulomb stress change is given by (e.g., Kilb et al., 2001):

$$\Delta CFS = \Delta\tau - \mu\left[\Delta\sigma_n - \Delta P\right] \quad , \qquad (1)$$

where $\Delta\tau$ is the change in shear stress on the failure plane (positive in the rake direction), $\Delta\sigma_n$ is the normal stress (positive in compression), ΔP is the pore-fluid pressure (positive in compression) and μ is the coefficient of friction.

In order to simulate microseismicity, we adopt a probabilistic approach. Our method assumes that failure only occurs when a critical value of the Coulomb Failure Stress is exceeded. Our time-dependent model uses 5-minute time steps and, for every node on a 5-m grid, there is a 20% probability of rupture occurring in a given time step if the Coulomb failure stress is greater than the threshold. Once rock failure has occurred at a grid node, it cannot fail again. In our model, we consider a fracture that grows slowly with time. The expansion of the tensile fracture takes place such that $c(t) \sim t^{1/2}$. This rate of crack growth implies a constant rate of volumetric expansion and means that the rate of increase of crack length slows down with time. The growth rate was selected to mimic fracture growth observed in previous studies, which can also be modelled as a diffusion process. When a simulated microseismic event occurs it is assigned a random magnitude that is subject to the condition that the complete set of modeled magnitude values satisfies the Gutenberg-Richter magnitude-recurrence relation with a *b* value of 1.5. A minimum magnitude of −3 was used in our simulations, but it should be emphasized that many of the modeled microseismic events may be too weak to be detected. In order to create a realistic simulation, we considered a magnitude-distance criterion for event detection based on real data. A representative time step from a simulation run is shown in plan view in Figure 1. The simulation produces a cloud of microseismic events that tracks the crack tip as it moves in the medium. The detected events, however, are significantly biased toward the receiver side.

Next, we consider representative source spectra for tensile events and compare these with expected source spectra for shear events, using formulas derived by Walter and Brune (1993). The far-field source displacement spectrum for both a tensile crack or a shear failure is assumed to have the idealized form

$$\left|\Omega^P(\omega)\right| = \frac{\Omega_0^P}{1+(\omega/\omega_c^P)^2} \quad , \quad \left|\Omega^S(\omega)\right| = \frac{\Omega_0^S}{1+(\omega/\omega_c^S)^2} \quad , \qquad (2)$$

where Ω^P and Ω^S are DC levels and ω_c denotes corner frequency. Although these idealized expressions are not derived from first principles, they fit well with many observed earthquake spectra and are considered to be the simplest forms of velocity spectra that have the correct asymptotic behaviour.

Examples

Figure 2a compares S/P spectral ratios for shear and tensile events, computed using the theory of Walter and Brune (1993) and based on two different scenarios. The parameters used in these calculations are summarized in Table 1. Case 1 corresponds to source parameters that are intended to represent a "high-frequency" case; the source diameter is relatively small (radius = 1.0 m), rupture is instantaneous and the seismic efficiency is relatively large (η = 0.1). Case 2 corresponds to source parameters that are intended to represent a "low-frequency" case; the source is relatively large (radius = 10 m), rupture is slow and the seismic efficiency is low ($\eta = 5\times10^{-5}$). This graph emphasizes that for either case 1 or case 2, the S/P spectral ratio is significantly greater for shear events than for P events. Amplitudes are averaged over the unit sphere and it is important to recognize that individual measurements are likely to be strongly influenced by amplitude radiation patterns. Nevertheless, this result points to a potentially diagnostic feature of tensile events (S/P spectral ratio) that is robust with respect to source diameter, seismic efficiency and rupture velocity.

Another factor that can affect the source spectra is the phenomenon of crack opening and closing in quick succession. Walter and Brune (1993) model this phenomenon as the superposition of positive

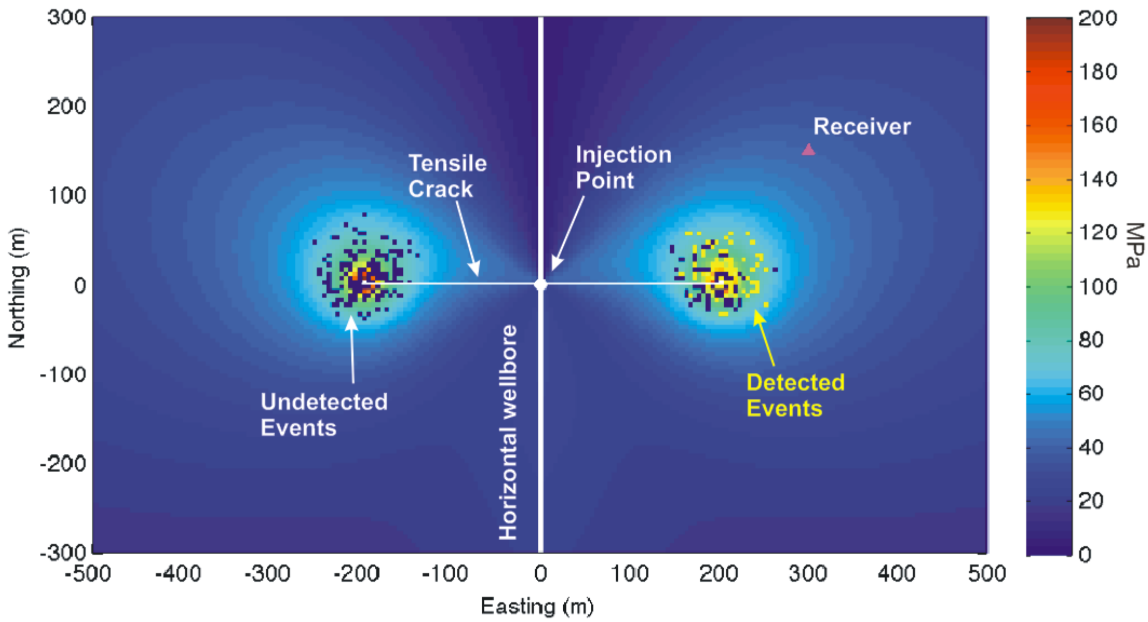

Figure 1: Time step from a simulation run. The white line shows a horizontal wellbore. As the crack propagates it produces a cloud of microseismic events. Detected events are shown in yellow.

negative spectra with a time offset, given by the parameter τ. This model leads to a filtering effect represented by a factor ξ that modifies the spectra as follows:

$$\Omega'(\omega) = \xi\,\Omega(\omega) = (2-2\cos\omega\tau)^{1/2}\Omega(\omega) \quad . \qquad (3)$$

Figure 2b shows the effects on the source spectrum for an opening-closing delay time of delay of τ ~ 50 ms. The filtering effect of opening/closing leads to amplification of the main spectral peak accompanied by a slight frequency shift. It also produces a characteristic pattern of spectral resonances and notches.

Conclusions

By combining recent empirical observations with classical theory from fracture mechanics and earthquake seismology, we have developed a simple geodynamical framework for simulation of frac-induced microseismicity. Using analytical expressions for crack-tip stress, we apply the Coulomb failure criterion to obtain a scalar field with units of stress, known as the Coulomb stress field. This approach is motivated by many previous studies in earthquake seismology that have demonstrated a possible causal link between Coulomb stress from co-seismic slip and the distribution of aftershocks. Subject to excedance of a threshold value of time-dependent Coulomb stress, microseismicity is postulated to occur at random locations and with event magnitudes that satisfy the Gutenberg-Richter magnitude-recurrence relation. In the second part of this paper, we consider Walter-Brune source spectra for both tensile and shear events, which are parameterized by P- and S-wave corner frequencies and DC displacement factors. As an alternative to moment-tensor analysis, pure tensile events may be distinguished from shear slip events based on low S/P amplitude ratios. Finally, a crack that opens and closes very quickly (delay time τ ~ 50 ms) is characterized by periodic resonance and spectral notches that may be diagnostic of this phenomenon.

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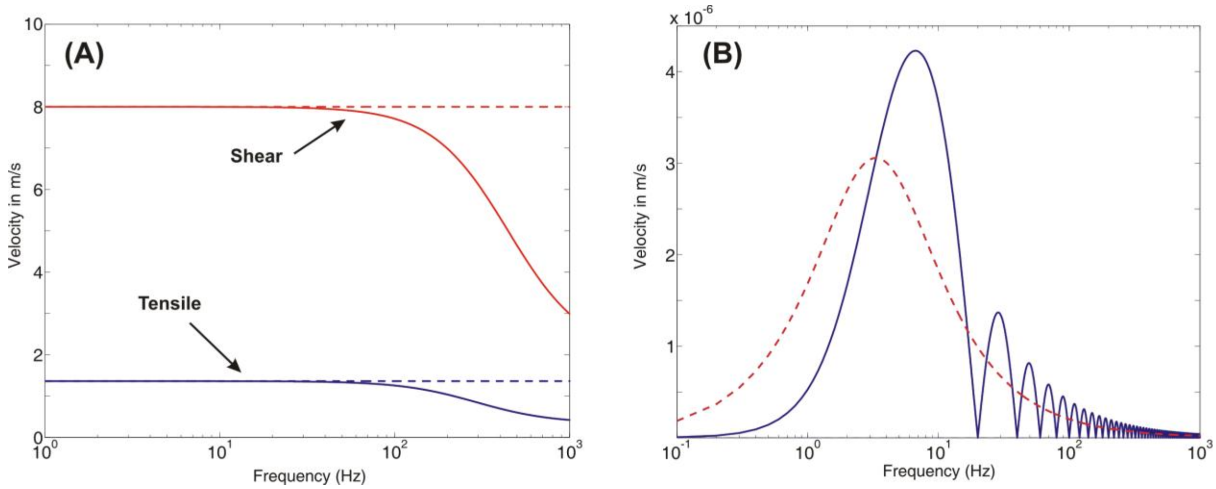

Figure 2: a) S/P spectral ratios for shear and tensile sources. b) P-wave spectra for tensile crack opening (dashed line) and open-and-closing with τ = 0.05 s.

Table 1. Parameters used to compute spectra in Figure 2; α and β denote P- and S-wave velocity, respectively, ρ is density, σ is treatment pressure, η is seismic efficiency, ζ is the ratio of P/S corner frequencies, R is the source-receiver distance and *a* is the crack radius.

	α (m/s)	β (m/s)	ρ (g/cm^3)	σ (MPa)	η	ζ	*R* (m)	*a* (m)
Case 1	4400	2200	2.5	50	0.1	2.0	400	1.0
Case 2	4400	2200	2.5	50	1×10^{-5}	1.0	400	10.0

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