Evaluation of a Fast Algorithm for the Eigen-Decomposition of Large Block Toeplitz Matrices with Application to 5D Seismic Data Interpolation

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Summary

We present a fast 5D (frequency and 4 spatial axes) reconstruction method that uses Multichannel Singular Spectrum Analysis / Cazdow algorithm. Rather than embedding the 4D spatial volume in a Hankel matrix, we propose to embed the data into a block Toeplitz form. Rank reduction is carried out via Lanczos bidiagonalization with fast block Toeplitz matrix-times-vector multiplications via 4D Fast Fourier Transforms. The presented algorithm leads to significant improvements in computational time with respect to the standard implementation via SVD that does not consider the special structure of block Toeplitz matrices.

Introduction

Algorithms for multidimensional seismic data reconstruction have proven to be an effective tool for seismic data preconditioning prior to seismic imaging, multiple suppression and detailed amplitude versus offset studies. In recent year, important advances were made in the development of interpolators based on Fourier synthesis operators. Multidimensional Fourier interpolators can be efficiently executed via optimization methods that heavily rely on Fast Fourier Transforms and/or a fast implementation of the non-uniform Discrete Fourier Transform (Sacchi and Liu, 2004; Abma and Kabir, 2006, Trad, 2009). New classes of interpolators that rely on rank reduction techniques have been proposed by Trickett and Burroughs (2009) and Oropeza and Sacchi (2011). Properly sampled multidimensional data can be embedded in a low rank block Toeplitz matrix. Missing data and noise increases the rank of the block Toeplitz matrix and therefore, reconstruction and denoising can be posed as a rank reduction problem. Rank reduction, however, requires the application of the Singular Value Decomposition (SVD) to estimate a few dominant eigenvectors of the Block Toeplitz form. This paper implements a fast method for rank reduction that uses the Lanczos bidiagonalization algorithm in conjunction with a fast matrix-times-vector multiplication implemented via the multidimensional Fast Fourier Transform (Strang, 1986). We compare rank reduction via SVD and the proposed algorithm. We show an important improvement in performance when rank reduction is carried out via the algorithm proposed in this paper.

Theory

The theory of seismic data reconstruction denoising and reconstruction using MSSA is presented in Sacchi (2007) and Oropeza and Sacchi (2010, 2011). In essence, data recovery is posed as a rank reduction problem. The computational burden of the method resides on the rank reduction stage of large block Hankel matrices that are form after embedding the spatial seismic volume. We simple replace Hankel matrices by Toeplitz matrices and use known properties of Block Toeplitz forms and circulant matrices to replace matrix-time-vector multiplication by FFT products. Notice that a similar strategy is utilized to speed up the high-resolution parabolic Radon Transform (Sacchi and Porsani, 2001).

Results

Figure 1 portrays the reconstruction error for a survey that consists of 8x8x8x8 traces and 512 samples per trace. In this simulation we have computed the reconstruction error versus the sparsity of the survey for the MSSA algorithm proposed in Oropeza and Sacchi (2011). The method was implemented using Matlab SVDS routine and the method proposed in this presentation (Lanczos with fast block Toeplitz matrix multiplications via 4D FFTs). In both cases only 5 eigenvectors were used for the low rank reconstruction. Figure 2 presents the computational time comparison for surveys of size NxNxNxN traces with 512 samples per trace. It is evident that the Lanczos algorithm outperforms the SVD. The important gain in efficiency is obtained because the matrix-time-vector multiplications required by the Lanczos bidiagonalization exploits the structure of the block Toeplitz form that arises after embedding the spatial volume. We complement this abstract with a real data example from the Western Canadian Sedimentary Basis. Figure 3 presents a small part of 5D seismic volume that consists of 12x12 CMP bins with 8x8 offsets per bin. The MSSA algorithm presented in Oropeza and Sacchi (2011) was extended to the 5D case and used to reconstruct the seismic volume.

Conclussion

MSSA/Cazdow denoising reconstruction can be efficientyly implemented by considering algorithms that exploit the block Toeplitz structure of the data embedding matrix. Our initial studies suggest that it is possible to develop rank reduction reconstruction methods that operate at FFT times and making their performance comparable to interpolation methods based on Fourier synthesis (Liu and Sacchi, 2005; Abma and Kabir, 2006; Trad, 2009)

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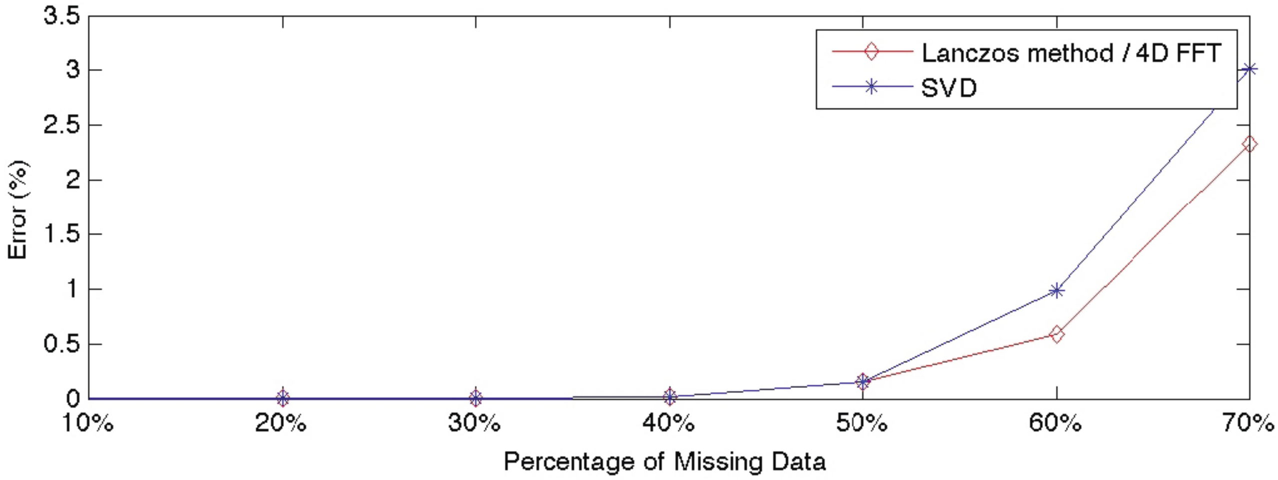

Figure 1. Multichannel Singular Spectrum Analysis (MSSA) reconstruction of a 5D synthetic data set. The survey consists of 8x8x8x8 traces, 512 samples, sampling rate is 2ms. The reconstruction was carried out in the band 0-80Hz. A rank k=5 was used to reconstruct the data. The vertical axis is the reconstruction error for the synthetic data set.

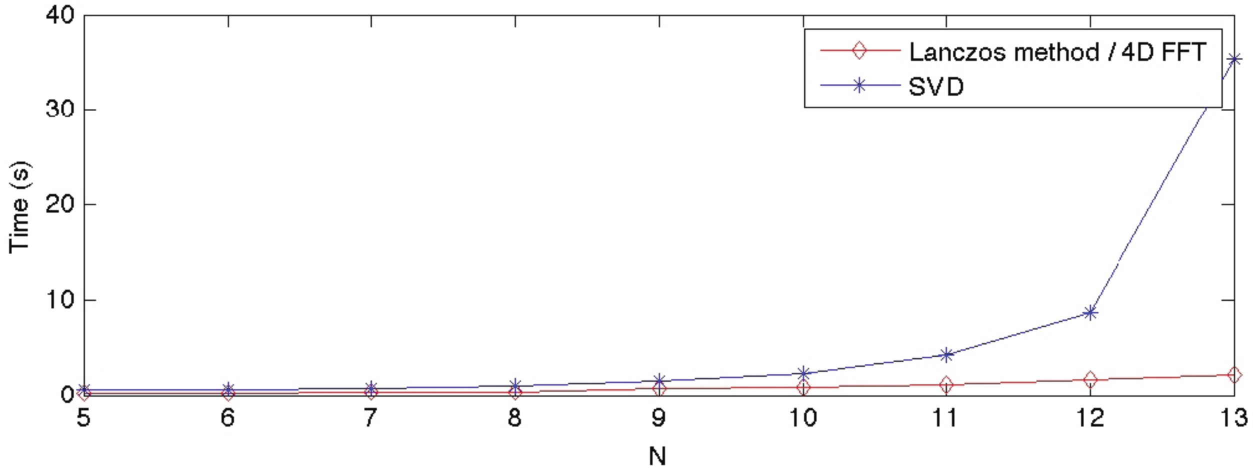

Figure 2. Evaluation of computational times of rank reduction. The survey consists of NxNxNxN traces, 512 samples, sampling rate is 2ms. The 4D cube was embedded in a Toeplitz matrix that was reduced to a rank k=5 matrix via the SVDS algorithm (Matlab) and the proposed Lanczos algorithms with fast matrix times vector multiplication via 4D FFTs.

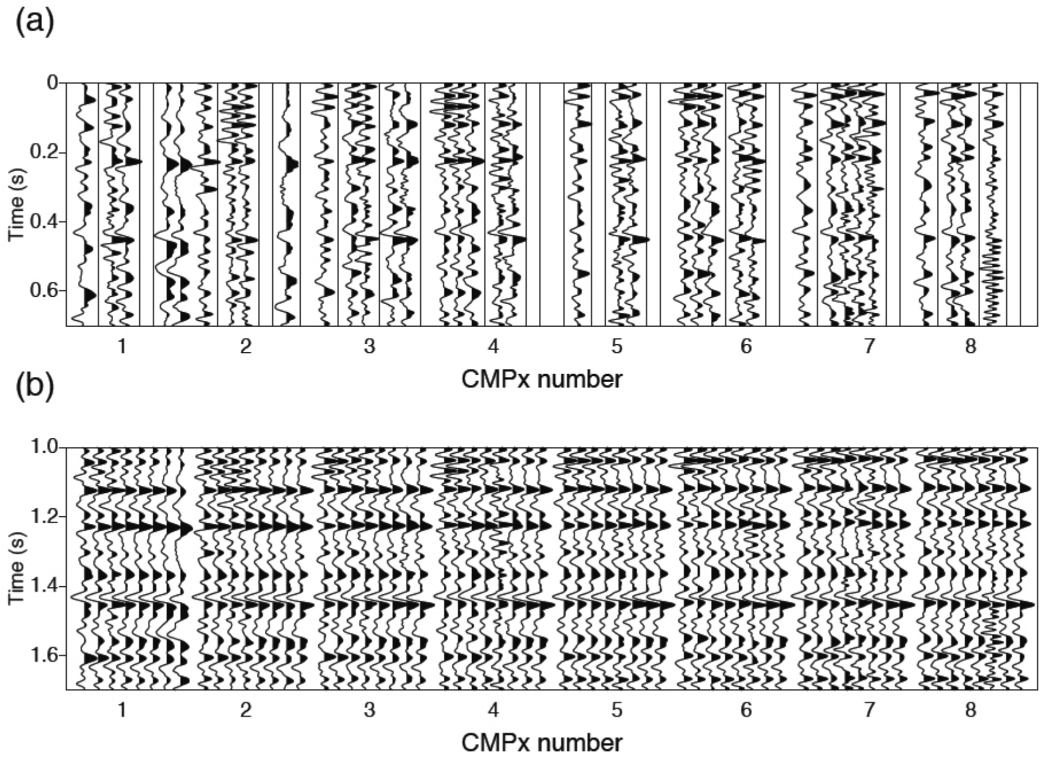

Figure 3. A segment of a 5D volume reconstructed via MSSA. The rank reduction required by the method was computed via the fast Lanczos biadiagonalization presented in this abstract. a) Original data. b) Reconstruction.

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